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ULTRASONIC PULSE DOPPLER METHOD IN BLOOD FLOW MEASUREMENT

ANDRZEJ NOWICKI

Institute of Fundamental Technological Research, Polish Academy of Sciences (Warszawa)

The phenomenon of the scattering of ultrasonic waves by the morphotic elements of blood is discussed. An analysis is presented of a correlation system for receiving Doppler signals, with consideration being given to the problem of the simultaneous measurement of the position and velocity of the blood corpuscles. The principles of operation of the device UDIMP, developed by IPPT PAN, is described and its basic technical parameters are given.

1. Introduction

Among the present diagnostic methods for the evaluation of the blood flow velocity in blood vessels, the ultrasonic Doppler method of blood flow [4] appears to be particularly interesting in promising further rapid development.

The Doppler phenomenon consists in the fact that a signal, reflected (or scattered) from a moving object, changes its frequency f_0 relative to the frequency of the transmitted signal f_n by a frequency f_d (termed subsequently the «Doppler frequency») which is proportional to the linear velocity of the object, according ot the expression

$$f_d = \frac{f_n v}{c}(\cos\theta_N + \cos\theta_0) \quad \text{for } \frac{v}{c} \ll 1, \tag{1}$$

where v denotes the velocity of the moving object (a blood corpuscle), c is the velocity of ultrasound in blood, θ_n – the angle between the direction of the blood velocity and the normal to the transmitting transducer, and θ_0 – the angle between the direction of the blood velocity and the normal to the receiving transducer.

In normal physiological conditions, the distribution of blood velocity across the vessel is a complicated function of the vessel radius, the flow pulsation and of the blood viscosity [6]. Thus each particle in the flowing blood produces a signal with a different frequency, so that the component signal, received by the receiving transducer, will contain a spectrum of Doppler frequencies over a range from $f_d = 0$ — for the particles at the walls of the vessel — to $f_d = f_{d\max}$ for the particles moving in the middle of the vessel.

The determination of the mean Doppler frequency $f_{d\acute{s}r}$ and, subsequently, the determination of the mean blood flow velocity, corresponding to the above frequency throughout the whole cross-section of the blood-vessel, is troublesome and sometimes quite imposible, especially for turbulent flow. Additional difficulties appear when we measure the velocity through the skin, beacuse the angles θ_n and θ_0 (Fig. 1) are unknown.

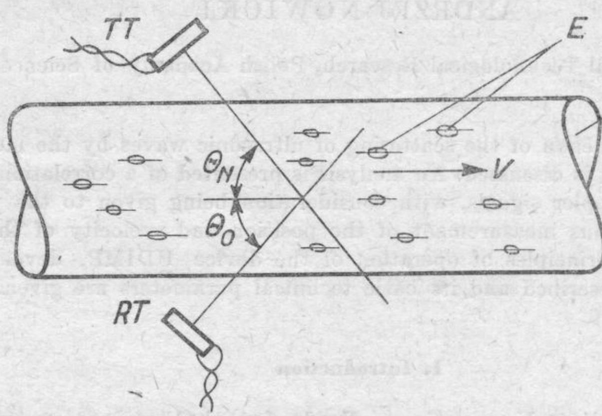

Fig. 1. The principle of operation of the ultrasonic blood flowmeter
TT – transmitting transducer, *RT* – receiving transducer, *E* – morphotic elements of blood, *WV* – blood vessel

For clinic diagnosis, the mean volume blood flow velocity Q, described by the relation

$$Q = v_{\acute{s}r} A, \tag{2}$$

is more important than the mean instantaneous blood flow velocity, $v_{\acute{s}r}$ being the instantaneous mean blood flow velocity, A — the cross-sectional area of the vessel, and Q — the instantaneous volume blood flow velocity.

The continuous wave Doppler method does not permit measurement of the vessel diameter and thus cannot be used for determination of the volume velocity of the blood flow.

The transition from qualitative to quantitative investigations has become possible due to a method which combines an ultrasonic Doppler technique with a pulse-echo technique. It permits, for example, experimental determination of the profiles of the blood flow velocity in different phases of the working cycle of the heart, the mean velocity, and also the volume blood flow velocity [11, 12] (Fig. 2).

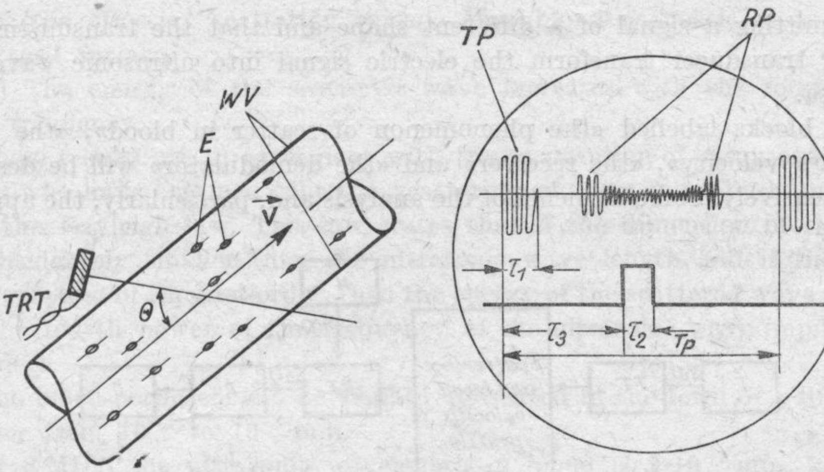

Fig. 2. The principle of operation of the pulsed ultrasonic Doppler flowmeter for measuring the blood flow velocity profile

TRT – transmitting and receiving transducer, *WV* – the walls of the vessel, *E* – morphotic elements of blood, *TP* – transmitted pulse, *RP* – pulses reflected from the walls of the vessel

The piezoelectric transducer emits, in the direction of the blood vessel, high-frequency pulses E of duration τ_1, frequency f_n and repetition frequency rate $F_p (F_p = 1/T_p)$. The ultrasonic wave is partially reflected and scattered by both the walls of the vessel and the blood corpuscles. As a result, during the time T_p between two subsequent transmitted pulses echoes return to the transducers corresponding to reflection from the walls of the blood-vessel and to the scattering from the blood corpuscles.

Knowing the angle θ between the direction of the incident ultrasonic wave and the blood-vessel, and also the propagation velocity of the ultrasound in tissue and blood, we can determine the transverse dimension of the vessel [2, 9]. Simultaneously, due to the use in the receiver of an electronic gate of adjustable duration τ_2 and time delay τ_3 relative to the transmitting pulse, it is possible to select the echoes that correspond to the scattering from the moving blood corpuscles at a selected depth.

2. The general model of the system for measuring blood flow velocity

Fig. 3 shows a general model of the system for measuring blood flow, consideration having been given to the macroscopic physical properties of blood and the nature of its flow in the circulating system.

The blocks labelled «the transmitter», «the transmitting transducer» and «the receiving transducer» do not require detailed discussion. Consideration will only be given in this paper to the problems related with the emission of a sequence of coherent pulses of high frequency. Nevertheless, it can be assumed that, without particular technical problems, it is possible to develop a transmitter emitting a signal of a different shape and that the transmitting and receiving transducer transform the electric signal into ultrasonic waves and vice versa.

The blocks labelled «the phenomenon of scatter in blood», «the profile of the flow velocity», «the receiver» and «the demodulator» will be described more extensively. Some elements of the analysis and, particularly, the approach

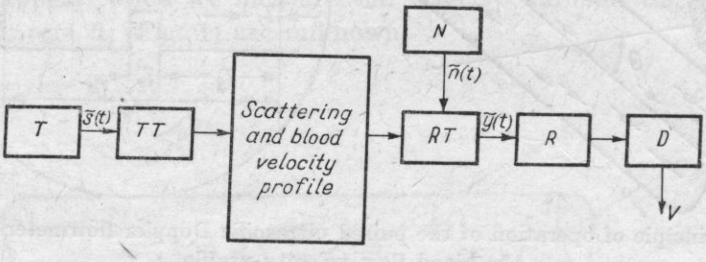

Fig. 3. A model of the system for measuring the blood flow velocity
T – transmitter, TT – transmitting transducer, N – noise, RT – receiving transducer, R – receiver, D – demodulator, v – the signal proportional to the flow rate

to the subject were taken from the theory of the radar techniques [16]. However, the differences encountered in blood flow measurement, compared with measurements using radar technique for determining the velocity of flying objects, make it impossible to consider ultrasonic Doppler flowmeters as analogous to Doppler radar.

From the physical point of view blood is a suspension of cells in a liquid plasma. These cells are red blood corpuscles (erythrocytes), white corpuscles (leucocytes), and blood lamellae (thrombocytes). Their mean number per cubic milimetre of blood is [1] about $5 \cdot 10^6, 10^4, 3 \cdot 10^5$, respectively.

The experimental investigations carried out by Reid [13] concerning the scattering uf ultrasonic wave by human blood over a frequency range from 4 MHz to 16 MHz gave the following results:

(a) The main source of scattering is the erythrocytes. At a frequency of 5 MHz the energy of the ultrasonic wave scattered by the thrombocytes was about 1000 times less than the energy scattered by the erythrocytes. It is practically undetectable for normal physiological structures.

(b) The quantity (acoustic pressure) of the scattered wave is proportional to the density (hematocryt) over the range from 7 % to 40 %, i.e. in the physiological range of blood density.

The hematocryt is defined by the expression:

$$\text{Hematocryt} = \frac{\text{volume of erythrocytes} \times 100\,\%}{\text{total volume of erythrocytes and plasma}}.$$

(c) The scattering is isotropic.

(d) The effective scattering surface of a blood coıpuscle is 10^{-4} of its geometrical surface.

(e) The energy of the scattered wave increases with the fourth power of the frequency.

These results are in agreement with the assumption of many authors (e.g. [3, 17]) who have assumed that the scattering of ultrasound by human blood obeys the Rayleigh law. This law states that if the dimension of the object are considerably smaller than the ultrasonic wave length and if the scatter is isotropic and of the first order, then the energy of the scattered wave increases with the fourth power of the frequency of the ultrasonic wave impinging on the object.

The blood corpuscle can be visually presented in the form of a disc of the diameter from 10^{-3} to 10^{-2} mm.

At 8 MHz, the ultrasonic wavelength in blood is $2 \cdot 10^{-1}$ mm, hence the ratio of the ultrasonic wavelength to the geometrical size of the blood corpuscle varies from about 20 to 200 and this permits, according to the Raleigh theory, its consideration as a point scatterer.

It was then assumed that the scattering is a random process. This means that at any moment the number of particles that scatter the ultrasonic wave will be subject to a particular function of probability density.

The assumptions of the first order and random scattering of the blood particles permit the supposition that the scattering process is subject to the Poisson distribution. With such an assumption, the probability that N of the blood corpuscles is contained in the volume V, defined by the ultrasonic field, is given by the formula

$$P(N/V) = P(N \text{ blood corpuscles in a volume } V) = \frac{(\varrho V)^N e^{-\varrho v}}{N!}, \quad (3)$$

where ϱ denotes the mean number of blood corpuscles per cubic millimetre.

The expected value of the number of blood corpuscles in the volume V is equal to the first moment of the distribution $P(N/V)$, and amounts to ϱV.

The description of the first orderly mathematical model of blood flow was presented by WOMERSLEY [6] who introduced expressions describing the instantaneous velocity $v_{\acute{s}r}$ and the volume velocity Q for a pressure gradient varying sinusoidally in time along the axis of an elastic tube (assuming that only radial motion of the tube walls occurs). WOMERSLEY also proved that the flow velocities, described in this manner, are identical with the corresponding flow velocities in a straight tube with rigid walls,

$$v(y) = \frac{M}{j\omega\varrho_0}\left[1 - \frac{J_0(aj^{3/2}y)}{J_0(aj^{3/2})}\right] e^{j\omega t}, \quad (4)$$

$$v_{\acute{s}r} = \frac{M}{j\omega\varrho_0}\left[1 - \frac{2J_1(aj^{3/2})}{j^{3/2}aJ_0(aj^{3/2})}\right]e^{j\omega t}, \tag{5}$$

$$Q = V_{\acute{s}r}A, \tag{6}$$

where $j = \sqrt{-1}$, $y = r/R$ (R is the radius of the tube, r – the radial coordinate in the cylindrical coordinate system (r, θ, z) with its origin at the centre of the tube), $a = R\sqrt{\omega/\nu}$ (ν is the kinematic viscosity, ω – the pulsation of the pressure wave, ϱ_0 – the specific mass of liquid, J_0 and J_1 are the Bessel functions of the zero and first order, M is the pressure gradient, and A – the tube cross-sectional area.

Expressions (4), (5) and (6) were determined on the basis of the following assumptions:

1. we have an incompressible Newtonian liquid;
2. the system is axially symmetric;
3. the flow is laminar;
4. the thickness of the tube walls is thin relative to the radius of the tube, i.e. $h/R \leqslant 0{,}1$;
5. the transverse and longitudinal deformations of the tube wall during flow are negligible;
6. the tube wall is isotropic and homogeneous;
7. the tube is infinitely long and of uniform shape (the transverse components of the velocity are thus only related to the transverse motion of the walls);
8. the velocity of the pressure wave c_0 is considerably higher than the flow velocity v, $v/c_0 \leqslant 1$.

Not all of these assumptions are satisfactorily fulfilled by the blood system.

The critical Reynold's number (≈ 2300) can be considerably exceeded during systole particularly in the aorta, thus preventing laminar flow [6]. Newertheless, in the lower vessels (carotid, femoral artery, descending aorta etc.) we may generally assume laminar flows [1, 6]. The condition $h/R \leqslant 0.1$ is satisfied only for large vessels h/R varies from 0.1 to 0.15).

Arteries are not infinitely long but, for the large vessels, the segments between successive bifurcations can be regarded as cylindrical tubes, identical throughout their length. The condition $v/c_0 \leqslant 1$ is generally met. The mean velocity c_0 of the pressure wave in the blood system lies between 8 and 12 m/s and this, for the range of instantaneous velocities v of blood flow varying from about 1 cm/s to about 200 cm/s (the latter in the ascending aorta), gives a value of the ratio v/c_0 between 0.001 and 0.2.

A more general characteristic of blood flow was proposed by Brody [3] who introduced the scattering function $p(v, r)$ describing the probability density of the appearance of a scattering blood corpuscle at a position r moving at a speed v (at the assumption that the position and velocity of the blood corpuscle are random variables).

The scattering function can be represented in the form

$$p(v, r) = p(v/r)\,p(r), \tag{7}$$

where $p(v/r)$ denotes the conditional probability density function for a blood corpuscle scattering at a speed v, provided that this blood corpuscle appears in the position r, $p(r)$ is the probability density function for the blood corpuscle in position r.

The function $p(r)$ describes the geometrical distribution of particles in the blood vessel. For a homogeneous medium and a cylindrical vessel we have

$$p(r) = p(r, \theta, z). \tag{8}$$

The condition

$$\int_{-\infty}^{+\infty} p(r)\,dr = 1$$

implies that for a homogeneous distribution of blood corpuscles we have

$$p(r) = \varrho\,\frac{r}{N}, \tag{9}$$

where ϱ denotes the density of blood corpuscles, and N is the total number of blood corpuscles in the region in which the probability density function $p(r)$ is defined.

Equation (9) states that the distribution of particles across the vessel is linear. Nevertheless, it should be pointed out here that, as LIH [5] has already shown, the blood corpuscles tend to migrate towards the middle of the vessel.

When the motion of blood corpuscles is caused only by the convection of plasma, the first factor in expression (7) can be written in the form

$$p(v/r) = \delta\big(v - v(r)\big), \tag{10}$$

where δ is the Dirac distribution. This distribution is thus referred to as the velocity distribution across the vessel, i.e. the velocity profile $v(r)$.

Diffusive flow, occurring in the vessel, determines the displacement of the blood corpuscles due to random collisions (Brownian motion) and can cause some fluctuation of the value of $v(r)$, but it does not contribute quantitatively to the blood flow velocity.

In conclusion, it should be stressed that a knowledge of $p(v, r)$ permits satisfactory determination of all flow parameters (velocity profile, flow rate etc.).

The function $p(v, r)$ can obviously be represented in the form of a function in which the independent variables are the quantities measured by a pulsed ultrasonic Doppler flowmeter. Thus, when the analogue of v is the Doppler frequency f_d and the position v corresponds to t_0, the transmitting transducer – blood corpuscle – receiving transducer delay.

Assuming new variables, the scattering function can be written in the form $p(f_d, t_0)$.

3. The correlative receiver

It will be assumed that a signal $\tilde{s}(t) = s(t)e^{j\omega nt}$ of the duration τ_1 and the repetition time T_p is emitted in the direction of a fixed object. After a time t_{01} the reflected (or scattered) wave returns from the fixed object and has the form $\tilde{y}(t) = \widetilde{ks}(t-t_{01})+\tilde{n}(t)$, where $\tilde{n}(t)$ is the noise with a random Gaussian distribution of the amplitude in the same frequency band as that of the transmitting signal and noise power density in watts per hertz equal to b.

The determination of t_{01} is reduced to the problem of finding the time t_0 for which the expression

$$C(t_0) = \frac{k}{b}\int_{\tau_1} \tilde{s}(t-t_0)\tilde{s}(t-t_{01})\,dt + \frac{k}{b}\int_{\tau_1} \tilde{s}(t-t_0)\,n(t)\,dt, \qquad C(t_0) = C_u(t_0)+C_b(t_0) \tag{11}$$

attains its maximum.

The term $C_u(t_0)$ of expression (11) is (with a coefficient $k\tau_1/b$) the auto-correlation function of the envelope of the transmitted signal $\tilde{s}(t)$ and attains its minimum for $t_0 = t_{01}$. This confirms *a priori* obvious fact that in the absence of noise the most probable position of the measured object is its real position.

The shape of the auto-correlation function $C_u(t_0)$ (i.e. its distribution about the time t_{01}) depends only on the square of the modulus of the Fourier transform, $|F(f)|^2$, of the signal $\tilde{s}(t)$, since the Fourier transform of the function $C_u(t_0)$ is equal to $|F(f)|^2$.

The maximum of the function $C_u(t_0)$ for $t_0 = t_{01}$ is equal to

$$[C_u(t_{01})]_{\max} = \frac{k}{b}\int_{\tau_1} = [\tilde{s}(t_0-t_{01})]^2\,dt, \tag{12}$$

where $[\tilde{s}(t-t_{01})]^2$ is equal to the power of the received signal.

The integral of the power in time τ_1 is equal to the energy E of the received signal, thus

$$C_u(t_0) = \frac{E}{b} = R, \tag{13}$$

where R determines the signal/noise energy ratio.

The term $C_b(t_0)$ in expression (11) shows the effect of noise on the accuracy of the measurement of the position of the object reflecting (scattering) the transmitted signal $\tilde{s}(t)$.

After suitable transformations we can determine the mean standard deviation $\sqrt{\overline{\Delta t^2}}$ of the measurement of the time t_{01} in the function R and also characteristic parameters of the transmitted signal (16),

$$\sqrt{\overline{\Delta t^2}} = \frac{1}{2\pi B\sqrt{R}}, \tag{14}$$

where

$$B^2 = \frac{\int_{\Delta f} (f-f_n) F^2 (f-f_n) df}{\int_{\Delta f} F^2 (f-f_n) df}. \tag{15}$$

If the signal $\tilde{s}(t)$ has the form of a sequence of high frequency pulses, B is approximately equal to

$$B = 0.43/\tau_1. \tag{16}$$

Remembering that

$$t = \frac{2d}{c}, \tag{17}$$

where d denotes the distance of the transducer from the reflecting object, and c is the propagation velocity of the ultrasonic wave in the medium, we can determine from expressions (14) and (16) the mean standard deviation $\sqrt{\overline{\Delta d^2}}$ of the measurement of the distance d:

$$\sqrt{\overline{\Delta d^2}} = 0.18 \frac{c\tau_1}{\sqrt{R}}. \tag{18}$$

Thus, for a given signal to noise ratio, the precision with which we can measure the position of a reflecting object is inversely proportional to the duration of the transmitted pulse.

If the scattering or the reflection of the ultrasonic wave occurs from the moving object, then we are concerned with the Doppler-Fizeau effect which is described in the introduction to this paper.

In the case of the emission of a continuous signal, the measurement of the frequency f_d and also of the velocity v can be performed with great precision, depending on the type of frequencymeter used. However, in this case it is impossible to determine the position of the scattering object.

For pulsed emission where, as a result of the scattering by the moving object, the whole spectrum of the transmitted signal is subjected to a Doppler shift, the precision of the measurement of f_d, with the simultaneous measurement of t_0, is subject to a limitation defined generally by the ambiguity function $\chi(\theta, f_d)$. It should be stressed that the maximum Doppler frequency $f_{d\max}$, measured explicitly, must satisfy the condition

$$f_{d\max} \leqslant \left| \frac{F_p}{2} \right|, \tag{19}$$

where $F_p = 1/T_p$ denotes the repetition function of high frequency pulses.

This relation is consistent with the Shannon law stating that in sampling systems the maximum measured frequency must be at last two times lower than the sampling frequency.

According to WOODWARD [18], in pulsed emission the mean standard deviation of the measured frequency f_d is expressed by the formula

$$\sqrt{\overline{\Delta f_d^2}} = \frac{3}{\pi \tau_1 R}, \tag{20}$$

and this corresponds to a mean standard deviation, for the measurement of the velocity v, of

$$\sqrt{\overline{\Delta v^2}} = \frac{c\sqrt{3}}{2\pi f_n \tau_1 R}. \tag{21}$$

For a coherent system, i.e. a system in which the phase of the subsequent pulses radiated into the medium is constant during the measurement, formulae (20) and (21) are modified to

$$\sqrt{\overline{\Delta f_d^2}} = \frac{\sqrt{3}}{\pi \tau_1 \sqrt{nR}} \tag{22}$$

and

$$\sqrt{\overline{\Delta v^2}} = \frac{c\sqrt{3}}{2\pi f_n \tau_1 \sqrt{nR}}, \tag{23}$$

where n denotes the number of pulses contained in the time during which the velocity of the object is quasi-constant.

Thus we see that the precision, with which the velocity of the object can be measured, is directly proportional to the duration of the transmitted pulse, the inverse being the case with the precision of measurement of the position. The ambiquity function is introduced into the theory of the radar and sonar systems as a measure of the quality of the signal repetition system. The ambiguity function takes the form

$$\chi(\theta, f_d) = \int_{-\infty}^{+\infty} \tilde{s}(t)\tilde{s}^*(t-\theta)e^{2\pi j f_d t}\,dt, \tag{24}$$

where $\theta = t_0 - t_{01}$ is the two-dimensional correlation function for the delay time t_0 and the Doppler frequency of the signal scattered by a particle which is at a distance d_1 (t_{01} is the delay time for the ultrasonic transducer – particle – ultrasonic transducer path) from the receiver and moving with a velocity v that corresponds to a change of the transmitter frequency f_n by a Doppler frequency f_d, delayed in time by t_0.

The simplified diagram of operation of the correlative receiver is shown in Fig. 4. The process is described by

$$\tilde{u}(t) = \tilde{y}(t)^* \tilde{s}(t) = \int_{\tau_2} \tilde{y}(t)\tilde{s}^*(t-t_0)\,dt, \tag{25}$$

where $\tilde{y}(t) = \tilde{s}(t-t_{01})$, when noise and interference are neglected.

In the case where the signal is scattered by only one particle, we can replace $\tilde{u}(t)$ by $\tilde{u}_1(t)$.

Substituting

$$\tilde{s}(t-t_0) = s(t-t_0)e^{j\omega_n(t-t_0)} \tag{26}$$

and

$$\tilde{y}(t) = s(t-t_0)e^{j\omega_1(t-t_{01})}, \tag{27}$$

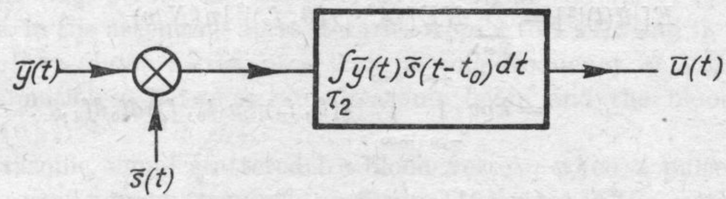

Fig. 4. Simplified diagram of the correlative receiver

and also introducing new variables $u = t-t_{01}$ and $\theta_1 = t_0 - t_{01}$, we obtain

$$\tilde{u}_1(t) = e^{j\omega_n\theta_1}\int_{\tau_2} s(u)s^*(u-\theta)e^{2\pi jf_{d1}u}du, \tag{28}$$

where

$$2\pi f_{d1} = \omega_1 - \omega_n.$$

The envelope of the signal output from the correlator takes the form $|\tilde{u}_1(t)|$, where

$$|\tilde{u}_1(t)|^2 = \chi(\theta_1, f_{d1})\chi^*(\theta_1, f_{d1}). \tag{29}$$

It can be seen that the signal output from the correlator can be determined from the ambiguity function.

In order to generalize the problem to N particles scattering the transmitted signal $\tilde{s}(t)$, we must introduce additional assumptions:

1. A signal scattered by any particle takes the form

$$y_i(t) = \tilde{A}_i s(t-t_{0i})e^{j2\pi f_i(t-t_{0i})}, \tag{30}$$

where $\tilde{A}_i$ is a complex scattering factor.

2. The scattering is isotropic and of the first order (i.e. independence of events), thus for N particles

$$\tilde{u}^N(t) = \sum_{l=1}^{N}\tilde{u}_i(t). \tag{31}$$

3. The scattering is a random process for which the mean value (expected value) of the envelope of the signal output is $E[u^N(t)]$.

When we assume that the number of scattering particles is a random variable, the square of the envelope of the global signal $|\tilde{u}(t)|^2$ takes the form

$$|\tilde{u}(t)|^2 = \sum_{N=1}^{\infty} |\tilde{u}^N(t)|^2 p(N/v), \tag{32}$$

while its expected value is

$$\begin{aligned} E[|\tilde{u}(t)|^2] &= \sum_{N=1}^{\infty} NE[|\tilde{A}^2 \cdot |\chi(\theta, f_d)|^2] p(N/v) \\ &= k\varrho v \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\chi(\theta, f_d)|^2 p(t_0, f_d)\, dt_0 df_d, \end{aligned} \tag{33}$$

where

$$k = E[|\tilde{A}|^2], \quad \varrho v = \sum_{N=1}^{\infty} NP(N/v),$$

$p(t_0, f_d)$ denotes the probability density function of the appearance of a particle at a time t_0 which is scattering with the frequency f_d.

Expression (33) represents the relation between the signal output from the correlator, the ambiguity function and the scattering function.

The most suitable for the purposes of profilometry (the determination of the velocity v as a function of the radius r of the blood vessel) would be an ambiguity function with the Dirac distribution, thus having the form $|\chi(t_0, f_d)|^2 = \delta(t_0, f_d)$, since expression (33) would then become

$$E[|\tilde{u}(t)|^2] = k\varrho v p(t_0, f_d). \tag{34}$$

This case, however, is not physically feasible.

Several systems have an ambiguity function with a distribution to ensure the required precision of both t_0 and f_d in one measurement, e.g.: noise emission with a Gaussian distribution, pseudo-noise with a band limited Gaussian distribution («coloured» noise), and the coherent emission of high-frequency pulses.

4. The demodulation and measurement of the Doppler frequency

It has been shown in section 2 that a satisfactory condition for the determination of the instantaneous velocity flow profile, and thus also of the instantaneous mean flow velocity and the delivery, there is the determination of the function $p(f_d, t_0)$. The measurement of the delay time t_0 has been explained in section 3.

The problem of the measurement of the Doppler frequency f_d requires a separate discussion.

It can be generally assumed that the blood volume, as seen by the receiving transducer in a time τ_2 equal to the duration of the analyzing gate, has a finite value and that the erythrocytes, contained in this blood volume, are moving at different speeds. Consequently, the signal scattered by the erythrocytes consists of components with different frequencies. It is thus necessary to replace the discrete Doppler frequency f_d by a frequency spectrum Δf_d.

When using a continuous wave Doppler ultrasonic flowmeter, the problem of the measurement of the mean Doppler frequency is very complicated. This follows from the fact that the spectrum of the signal scattered by the blood-corpuscles flowing across the whole cross-section of the vessel is very wide. For example, in the ascending aorta it varies from 0 to 7 kHz and in the carotid artery from 0 to about 4 kHz, at a transmitting frequency of 8 MHz and an angle of inclination θ between the ultrasonic beam and the blood vessel of about 60° [10].

The ultrasonic signal scattered by blood vessels, when a pulse technique is used, has a very narrow frequency spectrum [10], since in the section selected for measurement (which is determined by the duration of the transmitted pulse and the analyzing gate) the distribution of the blood flow velocity is small.

The received signal is a random one with a Gaussian distribution whose amplitude is stationary (for continuous flow) or quasi-stationary (for real blood flow).

Thus, according to the definition of Rice [14] of the mean frequency of a signal with a given finite, fixed, continuous frequency spectrum, the mean frequency of the signal scattered by the flowing blood is given by

$$f_{\acute{s}r} = \frac{\int\limits_{-\infty}^{+\infty} fP(f)\,df}{\int\limits_{-\infty}^{+\infty} P(f)\,df}, \tag{35}$$

where f denotes the frequency of the signal scattered, while $P(f)$ is the signal spectral density.

The mean Doppler frequency is given by the expression

$$\Delta f_{d\acute{s}r} = f_{\acute{s}r} - f_n. \tag{36}$$

Substituting $f_n + \Delta f$ for f in (35), and then substituting expression obtained in (36), we obtain

$$\Delta f_{d\acute{s}r} = \frac{\int\limits_{-\infty}^{+\infty} \Delta f P(f_n+\Delta f)\,d\Delta f}{\int\limits_{-\infty}^{+\infty} P(f_n+\Delta f)\,d\Delta f}. \tag{37}$$

On the basis of the results obtained be Reevros [15] an original measuring system (Fig. 5) was developed which directly measures the mean Doppler frequency given by expression (37) [8].

The system measuring the mean Doppler frequency has not so far found widespread use due to its complex structure.

Electronic frequency meters generally make use of the techniques for counting the number of crossings of the measured signal through the zero level. This is the so-called zero-crossing technique.

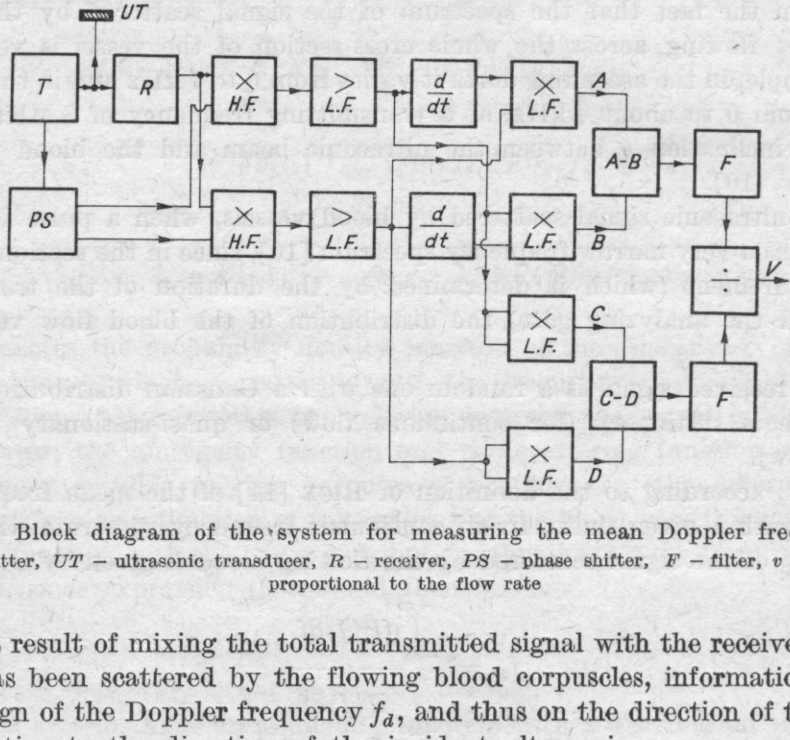

Fig. 5. Block diagram of the system for measuring the mean Doppler frequency
T – transmitter, UT – ultrasonic transducer, R – receiver, PS – phase shifter, F – filter, v – the signal proportional to the flow rate

As a result of mixing the total transmitted signal with the received signal which has been scattered by the flowing blood corpuscles, information is lost on the sign of the Doppler frequency f_d, and thus on the direction of the blood flow relative to the direction of the incident ultrasonic wave.

In order to obtain this information, McLeod [7] and Peronneau [10] independently proposed a zero-crossing system with phase-sensitive detection.

The measurement of the frequency by means of the zero-crossing technique is, however, burdened with an error. Rice [14] has shown that the expected value $E[N]$ of crossings through the zero level of a signal with a random Gaussian distribution of amplitude is given by the formula

$$E[N] = 2\left[\frac{\int_{-\infty}^{+\infty} f^2 P(f)\,df}{\int_{-\infty}^{+\infty} P(f)\,df}\right]^{1/2}, \tag{38}$$

and the measured Doppler frequency is determined by the relation

$$f_d = \tfrac{1}{2}E[N]. \tag{39}$$

The response of the zero-crossing system to an input signal with a finite spectrum is thus proportional to the second moment of the spectrum of the measured signal.

Additional errors in the measurement of the Doppler frequency by the zero-crossing technique are accounted for by interference, by the noise of the input stage of the high frequency amplifier, the level of the release of the Schmitt flip-flop multivibrators, and also by the ratio of the ultrasonic bandwidth to the cross-section of the blood vessel.

5. The block diagram and description of the operation of the pulsed ultrasonic Doppler flowmeter

The results of the theoretical analysis permitted the construction and development of a pulsed ultrasonic Doppler flowmeter, model type UDIMP-1 whose block diagram is shown in Fig. 6.

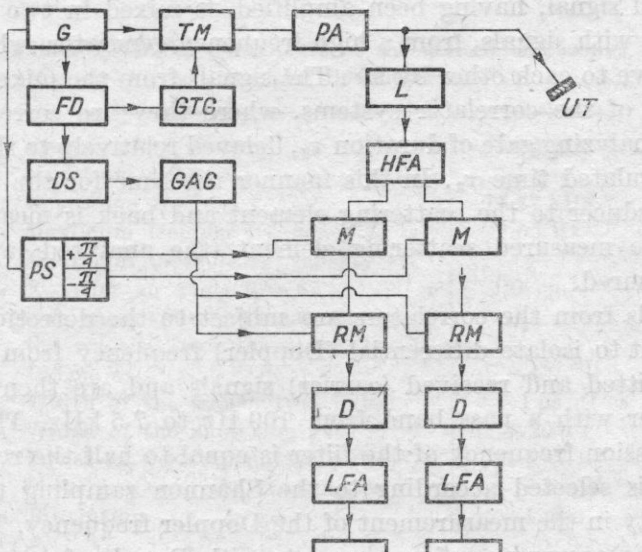

Fig. 6. Block diagram of the pulsed ultrasonic flowmeter type UDIMP-1

G – generator, FD – frequency divider, GTG – the generator of the transmitter gate, TM – the modulator of the transmitter, PA – power amplifier, L – limiter, HFA – high frequency voltage amplifier, M – mixer, PS – phase shifter, DS – delay system, GAG – the generator of the analyzing gate, RM – the modulator of the receiver, D – detector, LFA – low frequency amplifier, BF – band-pass filter, $DZCH$ – directional zero-crossing meter, LPF – low-pass filter, R – recorder, L – loudspeaker, UT – ultrasonic transducer

A signal from the high frequency generator $f_N (f_N = 7{,}8$ MHz) is supplied to the frequency divider system (division by 2^9) to produce a signal in the form of a sequence of pulses with a repetition frequency $F_p (F_p \cong 15.2$ kHz).

The high frequency signal from the generator is modulated in the modulator by a signal from the transmitter gate generator. The high frequency pulses f_n, with a repetition frequency of F_p obtained in this manner, having been amplified in a wideband aperiodical power amplifier, are supplied to the ultrasonic transducer where they are transformed into ultrasonic signals and emitted in the direction of the blood vessel. These signals, having been reflected from the walls of the blood vessel and scattered by the blood corpuscles, return to the ultrasonic transducer where they are transformed into electric signals.

When the ultrasonic wave is scattered by the flowing blood, the frequency of the scattered signal changes proportionally to the velocity of the moving blood corpuscles. This signal is supplied via a limiter to a high frequency amplifier. The limiter used in the input of the high frequency amplifier prevents from the saturation of the receiver during the transmitting pulse (the so called dead zone).

The useful signal, having been amplified, is mixed in two identical summation mixers with signals, from a high frequency generator, which are shifted in phase relative to each other by 90°. The signals from the mixers are supplied to two tracks of the correlator systems, where they are correlated with the pulses of the analyzing gate of duration τ_2, delayed relatively to the transmitting pulse by a regulated time τ_3. In this manner the time for the pulse to travel from the transducer to the scattering element and back is measured, and the position of the measured scattering element (the analyzed layer of flowing blood) is measured.

The signals from the correlators are subject to the detection in a sample and hold circuit to isolate differential (Doppler) frequency from the frequency of the transmitted and received (carrier) signals and are then supplied to a band-pass filter with a pass band from 100 Hz to 7.5 kHz. The upper limit of the transmission frequency of the filter is equal to half the repetition frequency F_p and is selected according to the Shannon sampling theory, and to avoid ambiguity in the measurement of the Doppler frequency. This ambiguity results from the occurrence of components with Doppler frequencies near the successive spectral lines of the transmitted signal $f_n + nF_p$ (where $n = 0, \pm 1, \pm 2, \ldots$).

Thus signals of frequencies $f_d \pm nf_p$ develop at the output of the detector, one of which at the Doppler frequency f_d carries the unambiguous information on the flow velocity.

The attenuation of the filter above 7.5 kHz is 80 dB/octave.

The attenuation below the lower limiting transmission frequency of the filter is not critical and is selected experimentally to optimize isolation from the industrial interference of the mains and the Doppler frequency which results from the reflection of the ultrasonic wave from the pulsating walls of the blood vessel. This limits the lowest measurable flow velocity to about 1 cm/s.

The signals from the band-pass filters are supplied to a phase-sensitive zero-crosing system. In this system the frequencies of the Doppler signal are transformed into a voltage whose amplitude is proportional to the measured Doppler frequency, with a positive or negative sign depending on the direction of the blood flow in the vessel. This signal, after filtration in the band 0-25 Hz, is supplied to a $y-t$ recorder that records the constant component.

The oscilloscope coupled to the device permits simultaneous observation of the echoes of the signal reflected from the walls of the blood vessel and of the position of the pulse of the analyzing gate.

Provision has also been made in the device to permit audiomonitoring of the Doppler frequency signal and thus of the blood flow.

The main technical parameters of the described apparatus are given in the Table.

Table. Technical parameters of the pulsed ultrasonic Doppler flowmeter

Frequency of receiver	7.8 MHz
Maximum range d_{max}	5.1 cm
Repetition frequency F_p	15.23 kHz
Maximum Doppler frequency f_d	7.6 kHz
Maximum measurable velocity V_{max}(for an angle $\theta = 67°$)	185 cm/s
Non-linearity in the measurement of the velocity over the range 2 cm/s-185 cm/s	~1%
Width of the transmitted pulse	1 μs
Width of the analyzing gate	1-20 μ
Power in the transmitted pulse	1-7 W
Mean power of the transmitter	15-100 mW
Resolution	~1 mm

6. Conclusions

The experience gained after a comparatively short period (2 1/2 years) of laboratory and clinical investigations of the pulsed ultrasonic Doppler flowmeter type UDIMP-1 has fully confirmed its practicability in the diagnosis of vessel disease.

Among the particular advantages of the device let us note its ability to make transcutaneous measurements of the blood flow velocity and of the diameter of the examined vessel, and thus of the volume blood flow in the blood vessel.

The investigations have confirmed the suitability of the device as a means of investigating the volume flow capacity of the vessels and vessel protheses, and also in the recognition and location of artery and vein fistulae.

It should be stressed that the technique presented here permits information to be obtained on the flow in a selected vessel situated deep in the patient's body when in the way of the ultrasonic beam another vessel occurs which, if a continuous wave Doppler method was used, would produce a masking effect, thus preventing from the measurement of velocity in the chosen deeper vessel.

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