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MEASUREMENT OF THE VISCOELASTIC CONSTANTS OF THE CELLULOSE USED FOR A LOUDSPEAKER MEMBRANE AND THEIR EFFECT UPON THE ELECTRO-ACOUSTIC PARAMETERS OF THE LOUDSPEAKER

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The concept of a complex Young's modulus, characterizing the properties of materials with internal loss, has been introduced. The methodology of the measurement of Young's modulus and a detailed description of the method of measuring this quantity for cellulose used for a loudspeaker membrane is given. Results of measurements of the viscoelastic parameters of six kinds of cellulose are presented, and a marked influence of the absolute value of the complex Young's modulus for cellulose on the upper frequency limit of a loudspeaker with a membrane made of this cellulose is shown. Subjective investigations of loudspeakers have confirmed this conclusion.

1. Introduction

In previous works on the theory and design of loudspeakers it has been assumed that Young's modulus of the membrane material is a constant and real. No consideration has been given to the losses involved and it has also been assumed that the modulus has a constant value which is independent of frequency.

The purpose of this paper is to show the dependence of the complex Young's modulus and loss factor on the frequency. Thereafter, a method is presented for the measurement of the mechanical properties of materials used for the production of loudspeaker membranes and the results of measurements of the absolute value of the complex Young's modulus and loss factor (loss angle) and their dependence on the frequency of vibration for six different kinds of cellulose. Finally, the results of an experiment are presented which was aimed at determining the effect of the material of which the loudspeaker membrane is made on its acoustic properties. The experiment involved the construction of 18 loudspeakers of the same type, differing only in the kind of cellulose from which their membranes were made (from each kind of cellulose, three loudspeakers were made).

2. The Young's complex modulus

For linear elastic bodies we have the relation called *Hooke's law*. This relation in a Cartesian system of coordinates, using the notation of tensor calculus, takes the form

$$\sigma_{ij} = C_{ijkl} e_{kl}, \tag{1}$$

where σ_{ij} represents the stress tensor, e_{kl} – the strain tensor, and C_{ijkl} – the tensor of elasticity constants.

In the case of an isotropic material the number of independent components of the tensor of elasticity constants is reduced to two. Equation (1) can then take the form

$$\sigma_{kk} = G_2 e_{kk}, \quad \sigma'_{ij} = G_1 e'_{ij}, \tag{2}$$

where G_1 denotes the shear modulus, G_2 – the modulus of all-sided (all-round) compression, e'_{ij} – the strain deviator, and σ'_{ij} – the stress deviator.

The deviators of strain and stress are given by equations (2a) and (2b) respectively:

$$e'_{ij} = e_{ij} - \frac{1}{3} e_{kk} \delta_{ij}, \tag{2a}$$

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}. \tag{2b}$$

It is possible to present equation (1) as follows:

$$e_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij}. \tag{3}$$

Between the constants G_1 and G_2, Young's modulus E and Poisson's ratio ν there are the explicit relations (4):

$$E = \frac{3G_1 G_2}{2G_2 + G_1}, \quad \nu = \frac{G_2 - G_1}{2G_2 + G_1}. \tag{4}$$

For lossy materials these relations become more complicated, and the interdependence of the stress and the strain at a given moment depends on the whole of the preceding relations between these two magnitudes. Hooke's law for materials of this type can be expressed using an integral convolution

$$\sigma_{ij}(t) = \int_{-\infty}^{t} G_{ijkl}(t-\tau) \frac{\partial e_{kl}(\tau)}{\partial \tau} d\tau, \tag{5}$$

where $G_{ijkl}(t-\tau)$ is the tensor relaxation function. Equation (2) can be written in the form

$$\sigma_{kk}(t) = \int_{-\infty}^{t} G_2(t-\tau)\frac{\partial e_{kk}(\tau)}{\partial\tau}d\tau, \qquad \sigma'_{ij}(t) = \int_{-\infty}^{t} G_1(t-\tau)\frac{\partial e'_{ij}(\tau)}{\partial\tau}d\tau. \tag{6}$$

It is assumed that the strain $e_{ij}(\tau)$, and thus also the strains $e'_{ij}(\tau)$ and $e_{kk}(\tau)$ are sinusoidal functions of time,

$$e_{kk}(\tau) = \bar{e}_{kk}e^{j\omega\tau}, \quad e'_{ij}(\tau) = \bar{e}'_{ij}e^{j\omega\tau}, \tag{7}$$

where $\bar{e}'_{ij}$ and $\bar{e}_{kk}$ are independent of time. It is also assumed that the motion lasts infinitely long, thus nonstationary states can be neglected. Substituting $\xi = t-\tau$, we obtain

$$\sigma_{kk}(t) = j\omega e_{kk}e^{j\omega t}\int_{-\infty}^{+\infty} G_2(\xi)e^{-j\omega\xi}d\xi, \qquad \sigma'_{ij}(t) = j\omega e'_{ij}e^{j\omega t}\int_{-\infty}^{+\infty} G_1(\xi)e^{-j\omega\xi}d\xi. \tag{8}$$

The integrals in formula (8) are Fourier transformations of the relaxation functions $G_2(t)$ and $G_1(t)$. These transformations, when multiplied by $j\omega$, are written respectively as

$$j\omega\int_{-\infty}^{+\infty} G_2(\xi)e^{-j\omega\xi}d\xi = \bar{G}_2(j\omega), \qquad j\omega\int_{-\infty}^{+\infty} G_1(\xi)e^{-j\omega\xi}d\xi = \bar{G}_1(j\omega). \tag{9}$$

The quantities $\bar{G}_2(j\omega)$ and $\bar{G}_1(j\omega)$ are called respectively the *complex compression modulus* and *complex shear modulus*.

While for elastic materials the elasticity constants E and ν are used, for viscoelastic materials one can introduce a complex Young's modulus $\bar{E}(j\omega)$ and a complex Poisson's ratio $\bar{\nu}(j\omega)$:

$$\bar{E}(j\omega) = \frac{3\bar{G}_1(j\omega)\bar{G}_2(j\omega)}{2\bar{G}_2(j\omega)+\bar{G}_1(j\omega)}, \qquad \bar{\nu}(j\omega) = \frac{\bar{G}_2(j\omega)-\bar{G}_1(j\omega)}{2\bar{G}_2(j\omega)+\bar{G}_1(j\omega)}. \tag{10}$$

Poisson's ratio over a frequency range of about 20 kHz [1] is real and independent of the frequency. The phase angle δ of Young's modulus is called *loss angle* while the tangent of this angle is termed the *loss factor*.

3. Measurement of the complex Young's modulus

The method of measuring the complex Young's modulus is based on the measurement of resonance frequencies and of widths of the resonance bands of a bar of the material to be tested. This bar is free at one end, clamped at the other and induced to vibrate transversely [4]. The measurement system is shown in Fig. 1. A specimen of the material is placed in the exciter which

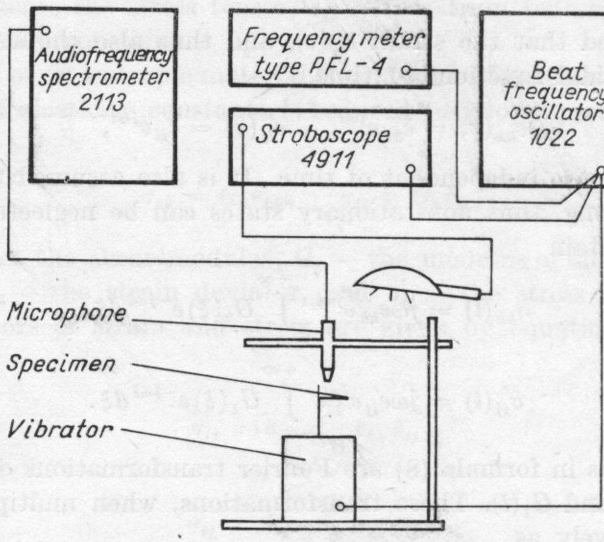

Fig. 1. System for measuring the complex Young's modulus

is driven by a generator of sinusoidal vibrations. The vibrating bar produces in its proximity an acoustic pressure proportional to the amplitude of vibrations. This pressure is measured using a microphone and a spectrometer. With the aid of a stroboscope one seeks, while tuning the generator by hand, successive modes of vibration. The frequencies of these modes are read by means of a precise frequency meter. By detuning the generator one obtains, in the proximity of a given mode of vibration, two frequencies for which the pressure level drops by 3 dB below the pressure level at resonance. These frequencies determine the band width of a given resonance. The complex Young's modulus is evaluated from the formula

$$\underline{E} = (1+jd)\cdot 4.8\pi^2 \varrho \left(\frac{l^2}{h^2}\frac{f_n}{k_n}\right)^2, \tag{11}$$

where ϱ denotes the density of the material [kg/m³], l – the active length of the bar [m], h – the thickness of the bar [m], f_n – the frequencies of successive modes of vibration [Hz], and d – the loss factor.

The values of the coefficients k_n depend only on the number n of the mode of vibration and are stated in Table 1.

Table 1. Values of the coefficient k_n for the first eight modes of vibration

Mode No.	1	2	3	4	5	6	7	8
k_n	3.52	22.0	61.7	121	200	299	417	555

4. Measurement of the Young's complex modulus of cellulose used for loudspeaker membranes

Measurements of the complex Young's modulus were made on prepared specimens of six grades of cellulose used for the production of loudspeaker membranes. The specimens were cut in the form of cellulose discs in the manner shown in Fig. 2. For each kind of cellulose about 60 specimens were cut out.

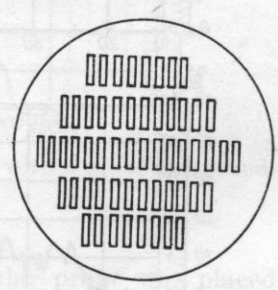

Fig. 2. The method of cutting out specimens for measurement from a pressed cellulose disc

Of these 60 specimens 10 were selected, the density and thickness of which corresponded with the prescribed tolerances to the density and thickness of loudspeaker membranes GDN 16/10.

All the specimens were cut out by means of one punch die. The shape and dimensions of the specimens are presented in Fig. 3. The specimen was

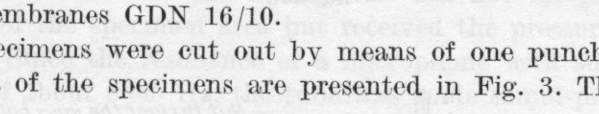

Fig. 3. Measurement specimen

then placed in the vibrator. For the measurements the vibrator produced by WZG TONSIL was used. It is constructed on the basis of a high-power loudspeaker.

In an anechoic chamber the pressure to frequency characteristics of the sound radiated by the exciter was measured. During this measurement the microphone was located at the same place as it was during measurements of the specimens. In order to ensure a correct measurement, the level of the sound emitted by the vibrator should be smaller by at least 20 dB than the level of the pressure produced by the specimen at the location of the measuring microphone. Since the vibrator did not satisfy this condition, sound emitted by the vibrator was attenuated. The method of attenuation is shown in Fig. 5. The frequency characteristics of the sound emitted by the vibrator before and after attenuation is presented in Fig. 4.

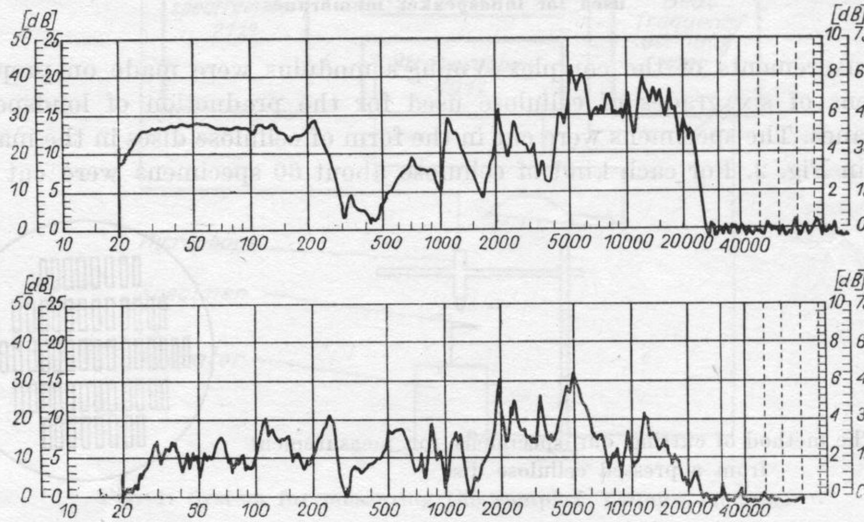

Fig. 4. Frequency characteristics of the sound emitted by the exciter before and after attenuation

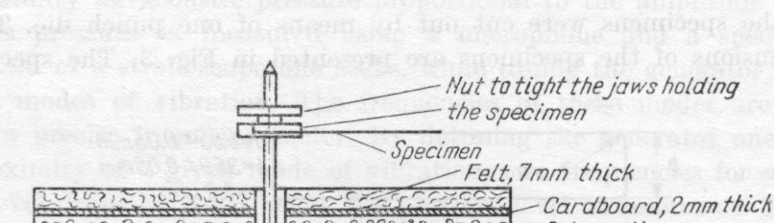

Fig. 5. The method of attenuating the exciter and of mounting the specimen in the exciter

The specimens were placed in the jaws of the vibrator with a constant pressure. Due to the lack of a torgue wrench this was done by tightening the nut so that a mark on the nut was always in the same position. The precision of the constant pressure was thus of the same order as the precision in selecting the specimen thickness. Nevertheless, the inaccuracy in establishing the pressure force was chiefly the main reason for measurement errors. In future measurements with torgue wrenches it will be absolutely necessary to obtain a certain constant pressure force. The procedure of mounting the specimen in the vibrator is shown in Fig. 5.

Measurement of the pressure level produced by the specimen was performed with a 1/2 in. microphone B&K type 4134. Since the front of the microphone

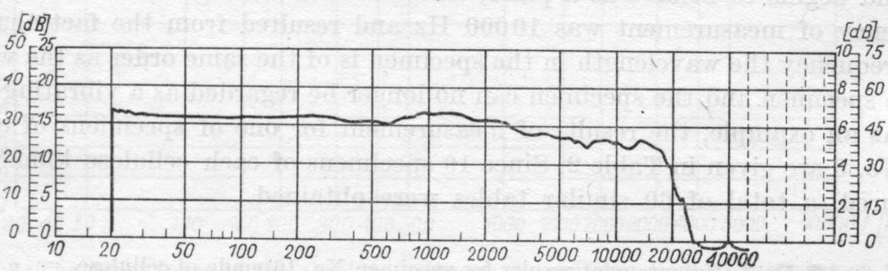

Fig. 6. Frequency characteristics of the efficiency of a microphone with a probe and sound-proof tube

was overlarge in relation to the specimen width, a tubular probe was placed on the microphone. This greatly facilitated the handling of the microphone over the specimen during the measurements, reduced disturbances due to the microphone and meant that the microphone did not integrate the pressure over the whole of the specimen area but received the pressure approximately from one point. Since the resonance of a microphone with such probe occurs at a frequency of about 5500 Hz, the tube was made sound-proof with fibrous material.

The frequency characteristics of the efficiency of the microphone with its probe and sound-proof tube is shown in Fig. 6. The measuring microphone is placed over the specimen and then moved along the specimen to the place where the level of the pressure produced by the specimen was highest. E.g. for the second mode of vibration, the position of the microphone over the specimen is shown in Fig. 7.

Since the resonance frequencies of free vibration of the bar are distributed rather rarely, the specimen was shortened during the measurement process. The length of the shortened specimen was measured with an accuracy of 0.1 mm, and the measurements were repeated. In this manner new resonance frequencies ranging between the resonance frequencies of the specimen before its shortening were obtained. When the number of measuring points was still

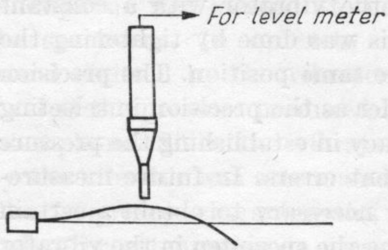

Fig. 7. An example of placing the microphone over the specimen at modes of vibration other than the fundamental

too small, the specimen was again shortened. The specimen not be shortened to more than half of its original length because it then stops vibrating as a bar and begins to behave as a plate, thus introducing large errors. The upper frequency of measurement was 10000 Hz and resulted from the fact that at this frequency the wavelength in the specimen is of the same order as the width of the specimen and the specimen can no longer be regarded as a vibrating bar.

As an example, the results of measurement for one of specimens of cellulose No. 6 are given in Table 2. Since 10 specimens of each cellulose kind were measured, a total of 60 similar tables were obtained.

Table 2. Measurement results for specimen No. 16 made of cellulose No. 6

ϱ [kg/m^3]	h [m]	f_n [Hz]	Δf_n [Hz]	d	l [m]	Mode No.
		95.1	8.0	0.08412	0.03375	1
		161.7	13.0	0.08040	0.02575	1
		284.0	20.1	0.07074	0.02000	1
		420.1	27.9	0.06641	0.01765	1
		652.0	35.0	0.05368	0.03375	2
		1071.0	47.5	0.04435	0.02575	2
454	0.000426	1847.0	66.5	0.03600	0.02000	2
		1851	61	0.03300	0.03375	3
		2960	133	0.04493	0.02575	3
		3479	130	0.03737	0.03375	4
		5965	262	0.04392	0.03375	5
		8521	330	0.03873	0.03375	6

The values of the loss factor have been calculated directly from the resonance curves, while the absolute values of the complex Young's modulus have been calculated from formula (11). As an example, the results of measurements in the form of graphs for cellulose No. 6 are shown together with measuring points in Figs. 8 and 9. It can be seen from these figures that the scatter of the measurements is considerable. For this reason the measured points for all cellulose kinds are not given in this paper. Only averaged mean curves of

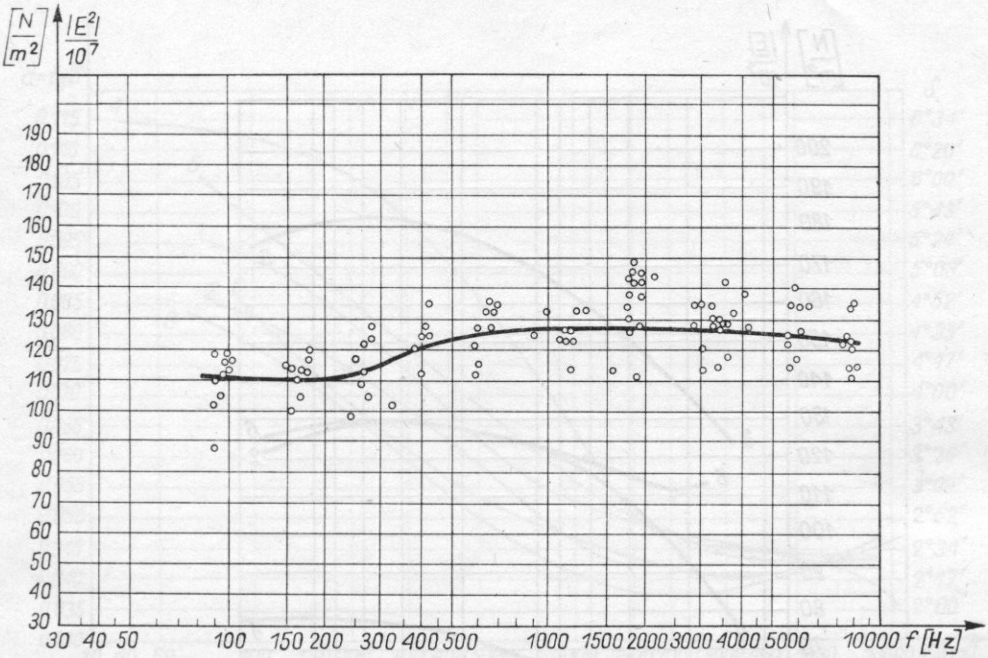

Fig. 8. The dependence of the absolute value of the complex Young's modulus on the frequency of vibration for cellulose No. 6

Fig. 9. The dependence of the value of the loss factor on the frequency of vibration for cellulose No. 6

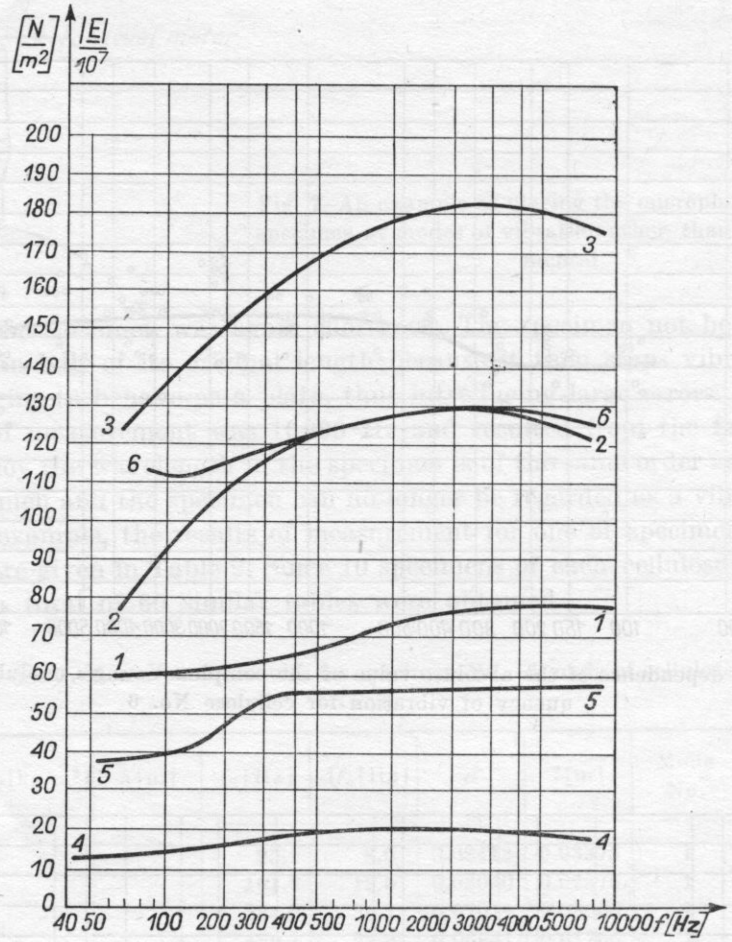

Fig. 10. The dependence of the absolute value of the Young's complex modulus on the frequency of vibration for cellulose kinds from 1 to 6

the absolute value of the complex Young's modulus and the loss angle are given, as a function of frequency, for all six cellulose kinds numbered from 1 to 6 (Figs. 10 and 11). From these figures it can be seen that the results for the individual cellulose kinds differ considerably from one another in terms of their complex Young's moduli, whereas the loss factors are approximately the same.

5. Influence of the membrane material on the properties of the loudspeaker

Loudspeakers were manufactured by WZG TONSIL from the kinds of cellulose tested, according to loudspeaker specifications GDN 16/10. All the design requirements of loudspeakers were strictly met, so that differences

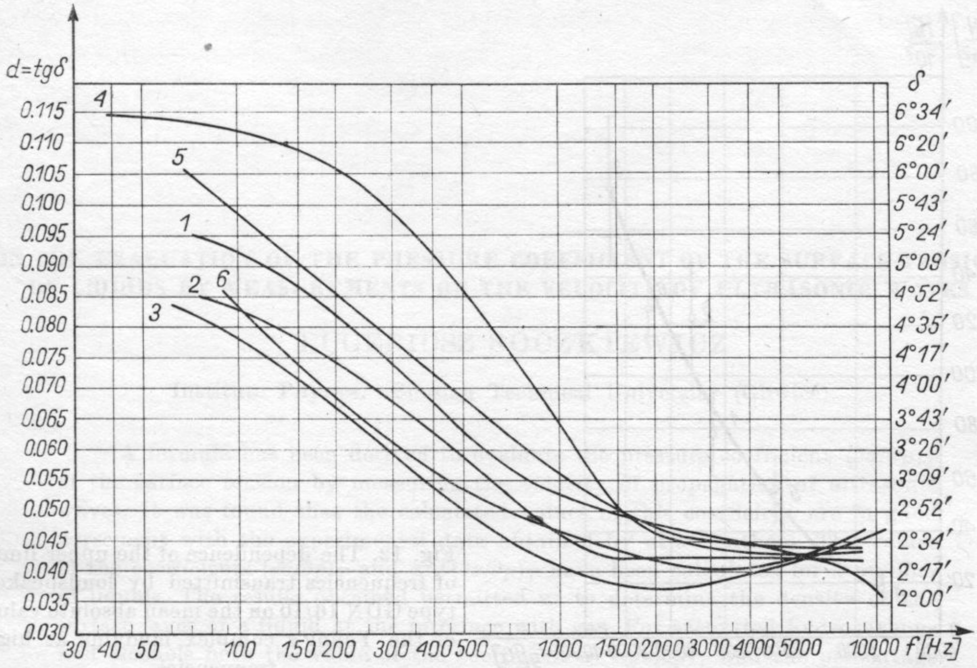

Fig. 11. The dependence of the loss factor on the frequency of vibration for cellulose kinds from 1 to 6

between loudspeakers could arise only from the varying properties of the membrane materials. Parameters of these loudspeakers were measured and from the analysis of the measurements on the loudspeakers it has been established that the Young's modulus has an important effect on the upper frequency of loudspeaker. This is shown in Fig. 12. In view of the approximated values of the loss coefficients, no relation was found to exist between the loudspeaker parameters and the magnitude of the loss factor.

No influence was found of the parameters of the viscoelastic materials used for loudspeaker membranes that were measured upon other electroacoustic parameters of the loudspeaker, except the upper frequency limit. Since the upper frequency limit increases with an increase in the absolute value of Young's modulus, the loudspeakers with membranes made of material with the larger values of Young's modulus should have the better properties. This thesis was confirmed by subjective investigations [5]. The results obtained are shown in Table 3.

Subjective measurements have supported the thesis that the quality of a loudspeaker depends on the complex Young's modulus and increases with an increase in the absolute value of the latter.

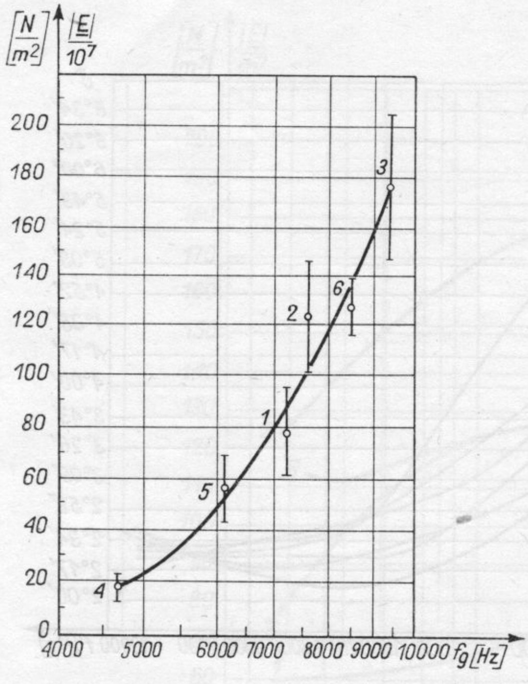

Fig. 12. The dependence of the upper limit of frequencies transmitted by loudspeaker type GDN 16/10 on the mean absolute value of the Young's complex modulus at high frequencies

Table 3. Results of the acoustic investigation of loudspeakers with membranes made from the cellulose kinds tested

Loudspeaker with membrane of cellulose No.	1	2	3	4	5	6
Fraction of voice in subjective measurements	0.51	0.63	0.70	0.13	0.40	0.62

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