

CORRELATION METHOD OF MEASUREMENTS OF SOUND POWER IN THE NEAR FIELD CONDITIONS

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The results of sound power measurements under near field conditions have been compared with the results obtained for far field conditions. For near field measurements correlation and phase methods were used. Far field conditions were investigated by means of reverberation and free-field methods. The experiments were performed on a piston in an infinite baffle, the situation assumed in the calculations.

Quite good agreement of the results was obtained at low frequencies. For the higher frequencies this agreement appeared to be worse although this disagreement has been explained theoretically.

The usefulness of a correlation method for the measurement of sound power under near field conditions was confirmed. The method can be used in practice for the evaluation of the sound power generated by different parts of machinery.

1. Introduction

One of the main problems of machinery noise is the identification of its sound sources, i.e. the location of the main sound sources of the machine, the determination of their sound power, and finding their frequency response.

In a previous paper [1], the fundamentals of the application of the correlation method to the evaluation of the sound power of vibroacoustical sources were presented. The method was based on the measurement of the cross-correlation function between the mechanical velocity of a vibrating element and the sound pressure in the near-field of its radiation.

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The main errors which can occur in the near-field measurements are: an increase of sound pressure due to reflections between the microphone and the vibrating surface, the influence of the phase shift between velocity and acoustic pressure [2] which is dependent on the distance of the microphone from the vibrating elements and the phase shifts of the measuring channels. Moreover, the investigation of the sound power near the source is intended to restrict the influence of radiation from neighbouring sources on the acoustic pressure being measured. These studies are thus of essential for the quantitative estimation of the acoustic power radiated by particular elements of the machinery or device under test.

The aim of this paper is to compare the results of the estimation of acoustic power using the correlation method in the near-field to the results obtained in the far field using the free field [3, 4] and reverberation methods.

The experiments were performed for a vibrating circular piston placed in a large baffle. This permitted verification of the experimental results by analytical calculations. The calculations were based on theoretical relations [5, 8] and on measurements of the phase shift between the vibration velocity of the source surface and the sound pressure.

2. The acoustic power and characteristic impedance of a piston vibrating in an infinite baffle

The overall sound power of a circular piston vibrating in an infinite baffle can be represented, for a sinusoidal signal, by the expression

$$W_a = \frac{1}{T} \int_0^T \left[\operatorname{Re} \int_S v_{am} e^{j\omega t} p_{am} e^{j(\omega t + \varphi_{pvt})} dS \right] dt, \quad (1)$$

where T — the period of vibration, Re — the real part of the expression, v_{am} and p_{am} — the amplitudes of acoustic velocity and sound pressure near to the piston, respectively, $S = \pi a^2$ — the piston area, a — the radius of piston, φ_{pvt} — the phase shift between the sound pressure and the acoustic velocity. It is the argument of the acoustic impedance Z of the piston, which is given by

$$Z = \frac{\hat{p}_{am}}{\hat{v}_{am}} = |Z| e^{j\varphi_{pvt}} = R_p + jX_w, \quad (2)$$

where R_p — the radiation resistance, X_w — the imaginary part of the piston impedance. The phase shift angle φ_{pvt} can be determined from the relation

$$\varphi_{pvt} = \tan^{-1} \frac{X_w}{R_p}, \quad (3)$$

which requires calculation of the components of the impedance of the piston loading.

In general, it is difficult to calculate the impedance of a piston with a non-uniform distribution of vibration velocity on its surface [6,7]. Assuming one of kinds of the boundary conditions of piston in dependence on the method of its fixing, it is possible to determine the eigenmodes of the piston. For the frequencies of eigenmodes the maximum non-uniformity of velocity distribution will appear. The lowest frequency of an eigenmode should be treated as a limit below which it is possible to assume that the distribution of vibration velocity on the piston surface is approximately constant. These frequencies are

— for a supported piston

$$f_{01} = 0.21 \frac{b}{a^2} \sqrt{\frac{E}{\rho(1-\nu^2)}}, \quad (4)$$

— for a rigidly fixed piston

$$f_{01} = 0.47 \frac{b}{a^2} \sqrt{\frac{E}{\rho(1-\nu^2)}}, \quad (5)$$

where b — the thickness of the piston, a — its radius, E — Young's modulus, ρ — the density, ν — Poisson's ratio.

To eliminate the non-uniformity of the velocity distribution on the piston surface the area of piston can be replaced by an equivalent area S_{eq} with a constant distribution.

For the fundamental eigenmode frequency we have, for a

— supported piston [8]

$$S_{eq} = 0.43 S, \quad (6)$$

— and for a rigidly fixed piston

$$S_{eq} = 0.29 S. \quad (7)$$

This means that the level of the sound power radiated by the piston is 3.7 dB lower than the level of the power calculated for a constant velocity distribution over a supported plate, and 5.4 dB lower than for a rigidly fixed plate. It follows that the agreement between the calculated and measured results is attained only below the lowest eigenmode frequency f_{01} .

Neglecting the non-uniformity of the velocity distribution at frequencies near f_{01} and above it, gives too high results for the calculated sound power.

Let us assume that the acoustic velocity v_a at a small distance from the piston surface is equal to the velocity v_0 of the vibrating piston which has a constant velocity distribution.

Then, after integrating expression (1), we obtain the following expression for the real part of sound power radiated by the piston:

$$W_a = \frac{1}{2} p_{am} v_m S \cos \varphi_{pvt}. \quad (8)$$

Using (2) and (6), expression (8) can be transformed to a form convenient for calculations:

$$W_a = \frac{1}{2} v_m^2 R_p S. \quad (9)$$

The radiation resistance R_p and phase shift φ_{pvt} can be determined from the well-known formula for the impedance of a piston in an infinite baffle,

$$Z = \varrho_0 c \left[1 - \frac{J_1(2ka)}{ka} \right] + j \varrho_0 c \left[\frac{K_1(2ka)}{2(ka)^2} \right] = R_p + jX_w, \quad (10)$$

where $k = \omega/c$ — wave number, a — radius of piston, ϱ_0 — air density, c — sound speed, $J_1(2ka)$ — Bessel function of the 1st kind and 1st order, $K_1(2ka)$ — Rayleigh function.

If $ka < 1$, then

$$R_p = \varrho_0 c \frac{k^2 a^2}{2}, \quad (11)$$

$$X_w = \varrho_0 c \frac{8ka}{3\pi}, \quad (12)$$

$$\varphi_{pvt} = \tan^{-1} \frac{16}{3\pi ka}. \quad (13)$$

If $ka > 1$

$$R_p = \varrho_0 c, \quad (14)$$

$$X_w = \varrho_0 c \frac{2}{\pi ka}, \quad (15)$$

$$\varphi_{pvt} = \tan^{-1} \frac{2}{\pi ka}. \quad (16)$$

The level of the real part of the sound power in relation to the reference power $W_0 = 10^{-12} \text{ W/m}^2$ is

$$L_t = 10 \log \frac{W_a}{W_0} = 10 \log \frac{\varrho_0 c}{4} \pi k^2 a^4 v_m^2 \times 10^{12} \text{ [dB]} \quad \text{for } ka < 1, \quad (17)$$

and

$$L_t = 10 \log \frac{W_a}{W_0} = 10 \log \frac{\varrho_0 c}{2} \pi a^2 v_m^2 \times 10^{12} \text{ [dB]} \quad \text{for } ka > 1. \quad (18)$$

For a constant velocity distribution on the piston surface, the distribution of acoustic pressure near the piston should be constant. At longer distances from the vibrating surface the directional piston characteristics will differ more and more from a hemispherical one as the value of ka increases. This effect

occurs for values of $ka > 1$ and causes a decrease in the mean value of the sound pressure level in the far field. This means that for $ka > 1$ the values of power measured under far field conditions will be smaller than the values for near field conditions.

3. Measuring conditions and testing method

The schematic diagram of the testing arrangement is shown in Fig. 1. It includes: the driving system and the emitting source in the form of a circular piston in a baffle. The piston 1 with a smooth and flat surface is fixed to the

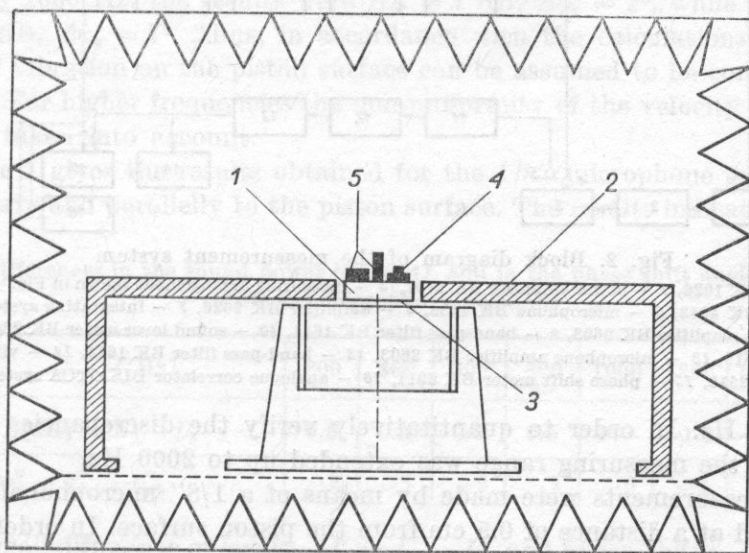


Fig. 1. Schematic diagram of the experimental arrangement for measurements of the acoustic power of a piston in a baffle

1 - piston, 2 - baffle, 3 - driver BK 4801, 4 - acceleration pick-up, 5 - microphone

head of an electrodynamic driver 3 (BK 4801). The driver base is attached to the support structure located in an anechoic chamber. The piston plate forced by the driver head is capable of vibrating in the circular hole of the baffle.

A steel piston with a diameter of $2a = 14$ mm was used. It was driven with a constant amplitude of vibration velocity $v_m = 5$ cm/s within the range of measuring frequencies. The piston thickness was $b = 0.5$ cm.

For steel $E = 2 \times 10^{11}$ N/m², $\rho = 0.78$ kg/m³, $\nu = 0.27$. In the system under consideration the boundary conditions are not distinctly determined. However, it will be assumed that the best approximation to the boundary conditions are those corresponding to the supported plate. With this assumption and with data of the piston given above, from (4) we obtain the fundamental

eigenmode frequency $f_{01} = 1100$ Hz. The condition $ka = 1$ for the piston under consideration corresponds to a frequency of 775 Hz.

The piston was placed in a square baffle of side length $l = 2$ m and thickness 0.5 cm. The baffle was made from plywood and was clamped at the edges, by means of steel rods, to a structure which was not rigidly connected to the base of the drivers.

The lowest measuring frequency was established to be 200 Hz in order to fulfil the condition of the baffle dimensions $l > \lambda$.

With the above parameters of the system the calculated and measured results for both near and far field should agree within the frequency range from

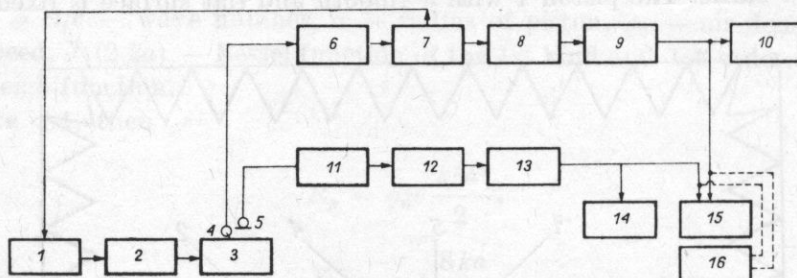


Fig. 2. Block diagram of the measurement system

1 - generator BK 1026, 2 - power amplifier BK 2707, 3 - measuring arrangement shown in Fig. 1, 4 - acceleration pick-up BK 4343, 5 - microphone BK 4138, 6 - amplifier BK 2626, 7 - integrating system BK 2625, 8 - microphone amplifier BK 2603, 9 - band-pass filter BK 1614, 10 - sound level meter BK 2203, 11 - pre-amplifier BK 2619, 12 - microphone amplifier BK 2603, 13 - band-pass filter BK 1614, 14 - vibration meter BK 2510, 15 - phase shift meter BK 2911, 16 - analogue correlator DISA TCA system

200 to 775 Hz. In order to quantitatively verify the discrepancies at higher frequencies the measuring range was extended up to 2000 Hz.

The measurements were made by means of a 1/8" microphone, type BK 4138, placed at a distance of 0.5 cm from the piston surface. In order to check the discrepancies in the results, produced by reflections between the microphone and the piston surface, measurements were performed for two positions of the microphone (Fig. 1), namely: perpendicular (a) and parallel (b) to the piston surface. The piston velocity was measured by means of an accelerometer pick-up, type BK 4443, in conjunction with an integrating unit, type BK 2625.

Signals were transmitted from the microphone and pick-up to r.m.s. meters through two measuring channels (Fig. 2). The power level was determined in dB referred to a power of $W_0 = 10^{-12}$ W:

$$L_{pv} = 10 \log \frac{p_{acf} v_{ef} \pi a^2 \cos \varphi_{pvt}}{W_0} \text{ [dB]}. \quad (19)$$

The sound power of the piston was determined by employing two methods:

- (a) a phase method using a phase meter BK 2971,
- (b) a correlation method using a DISA correlator, type 55D70, in accordance with the block diagram shown in Fig. 2.

During preliminary testing, the effect of the following factors that would cause possible disagreement between the measured and calculated results was considered:

- a non-uniform distribution of vibration velocity over the piston surface,
- the position of the microphone,
- the phase shifts in different measuring channels.

The non-uniformity of the distribution of vibration velocity over the piston surface was examined by changing the position of the vibration pick-up on the piston surface. The maximum deviation ΔL of the velocity level and the change of the phase shift $\Delta\varphi_v$, with respect to the values obtained with the pick-up positioned at the centre of the plate, were measured. At a frequency of 1000 Hz, the results were $\Delta L = 1$ dB, $\Delta\varphi_v = 2^\circ$, while at 2000 Hz, $\Delta L = 3$ dB, $\Delta\varphi_v = 7^\circ$. Thus, in accordance with the calculations, the distribution of vibration on the piston surface can be assumed to be constant below 1100 Hz. For higher frequencies the non-uniformity of the velocity distribution must be taken into account.

Table 1 gives the results obtained for the 1/8'' microphone arranged perpendicularly and parallelly to the piston surface. The results indicate that there

Table 1. Differences in the sound power levels ΔL and in the phase shift angles $\Delta\varphi_p$ at two different microphone positions

f [Hz]	200	400	600	800	1000	1200	1600
ΔL [dB] = $L_\perp - L_\parallel$	0.5	0.5	0.5	0.5	0.0	-1.0	-1.5
$\Delta\varphi_p^\circ = \varphi_{p\perp}^\circ - \varphi_{p\parallel}^\circ$	-2	-1	-2	-3	-6	-4	-6

exists a small difference of levels, $\Delta L = L_\perp - L_\parallel$, and phase shifts, $\Delta\varphi_p = \varphi_{p\perp} - \varphi_{p\parallel}$, for frequencies below 1000 Hz with a tendency for them to grow at higher frequencies.

The measured values of phase shift must be corrected due to the additional phase shifts (dependent on frequency) introduced by the measurement channels.

Denoting by φ_0 the phase shift value indicated by the phase meter, we can write the real value of phase shift between the sound pressure and the piston velocity as

$$\varphi_{pv} = \varphi_0 + \Delta\varphi, \quad (20)$$

where $\Delta\varphi$ is the error in the phase shift introduced by the experimental arrangement.

In the measuring arrangement used in these experiments the errors $\Delta\varphi$ consist of the following components: φ_p - from the amplifiers of the sound pressure measuring channel, φ_v - from the amplifiers of the velocity measuring

channel, φ_m — from the microphone, and φ_{pr} — from the delay of acoustic pressure as a result of the distance r between the microphone and the piston surface

$$\varphi_{pr} = \frac{\omega r}{c} \times 360^\circ. \quad (21)$$

Taking into account the signs of the phase shifts, we obtain

$$\Delta\varphi = \varphi_p - \varphi_v - \varphi_{pm} + \varphi_{pr}. \quad (22)$$

The calculated and measured values of phase shifts for the measuring frequencies are presented in Table 2.

Table 2. Results of measurements and calculations of phase shifts

f [Hz]	200	400	600	800	1000	1200	1600	2000
$\varphi_p^\circ - \varphi_v^\circ$	12	0	20	6	1	3	6	2
φ_{pm}°	0	0	0	0.5	1.5	2	3	5
φ_{pr}°	1.2	2.1	3.1	4.2	5.2	6.2	8.3	10.4
$\Delta\varphi^\circ$	10.8	-2.1	16.9	2.3	-2.7	-1.2	0.7	-3.4
φ_0°	70	76	46	57	55	36	32	22
φ_{pv}°	80.8	73.9	62.9	59.3	52.3	34.8	32.7	18.6
φ_τ°	80	69	62	66	59	40	33	19
φ_{prt}° (calculated)	81.5	74	65	59.5				
				57.5	48	45	36.5	31

4. The results of measurements and calculations of acoustic power under near field conditions

(a) *The phase method.* The sound power level of the piston was determined on the basis of the following measurements: the r.m.s. value of acoustic pressure p_{acf} , the r.m.s. value of velocity of the piston v_{ef} and phase shift φ_{pv} .

The results of the measurement of phase shifts as a function of frequency are presented in Fig. 3. The curve φ_0 represents the measured values without correction, φ_{pv} — with the correction according to (20), (21) and (22), φ_{prt} — the values calculated from (13) for frequencies below 775 Hz and from (16) — above this value.

The measurements of the sound power level obtained by the phase method on the basis of (19) are presented in curve L_{pv} of Fig. 4. The curve L_t in Fig. 4 shows the values calculated from (17) for frequencies below 775 Hz, and from (18) above this value.

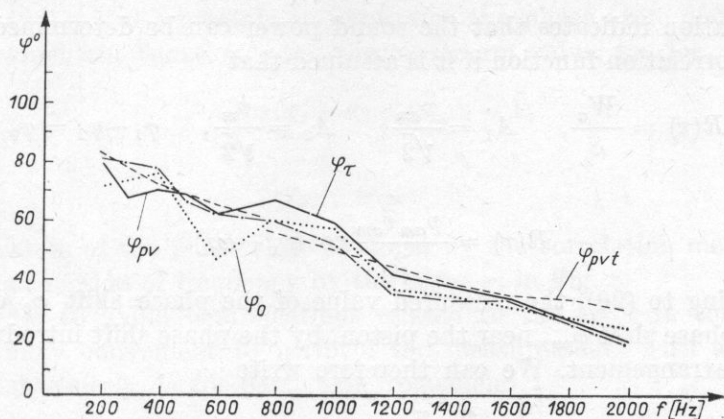


Fig. 3. Results of measurements and calculations of phase shifts φ as a function of frequency using: φ_{pv} — phase method with phase shift correction, φ_0 — the phase method without corrections, φ_τ — the correlation method, φ_{pvt} — theoretical calculations

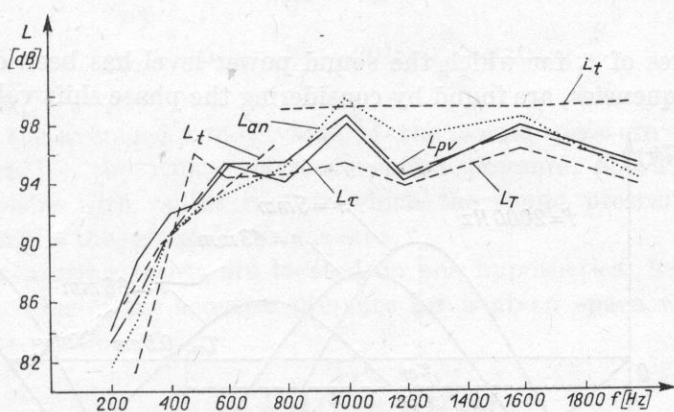


Fig. 4. Results of measurements and calculations of the sound power level of the piston as a function of frequency using: L_{pv} — the phase method, L_τ — the correlation method, L_{an} — the free field method ISO, L_T — the reverberation method, L_t — theoretical calculations on the basis of approximate formulae

(b) *The correlation method.* The cross-correlation function of two sinusoidal processes with amplitudes A_1 and A_2 and initial phases φ_1 and φ_2 takes the form

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T A_1 \sin(\omega t + \varphi_1) A_2 \sin(\omega t + \varphi_2 + \omega \tau) dt,$$

which, after being integrated, is simplified to the form

$$R(\tau) = A_1 A_2 \cos(\varphi_1 - \varphi_2 - \omega \tau). \quad (24)$$

This relation indicates that the sound power can be determined by means of a cross-correlation function if it is assumed that

$$R(\tau) = \frac{W_a}{S}, \quad A_1 = \frac{p_{am}}{\sqrt{2}}, \quad A_2 = \frac{v_m}{\sqrt{2}}, \quad \varphi_1 - \varphi_2 = \varphi_0, \quad (25)$$

and then

$$R(\tau) = \frac{p_{am} v_{am}}{2} \cos(\varphi_0 - \omega\tau). \quad (26)$$

According to (20), the measured value of the phase shift φ_0 differs from the actual phase shift φ_{pv} , near the piston, by the phase shift introduced by the measuring arrangement. We can therefore write

$$R(\tau) = \frac{p_{am} v_{am}}{2} \cos(\varphi_{pv} + \Delta\varphi - \omega\tau). \quad (27)$$

Hence, the radiated power described by equation (8) is determined if the condition

$$\omega\tau = \Delta\varphi \quad (28)$$

is satisfied.

The values of τ , for which the sound power level has been determined at particular frequencies, are found by considering the phase shift values presented

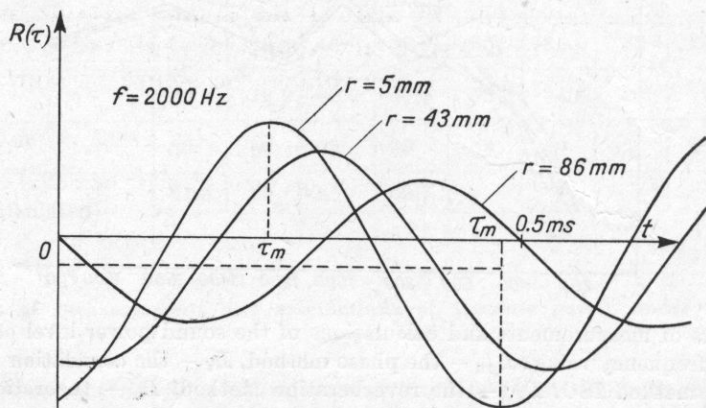


Fig. 5. An example of the cross-correlation curves used for the calculations of phase shift φ_τ and sound power level L_τ for different distances r between the microphone and the piston

in Table 2. Figure 5 represents a specimen of the correlation curves at 200 Hz for different distances r between the piston and the microphone. The sound power level obtained by the correlation method was calculated from the expression

$$L_\tau = 10 \log \frac{R(\tau)S}{W_0}, \quad (29)$$

and presented as the curve L_τ in Fig. 4.

From (27) it is also possible to determine the phase shift φ_τ for the value of τ_m for which the function takes the maximum value, i.e. for

$$\cos(\varphi_\tau + \Delta\varphi - \omega\tau_m) = 1, \quad (30)$$

hence

$$\varphi_\tau = \omega\tau_m - \Delta\varphi. \quad (31)$$

The values of the phase shift obtained by the correlation method are presented as a function of frequency by the curve φ_τ in Fig. 3.

In order to avoid complications related to the error in the phase shift $\Delta\varphi$, it is more convenient to perform the measurements with a phase shifter which cancels $\Delta\varphi$ for particular frequencies.

5. The results of measuring acoustic power under far field conditions

(a) *Free field conditions method.* According to the ISO recommendations, the acoustic power level of a source is calculated from

$$L_{\text{ISO}} = 10 \log \frac{W_a}{W_0} = 20 \log \frac{p_a}{p_{a0}} + 10 \log \frac{S}{S_0}, \quad (32)$$

where p_a is the averaged r.m.s. value of the sound pressure [N/m^2], $p_{a0} = 2 \times 10^{-5} [N/m^2]$ — the r.m.s. reference sound pressure, $S = 2\pi r^2$ — the area of a hemisphere with radius r , over which the sound pressure is measured, and $S_0 = 1 \text{ m}^2$ — the reference surface area.

If all measuring points are located on one hypothetical hemisphere, then the average value of the acoustic pressure for a given space would be determined by the relation

$$\frac{p_a}{p_{a0}} = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{p_{ai}}{p_0} \right)^2}, \quad (33)$$

where p_{ai} is the value of the sound pressure at the i -th measuring point, i — the number of a measuring point in an anechoic room (there were six positions of the microphone at a distance of $r = 1 \text{ m}$ from the piston), and n — the number of measuring points.

The measurements of the sound power level of the source, after averaging the sound pressures in free field conditions, are presented by curve L_{an} in Fig. 4.

(b) *The reverberation method.* The level of the sound power of the source in the reverberation field is described by

$$L_T = 10 \log \frac{W_a}{W_0} = 20 \log \frac{p_a}{p_{a0}} - 10 \log T + 10 \log V - 14 \text{ [dB]}, \quad (34)$$

where p_a is the averaged r.m.s. value of the sound pressure for a microphone placed in the far field, T — the reverberation time [s], V — the volume of the room [m^3].

The measurements were performed in a reverberation chamber with a volume of $V = 196 m^3$. The results of measurement of the reverberation time after installation of the equipment (from Fig. 1) are given in Table 3.

The measurement results of the sound power level of the piston obtained by the reverberation method are illustrated by curve L_T in Fig. 4.

Table 3. Characteristics of the reverberation time of the reverberation chamber as a function of frequency after the equipment has been placed in it

f [Hz]	200	400	600	800	1000	1200	1600	2000
T [s]	6.4	6.7	5.8	5.0	5.2	4.5	3.2	3.3

6. Discussion of the results

The comparison of the calculated and measured results of the sound power level of the piston obtained for near field conditions and for far field conditions shows that there is a very good agreement at low frequencies. However, for higher frequencies, this agreement become worse.

The disagreement for higher frequencies is due to two factors: non-uniformity of the distribution of the velocity on the piston surface, and non-uniformity of the directional characteristics of the sound pressure radiated by the piston.

The non-uniformity of the velocity distribution depends on the mechanical properties of the piston expressed by its eigenmode frequencies. It follows from theoretical considerations that the experimental results would be expected to be 3 to 5 dB smaller than those calculated for the fundamental eigenmode frequency.

The non-uniformity of the distribution of the sound pressure depends on the non-uniformity of the velocity distribution on the piston surface and on the value of ka . For $ka > 1$ the piston will exhibit directional properties, which will effect the non-uniformity of the distribution of the acoustic field in the far field and, therefore, thus cause an increase of measurement errors.

The experimental results confirm the above conclusions on the disagreement of the results at higher frequencies. Near the first eigenmode frequency of the piston, the values measured by different methods show the sound power level to be about 4 to 5 dB lower than the calculated values.

Moreover, for $ka > 1$ the dispersion of the results increased and, therefore, it was necessary to use a greater number of measuring points.

7. Conclusions

1. The near field method of measurement of the sound source power level is suitable for measuring the vibrating elements for the low frequency range.

The accuracy of the method becomes worse for the frequency range over the eigenmode frequencies of the elements under test, when the element has dimensions large compared to the wavelength.

2. At higher frequencies, it is necessary to average the measurements of the vibration velocity and sound pressure at several measuring points or to calculate an equivalent vibrating area.

3. The correlation method is a very simple way of measuring the level of the power of the sound source. However, it requires to take into account the phase shifts of the measuring arrangement.

4. It could be expected that the present work should enable the correlation method to be applied for measuring the sound power level generated by particular elements of machinery.

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