ARCHIVES OF ACOUSTICS
4, 3 (1979)

SOUND RADIATION PRODUCED BY A SHIP PROPELLER

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The theory of the generation of acoustic radiation by a ship propeller working in a stream with a non-uniform stationary distribution is briefly discussed in this paper. Relations permitting calculation of acoustic pressure values produced by a ship propeller working under given conditions are also derived. The acoustic method for determining the coefficients characterizing the load variation of ship propeller blades working in a non-uniform liquid stream with stationary characteristics is also discussed. This method can also be used for estimation of the velocity field non-uniformity for the flow of the medium in which the propeller works. Experimental measurements were carried out and calculations made, based on the equations derived, and the values were compared. Numerical computations were made, based on purpose-designed programmes. Spectral analysis was carried out numerically using an FFT algorithm.

Basic notation

a — effective radius of the ship propeller ($a = 0.8\, r_M$) [m]
b — ship propeller width [m]
c — sound velocity in water [m/s]
D — ship propeller diameter [m]
$f = F/2\pi a$ — force density distribution on the circumference of the propeller circle [N/m]
f_N — normal component of the force density f [N/m]
F — action of the ship propeller on the medium [N]
$G(\boldsymbol{R}|r, \omega)$ — Green's function
H/D — propeller pitch
i — imaginary number ($i = \sqrt{-1}$)
$J_q(x)$ — Bessel function of the first kind of order q
k — wave number [1/m]
K_M — torque coefficient
K_T — thrust coefficient
M — torque applied to the propeller [Nm]
m — number of harmonic of the sound pressure
n — propeller speed [rps]
L — sound pressure level [dB]
p — sound pressure [N/m²]
p_N — thrust-related sound pressure

p_S – torque-related sound pressure [N/m²]
p_m – sound pressure of m-th harmonic [N/m²]
r_M – ship propeller radius [m]
R – distance between the propeller centre and the observation point [m]
S/S_0 – area coefficient
T – propeller thrust [N]
z – number of the propeller blades
a_m – coefficient related to the load distribution along the chord
β_l – coefficient for the ship propeller load distribution
$\delta(x)$ – Dirac function
φ, ψ, θ – coordinates
λ – wavelength – $\lambda = 2\pi/k$ [m]
ϱ – density of the medium [kg/m³]
ω – angular velocity [rad/s]

1. Introduction

The theory of sound generation by rotary sources working in a gaseous medium has a comparatively large literature. Investigations in this field deal mainly with the problems related to the methods for noise level diminution in bladed machines. In particular, these papers describe the phenomena of the sound generation by an aircraft propeller, helicopter rotors, compressors, turbines, and also fans. Less attention has been paid, however, to sound generation by a ship propeller.

In the literature on the subject this problem, for the range of steady-state conditions, i.e. a steady load on a rotary system, has been solved by methods used in mathematical physics. The method of separation of the variables (Fourier method) has been used most frequently. These problems have also often been solved using integral transforms (the Hankel-Fourier transformation). In the present paper the phenomenon of sound radiation by a source of the propeller type will be solved by the Green function method, which is the most effective for solution of stationary cases in infinite region.

Reviewing the literature on the theory of the sound generation by rotary blade systems, mention should be made of the first paper, now classical, of GUTIN [12] describing the phenomenon of sound radiation by an aircraft propeller. The results given in this paper are valid even today, e.g. in the case of an aircraft propeller working in a uniform air stream [21].

LIGHTHILL's paper [17] in which the author presented the so-called *acoustic analogy approach*, now totally accepted, was another essential step in the development of the theory of sound generation by aerodynamic sources.

In his paper Lighthill did not specify precisely the form of the excitement source itself, assuming that it was *a priori* known. The idea of the acoustical analogy method lies in a representation of the phenomenon of sound generation by complicated physical processes, as a so-called *equivalent source* [3].

Further development of the theory of sound generation, as related to acoustical analogy, has been directed to the determination of the acoustic effects related to the action of a solid on the flow of a liquid. The first paper on this subject was published by CURLE [2].

Paper [6] is important from the viewpoint of the relation between the acoustical analogy and the action of the solid boundaries in the liquid stream flow. Further development of the theory of sound generation by aerodynamic sources is related to the investigation of the structure of noise produced by fans and compressors [7, 8, 18, 19], aircraft propellers, and helicopter rotors [13, 20, 22, 26, 27]. GOLDSTEIN presented a description of the phenomenon of sound generation by sources of the propeller type, with a relatively general approach. In his papers on aeroacoustics [9-11] he presented a theory of sound generation by aerodynamic sources and suggested its practical applications.

Mention should also be made of the papers of DOKUCHAEV [4, 5] which are related to a description of sound generation by concentrated forces moving on the circumference and helix.

As regards those papers whose content is strictly related to sound radiation by a ship propeller, only a few works have been published. The most interesting are those of TSAKONAS et al. [24, 25]. It should be mentioned here that the model of the phenomenon of sound generation by a ship propeller which was used in the above-mentioned papers is based on the analysis in BRESLIN's paper [1].

It follows from this review that only a few papers have been devoted to the description of phenomena involved in sound radiation by a ship propeller.

In the present investigation a model of the sound generation by blade systems was used where the action of the blades was replaced with the effect of concentrated forces rotating on the circumference of a circle.

2. Description of sound radiation by a ship propeller working in a uniform velocity field of water flow

Let us consider the problem of sound radiation by a ship propeller working in the "sub-cavitation" range. When the phenomenon of cavitation does not occur, sound radiation is mainly related to the action of the propeller blades on the medium around them. The radiation has almost purely a dipole distribution. In the following discussion we shall assume that the ship propeller has z blades and rotates with angular velocity ω.

The problem of sound generation by a ship propeller will be solved by a method which uses the properties of Green's function. The Green function for infinite region is determined from the solution of the heterogeneous Helmholtz equation of the form

$$(\nabla^2 + k^2) G(\boldsymbol{R}|\boldsymbol{r}, \omega) = -\delta(\boldsymbol{R}-\boldsymbol{r}) \tag{1}$$

with the SOMMERFELD condition [11, 23]

$$\lim_{|\boldsymbol{R}|\to\infty} |\boldsymbol{R}| \left[\frac{\partial G(\boldsymbol{R}|\boldsymbol{r},\omega)}{\partial |\boldsymbol{R}|} + ikG(\boldsymbol{R}|\boldsymbol{r},\omega) \right] = 0, \tag{2}$$

where $\boldsymbol{R}$ is the vector connecting the origin of coordinates with the observation point, $\boldsymbol{r}$ — the vector connecting the origin of coordinates with a point source, ω — the angular velocity, $\delta(x)$ — the Dirac function, ∇^2 — the Laplace operator, and k — the wave number.

We have

$$\nabla^2 = \operatorname{div}\operatorname{grad} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right) + \frac{1}{R^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2}{\partial\psi^2},$$

$$\delta(\boldsymbol{R}-\boldsymbol{r}) = \frac{\delta(R-r)\delta(\psi-\varphi)\delta\theta}{R^2\sin\theta}.$$

The solution of equation (1) with condition (2) has the following form:

$$G(\boldsymbol{R}|\boldsymbol{r},\omega) = \frac{e^{-ik|\boldsymbol{R}-\boldsymbol{r}|}}{4\pi|\boldsymbol{R}-\boldsymbol{r}|}. \tag{3}$$

Knowing the form of the Green function, one can determine the acoustic pressure produced by a moving body, with a preset force density distribution acting on the surrounding medium, using [11, 16] the relation

$$p(R,\psi,\theta,t) = \iint_S f_i G_i(\boldsymbol{R}|\boldsymbol{r},\omega)\, dS(\boldsymbol{x}), \tag{4}$$

where $G_i(\boldsymbol{R}|\boldsymbol{r},\omega)$ is the partial derivative with respect to the i-th coordinate, and f_i is the i-th component of the force density distribution.

The force density distribution per unit area blade is determined from the equation

$$f_i = \frac{f_i^{(1)}}{|\boldsymbol{n}_3^{(1)}|} + \frac{f_i^{(2)}}{|\boldsymbol{n}_3^{(2)}|}, \tag{5}$$

where $\boldsymbol{n}_3$ is a unit vector normal to the ship propeller blade surface, $i = \{x_3, \varphi\}$.

Figure 1 shows the action of the forces on the ship propeller blade.

Since in practice the analytical form of the expression describing ship propeller blade area is usually not known, the force generated by a ship propeller on the medium should be determined experimentally by measuring the thrust and the torque. Hence for the determination of acoustic effects produced by a ship propeller it is enough to present the force density distribution on the circumference of a circle with a radius, called the *effective radius* [13]. The force density distribution can be expressed by the formula

$$\boldsymbol{f} = \frac{F_1}{2\pi r}\sum_{m=-\infty}^{\infty} a_m e^{im(\omega t-\varphi)}\delta(r-a), \tag{6}$$

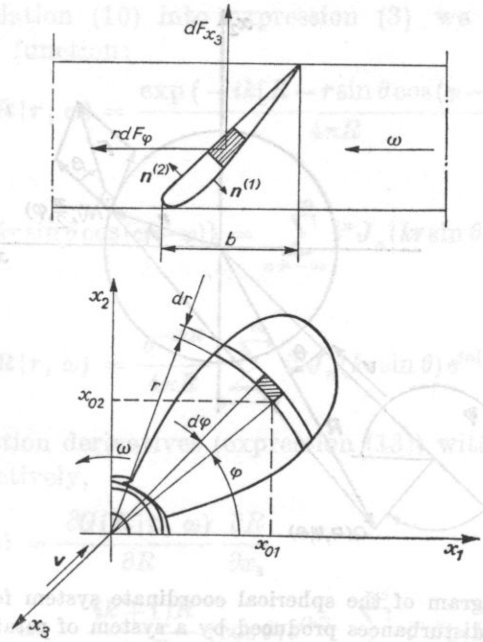

Fig. 1. The action of ship propeller blades on the surrounding medium

where a is the effective ship propeller radius ($a \simeq 0.8\, r_M$), r_M — the ship propeller radius, F_1 — the force of a ship propeller acting on the medium, a_m — the coefficient accounting for the finite width of the ship propeller blades.

If a ship propeller has z blades, equation (6) takes the form

$$f = \frac{F}{2\pi R} \sum_{m=\infty}^{\infty} a_m e^{imz(\omega t-\varphi)} \delta(r-a), \tag{7}$$

where z is the number of blades.

Expression (7) describes the force density distribution over unit circumference of a circle of radius $r = a$ for a propeller load which is constant as a function of the rotation angle.

The force density distribution can be represented as the vector sum of components related to the ship propeller thrust (normal to the surface of the propeller circle) and the torque (tangential to the circumference of the effective propeller circle). Figure 2 shows the acoustic pressure produced by a rotating system of concentrated forces, representing the action of the blades on the liquid medium.

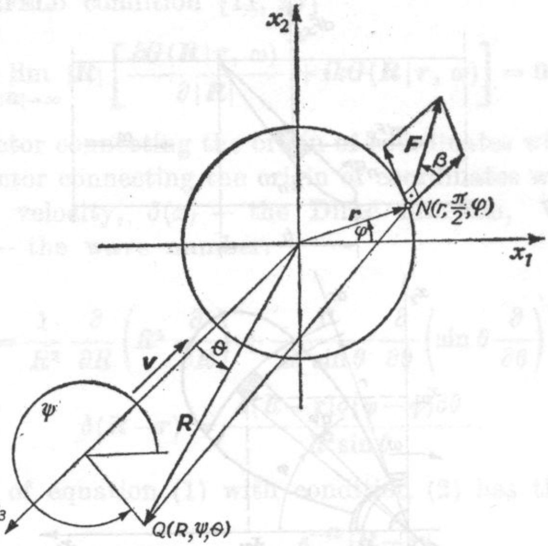

Fig. 2. A schematic diagram of the spherical coordinate system for determination of the distribution of acoustic disturbances produced by a system of rotating concentrated forces

The components of the force density distribution are represented by the following relations:

$$f_{x_3} = \frac{|F|}{2\pi R} \cos\beta \sum_{m=-\infty}^{\infty} a_m e^{imz(\omega t - \varphi)} \delta(r-a), \tag{8}$$

$$f_{\varphi} = \frac{|F|}{2\pi R} \sin\beta \sum_{m=-\infty}^{\infty} a_m e^{imz(\omega t - \varphi)} \delta(r-a). \tag{9}$$

It will be assumed further on that the acoustics pressure is calculated for distances R satisfying the relation $R \gg r$.

The coordinates of the observation point are

$$Q(R, \psi, \theta) = \{R\cos\psi\sin\theta, R\sin\psi\sin\theta, R\cos\theta\},$$

whereas the force action point coordinates are given by

$$N\left(r, \varphi, \frac{\pi}{2}\right) = \{r\cos\varphi, r\sin\varphi, 0\}.$$

The distance of the force action point from the observation point is

$$\begin{aligned} |\boldsymbol{R} - \boldsymbol{r}| &= [R^2 + r^2 - 2Rr\sin\theta\cos(\varphi-\psi)]^{1/2} \\ &\approx R - r\sin\theta\cos(\varphi-\psi). \end{aligned} \tag{10}$$

Substituting relation (10) into expression (3) we obtain the following form for the Green function:

$$G(\boldsymbol{R}|\boldsymbol{r},\omega) = \frac{\exp\{-ik[R - r\sin\theta\cos(\varphi-\psi)\}}{4\pi R}. \tag{11}$$

Since (see [23])

$$\exp\{ikr\sin\theta\cos(\varphi-\psi)\} = \sum_{n=-\infty}^{\infty} i^n J_n(kr\sin\theta)\, e^{in(\varphi-\psi)},$$

we have

$$G(\boldsymbol{R}|\boldsymbol{r},\omega) = \frac{e^{-ikR}}{4\pi R}\sum_{n=-\infty}^{\infty} i^n J_n(kr\sin\theta)\, e^{in(\varphi-\psi)}. \tag{13}$$

The Green function derivatives (expression (13)) with respect to variables x_3 and ψ are, respectively,

$$G_{x_3}(\boldsymbol{R}|\boldsymbol{r},\omega) = \frac{\partial G(\boldsymbol{R}|\boldsymbol{r},\omega)}{\partial R}\frac{\partial R}{\partial x_3}$$

$$= -\frac{ik+1/R}{4\pi R}(\cos\theta)\, e^{ikR}\sum_{n=-\infty}^{\infty} i^n J_n(kr\sin\theta)\, e^{in(\varphi-\psi)}, \tag{14}$$

$$G_\psi(\boldsymbol{R}|\boldsymbol{r},\omega) = \frac{1}{r}\frac{\partial G(\boldsymbol{R}\boldsymbol{r},\omega)}{\partial\psi}$$

$$= -\frac{e^{-ikR}}{4\pi R}\sum_{n=-\infty}^{\infty} n i^n J_n(kr\sin\theta)\, e^{in(\varphi-\psi)}. \tag{15}$$

A ship propeller with z blades produces an acoustic pressure given by the following relation:

$$p(R,\psi,\theta,t)$$

$$= -\iint_S \frac{|\boldsymbol{F}|}{2\pi r}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} a_m \frac{e^{-imzkR}}{4\pi R}\left[-\left(imzk+\frac{1}{R}\right)\cos\beta\cos\theta + \frac{in}{r}\sin\beta\right]\times$$

$$\times\, i^n J_n(mzkr\sin\theta)\exp\{i[mz\omega t - (mz-n)\varphi - n\psi]\}\,\delta(r-a)\, r\,dr\,d\varphi. \tag{16}$$

Expression (16) describes the acoustic pressure distribution around a system of z concentrated forces, rotating at an angular velocity ω. The resultant force of the ship propeller action on the surrounding medium is constant. After integration of expression (16) we obtain the following relation:

$$p(R,\psi,\theta,t) = -\frac{|\boldsymbol{F}|}{4\pi R}\sum_{m=-\infty}^{\infty} a_m\left[-\left(imzk+\frac{1}{R}\right)\cos\beta\cos\theta + \frac{imz}{a}\sin\beta\right]\times$$

$$\times\, i^{mz} J_{mz}(mzka\sin\theta)\exp\left\{imz\left[\omega\left(t-\frac{R}{c}\right)-\psi\right]\right\}. \tag{17}$$

The acoustic pressure described by relation (17) can be given by the sum

$$p(R,\psi,\theta,t) = p_{x_3}(R,\psi,\theta,t)+p_{\psi}(R,\psi,\theta,t), \tag{18}$$

where

$$p_{x_3}(R,\psi,\theta,t) = \frac{|\boldsymbol{F}|}{4\pi R}\sum_{m=-\infty}^{\infty} a_m\left[imzk+\frac{1}{R}\right][\cos\beta\cos\theta]\times \times i^{mz}J_{mz}(mzk\sin\theta)\exp\left\{imz\left[\omega\left(t-\frac{R}{c}\right)-\psi\right]\right\}, \tag{19}$$

and

$$p_{\psi}(R,\psi,\theta,t) = -\frac{|\boldsymbol{F}|}{4\pi R}\sum_{m=-\infty}^{\infty} a_m\frac{mz}{a}(\sin\beta)\times \times i^{mz}J_{mz}(mzka\sin\theta)\exp\left\{imz\left[\omega\left(t-\frac{R}{c}\right)-\psi\right]\right\}. \tag{20}$$

The possibility of a spatial division of the total acoustic pressure into the sum of pressures produced by forces normal and tangential to the propeller circle has an essential practical significance.

Expression (19) describes the acoustic pressure distribution produced by the axial force. The force can be represented by a continuous distribution of acoustic dipoles on the circumference of a circle with radius equal to the effective ship propeller radius. The dipole axes are parallel to the propeller rotation axis. The directional characteristic of the sound radiation by such a source is defined as the product of $\cos\theta$ and a Bessel function of the first kind of the mz-th order $[J_{mz}(x\sin)]$. The value of the acoustic pressure depends on the magnitude od the axial force, and thus on the ship propeller thrust. Figure 3a shows schematically the directional characteristic of the thrust-produced radiation.

Expression (20) describes the acoustic pressure distribution produced by a force tangential to a circle of radius a. An acoustic field is created by acoustic dipoles whose axes are tangential to the plane of the propeller circle. The directional characteristic of radiation has its maximum value on the plane of the propeller circle. The value of the acoustic pressure is related to the magnitude of the axial force, i.e. it is proportional to the torque applied to the ship propeller. Figure 3b shows schematically the characteristic of the radiation produced by a force tangential to the propeller circle.

Using the properties of complex series to express it in the trigonometric form resulting from the relation

$$\sum_{k=-\infty}^{\infty} W_k = W_0+\sum_{k=1}^{\infty}(W_{-k}+W_k), \quad i^n = e^{in\pi/2}, \tag{21}$$

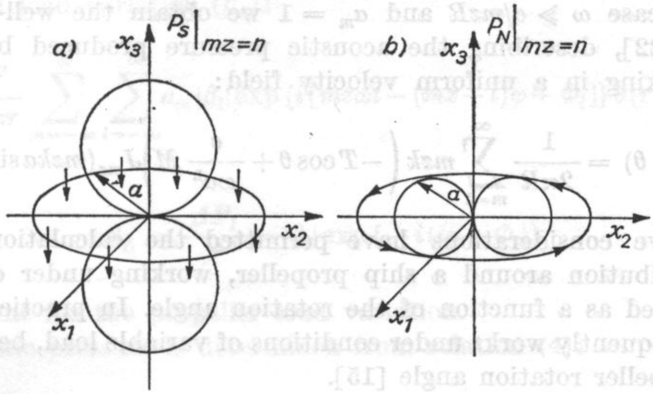

Fig. 3. Directional characteristics of the radiation of acoustic disturbances by a ship propeller, a – thrust produced radiation, b – torque produced radiation

and neglecting the term with index 0 which represents the constant behaviour, for expression (17) we get

$$p(R,\psi,\theta,t) = \frac{|F|}{2\pi R}\left\{\sum_{m=1}^{\infty} a_m\left(-mzk\cos\beta\cos\theta + \frac{mz}{a}\sin\beta\right)\sin\xi - \frac{\cos\beta\cos\theta}{R}\cos\xi\right\} J_{mz}(mzk\sin\theta), \quad (22)$$

where

$$\xi = mz\left[\omega\left(t-\frac{R}{c}\right)-\left(\psi-\frac{\pi}{2}\right)\right].$$

Using the relation between the axial force and the ship propeller thrust and between the tangential force and the torque, which can be written as

$$F_{x_3} = -|F|\cos\beta = -T, \quad (23)$$

$$F_{\varphi} = |F|\sin\beta = \frac{M}{a}, \quad (24)$$

and substituting them into relation (22), we obtain an expression for the acoustic pressure distribution in space in relation to the thrust and the torque:

$$p(R,\psi,\theta,t) = \frac{1}{2\pi R}\sum_{m=1}^{\infty} a_m\left\{\left(-mzkT\cos\theta + \frac{mz}{a^2}M\right)\sin\xi - \frac{T\cos\theta}{R}\cos\xi\right\} J_{mz}(mzka\sin\theta). \quad (25)$$

For the case $\omega \gg c/mzR$ and $a_m = 1$ we obtain the well-known Gutin formula [21, 22], describing the acoustic pressure produced by an aircraft propeller working in a uniform velocity field:

$$p(R, \theta) = \frac{1}{2\pi R}\sum_{m=1}^{\infty} mzk\left(-T\cos\theta + \frac{c}{\omega a^2}M\right)J_{mz}(mzka\sin\theta). \tag{26}$$

The above considerations have permitted the calculation of acoustic pressure distribution around a ship propeller, working under constant load, to be performed as a function of the rotation angle. In practice, a ship propeller most frequently works under conditions of variable load, being dependent upon the propeller rotation angle [15].

3. Sound radiation by a ship propeller working under conditions of variable load

Let us now consider the problem of sound radiation by a ship propeller whose load changes with variation in the rotation angle. This is a typical case where the velocity field distribution of water stream varies over the plane of the propeller circle. The case of the stream velocity variation as a function of the ship propeller rotation angle will be discussed here, with the distribution being invariable in time (stationary case).

In practice a ship propeller nearly always works under conditions of variable load. If a variable velocity field distribution occurs in the water, the ship propeller blades work at different attack angles depending on the angle of rotation. This causes variation in the thrust and torque. Figure 4 shows an

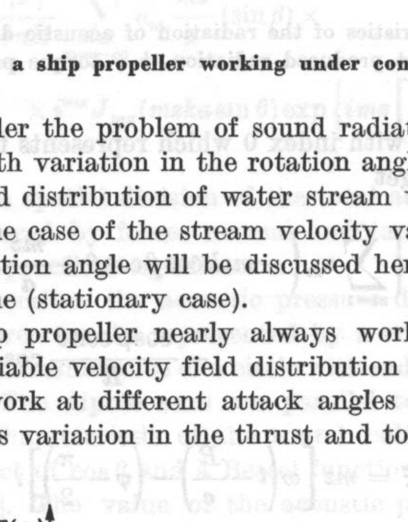

Fig. 4. An example of the ship propeller load distribution variation dependence on the rotation angle

example of a variable ship propeller load which is dependent upon the rotation angle.

The force density distribution on the ship propeller blades can be given a double complex Fourier series, taking into account the effects of finite blade width (a_m) and load variation (β_L):

$$f = \frac{F}{2\pi r} \sum_{m=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} a_m |\beta_l| \exp\{i[mz\omega t - (mz+l)\varphi + \Phi_l]\}\, \delta(r-a), \tag{27}$$

where

$$\beta_L = \frac{\Delta F_l}{F} = |\beta_l| \exp\{-i(l\varphi - \Phi_l)\}$$

is the coefficient for the propeller load variation.

The acoustic pressure is determined from relation (4):

$$p(R, \psi, \theta, t) = -\int_0^{r_M} \int_0^{2\pi} \frac{|F|}{2\pi r} \sum_{m=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{a_m |\beta_l|}{4\pi R} \left[-\left(imzk + \frac{1}{R}\right) \cos\beta \cos\theta + \frac{in}{r} \sin\beta \right] \times$$

$$\times i^n J_n(mzkr \sin\theta) \exp\left\{ i\left[mz\omega\left(t - \frac{R}{c}\right) - (mz+l-n)\varphi - n\varphi + \Phi_l \right] \right\} \delta(r-a)\, r\, d\varphi\, dr. \tag{28}$$

After integrating expression (28) we obtain

$$p(R, \psi, \theta, t) = \frac{|F|}{4\pi R} \sum_{m=-\infty}^{\infty} a_m \sum_{l=-\infty}^{\infty} |\beta_l| \left[-\left(imzk + \frac{1}{R}\right) \cos\beta \cos\theta + \frac{mz+l}{a} \sin\beta \right] \times$$

$$\times J_{mz+l}(mzka \sin\theta) \exp\left\{ i\left[mz\omega\left(t - \frac{R}{c}\right) - (mz+l)\left(\varphi - \frac{\pi}{2}\right) + \Phi_l \right] \right\}. \tag{29}$$

Using equation (21) and representing the double complex series in the form of a double trigonometric series, we obtain

$$p(R, \psi, \theta, t) = \frac{|F|}{2\pi R} \sum_{m=1}^{\infty} a_m \left\{ \sum_{l=0}^{\infty} |\beta_l| \left(\left[-mzk \cos\beta \cos\theta + \frac{mz-l}{a} \sin\beta \right] \times \right.\right.$$

$$\left. \times \sin\xi - \frac{\cos\beta \cos\theta}{R} \cos\xi_1 \right) J_{mz-l}(mzka \sin\theta) + \sum_{l=0}^{\infty} |\beta_l| \times$$

$$\times \left[-mzk \cos\beta \cos\theta + \frac{mz+l}{a} \sin\beta \right] \left(\sin\xi_2 - \frac{\cos\beta \cos\theta}{R} \cos\xi_2 \right) J_{mz+l}(mzka \sin\theta), \tag{30}$$

where

$$\xi_1 = mz\omega\left(t-\frac{R}{c}\right)-(mz-l)\left(\psi-\frac{\pi}{2}\right)+\Phi_l,$$

$$\xi_2 = mz\omega\left(t-\frac{R}{c}\right)-(mz+l)\left(\psi-\frac{\pi}{2}\right)+\Phi_l.$$

From the cylindrical properties of Bessel functions it is known that

$$J_{mz-l}(x) \gg J_{mz+l}(x), \tag{31}$$

where m and l are natural numbers.

On the basis of relations (31), (23) and (24), expression (30) can take the following form:

$$p(R,\psi,\theta,t) = \frac{1}{2\pi R}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty} a_m|\beta_l| \times$$

$$\times\left\{\left[-mzkT\cos\theta+\frac{mz-l}{a^2}M\right]\sin\xi_1-\frac{T\cos\theta}{R}\cos\xi_1\right\}J_{mz-l}(mzka\sin\theta). \tag{32}$$

For $\omega \ll c/mzR$, expression (32) becomes the following:

$$p(R,\psi,\theta,t) = \frac{1}{2\pi R}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty} a_m|\beta_l| \times$$

$$\times\left[\frac{mz-l}{a^2}M\sin\xi_1-\frac{T\cos\theta}{R}\cos\xi_1\right]J_{mz-l}(mzka\sin\theta). \tag{33}$$

Expression (33) describes the acoustic pressure distribution near a ship propeller in the case of a propeller rotating with a small angular velocity. This relation can be used for the analytical determination of the acoustic pressure in order to verify the data obtained from experiment carried out in an anechoic basin, for example.

For determination of the acoustic pressure at long distances from the source, one can use the relation

$$p(R,\psi,\theta,t) = \frac{1}{2\pi R}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty} a_m|\beta_l| \times$$

$$\times\left(-mzkT\cos\theta+\frac{mz-l}{a^2}M\right)J_{mz-l}(mzka\sin\theta), \tag{34}$$

where $\omega \gg c/mzR$.

Expression (32) describes the spatial acoustic pressure distribution produced by the rotation of a ship propeller with z blades, acting on the surrounding medium with a thrust of a mean strength T and a torque with a mean value M.

The ship propeller load variation is described by the coefficients β_L. The formula permits the determination of the acoustic pressure distribution if the thrust, the torque and the coefficients α_m and β_l are known. Figure 5 shows the method of determining the acoustic pressure under the assumption that $\varphi = \omega t$. Figure 6 shows the acoustic pressure amplitude variation as a function of distance.

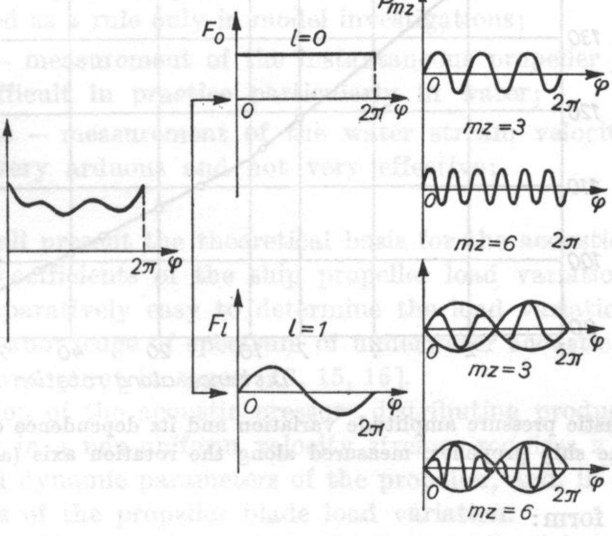

Fig. 5. Determination of the acoustic pressure components related to the ship propeller load variation as a function of the rotation angle (26)

These results were obtained from calculations using present propeller parameters. Experimental research most frequently measures the effective value of the acoustic pressure. The effective value of the m-th harmonic was determined from the following formula:

$$p_{msk}(R, \psi, \theta) = \frac{|\alpha_m|}{2\sqrt{2}\pi R} \times$$

$$\times \left\{ \sum_{l=0}^{\infty} \left[|\beta_l| \, \frac{-mzk\cos\theta + M(mz-l)/a^2}{\cos\varphi_1} \right]^2 \right\}^{1/2} J_{mz-l}(mzka\sin\theta), \qquad (35)$$

where

$$\varphi_1 = \operatorname{arc\,tg}\left[\frac{T\cos\theta}{R\left(mzkT\cos\theta - (mz-l)/a^2\right)M}\right].$$

Using the relations between the thrust and the torque and the thrust coefficient and torque coefficient respectively [15, 16], expression (35) takes

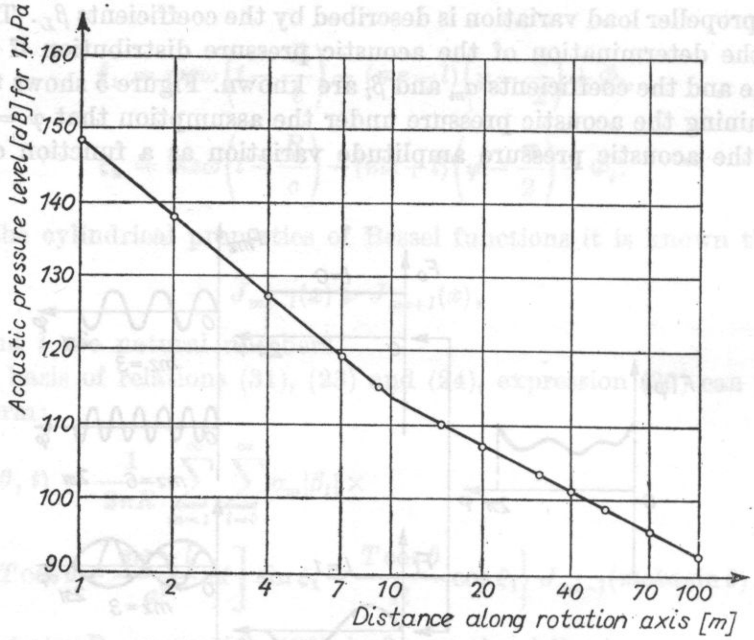

Fig. 6. The acoustic pressure amplitude variation and its dependence on the distance from the ship propeller, measured along the rotation axis (axis x_3)

the following form:

$$p_{msk}(R, \psi, \theta) = \frac{\sqrt{2}\varrho |a_m| \omega^2 r_m^4}{\pi^3 R} \times$$

$$\times \left\{ \sum_{l=0}^{\infty} \left[|\beta_l| \frac{-mzkK_T \cos\theta + 2.5(mz-l)K_M/a}{\varphi_1} \right]^2 \right\}^{1/2} J_{mz-l}(mzka \sin\theta). \quad (36)$$

Relations (35) and (36) are also valid for a thrust force equal to zero (e.g. for a propeller with ssymmetrical blade profiles working at a zero angle of attack), since the resultant force in the direction φ is different from zero. In this case the value of the acoustic pressure will, obviously, be considerably smaller than that for a propeller producing a given thrust force value.

4. Acoustic method for the determination of the coefficients of the ship propeller blade load variation

In practice the velocity field distribution of water stream flowing onto a ship propeller is, generally, unknown. Thus the variation of the velocity circulation on the blades, which depends on the propeller rotation angle, is unknown. It has been shown previously [14] that the theory of sound generation by a propeller type system, neglecting the instantaneous load variation, gives values for the acoustic pressure lower than those experimentally obtained. One of the conditions for the determination of real values of the acoustic pressure produced by a ship propeller under given working conditions, is a knowledge of coefficients of the ship propeller load variation [9, 11, 13, 15, 27].

The measurements of this variation can be made by one of the following methods:

dynamometric — permitting measurement of instantaneous thrust and torque values, used as a rule only in model investigations;

tensometric — measurement of the instantaneous propeller blade strains, comparatively difficult in practice particularly in water;

hydrodynamic — measurement of the water stream velocity within the propeller circle, very arduous and not very effective;

acoustic.

Below we shall present the theoretical basis for the acoustical method of determining the coefficients of the ship propeller load variation. Using this method it is comparatively easy to determine the load variation coefficients on the basis of a knowledge of spectrum of underwater acoustic disturbances, measured at a given point in space [13, 15, 16].

The calculation of the acoustic pressure distribution produced by a ship propeller working in a non-uniform velocity stream, requires a knowledge of the geometric and dynamic parameters of the propeller, and, in addition, that of the coefficients of the propeller blade load variation.

Let us consider the case of a variation in the velocity field which is stationary in time and random within the propeller circle (stationary space-time random field).

Green's function along the x_3 axis, related to the density distribution of acoustic dipoles whose axes are normal to the plane in the propeller circle (see the coordinate system in Fig. 7) has [15, 16] the form

$$G_{x_3}(\boldsymbol{R}\,|\,\boldsymbol{r},\,\omega) = -\frac{e^{-imzkR}}{4\pi R^2}\left[imzk\bar{x}_3 + \frac{\bar{x}_3}{R}\right], \tag{37}$$

where $R = [r^2 + \bar{x}_3^2]^{1/2}$, and $k = \omega/c$ is the wave number.

The acoustic pressure along the x_3 axis can be determined [16] from the relation

$$p(R, x_3, t) = -\iint\limits_S \frac{f_{x_3} e^{-imzkR}}{4\pi R^2}\left[imzk\bar{x}_3 + \frac{\bar{x}_3}{R}\right] dS(x), \tag{38}$$

which after integration takes the form

$$p(R_a, x_3, t) = T\sum_{m=-\infty}^{\infty} \frac{a_m|\beta_{mz}|}{4\pi R_a^2}\left[imzk\bar{x}_3 + \frac{\bar{x}_3}{R_a}\right] \exp\left\{imz\omega\left(t - \frac{R_a}{c}\right)\right\}, \tag{39}$$

where $R_a = [a^2 + \bar{x}_3^2]^{1/2}$.

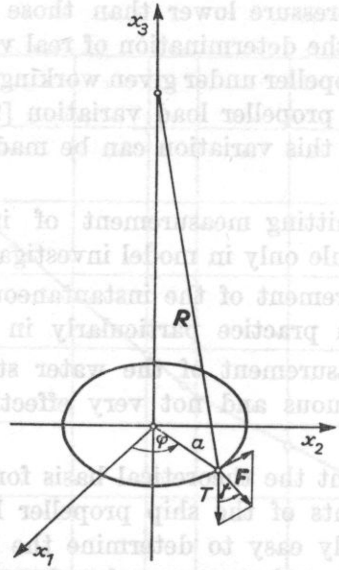

Fig. 7. The coordinate system used in the present investigations

Using the properties of the complex variable function and making further transformations of expression (39) we obtain the formula for the pressure of the m-th harmonic on the rotation axis at a distance $\bar{x}_3$ from the propeller centre,

$$p(R_a, x_3, t) = 2T\sum_{m=1}^{\infty}\frac{a_m|\beta_{mz}|}{4\pi R_a^2}\left[-mzk\bar{x}_3\sin\xi+\frac{\bar{x}_3}{R_a}\cos\xi\right], \quad (40)$$

where

$$\xi = mz\omega\left(t-\frac{R_a}{c}\right)+\Phi_{mz}.$$

The moduli of load variation coefficients in relation to the effective value of the m-th harmonic of the acoustic pressure are obtained from expression (40) in the following form:

$$|\beta_{mz}| = \frac{2\sqrt{2}R_a^2 p_{msk}}{T|a_m|}\left[(mzk\bar{x}_3)^2+\left(\frac{\bar{x}_3}{R_a}\right)^2\right]^{-1/2}. \quad (41)$$

This formula permits calculation of coefficients of the ship propeller load variation. The effective acoustic pressure values of the m-th harmonic are determined from measurements which are most frequently performed in the following way:

the acoustic pressure variation is recorded in real time,

a narrow band analysis of the acoustic pressure variation is made using an analogue or digital method.

Values of the m-th harmonic of the acoustic pressure are determined from the spectrum, using the relation

$$p_{msk} = p_{msk}(mnz) \quad (42)$$

where m is the order of the harmonic, n — the angular velocity of the ship propeller, p_{msk} — the effective acoustic pressure value of the m-th harmonic, and z — the number of ship propeller blades.

A measurement of the thrust force is made at the same time.

The load distribution along the chord is related to the coefficient a_m.

It has been assumed in this paper that the load distribution along the chord has a rectangular shape [16, 26]. In this connection, the coefficient a_m is determined from the relation

$$a_m = 2z\frac{\bar{\alpha}(r)}{2\pi}\sin c\,\frac{zm\bar{\alpha}(r)}{2\pi}, \tag{43}$$

where $\bar{\alpha}(r)$ is the angle between the projections of the leading and trailing edges of the blade onto the propeller circle plane.

This type of distribution is an idealization of the problem, since the propeller blade load along the chord is not constant. In addition, this distribution changes, depending on the attack of the angle of the propeller blade. It is obviously difficult to give an estimate of the error, since it is mainly dependent on the ship propeller blade width and the angle of the attack of the blade. For very narrow blades, the estimated error is comparatively small. It increases with increasing blade width. The angle of attack of the ship propeller blade also has a significant effect on the load distribution. Thus the exact pressure distribution on the propeller blade should be calculated individually for a given type of propeller and its working parameters.

5. The experimental part

Experiments were made in an anechoic basin. Instantaneous acoustic pressure values were registered, using a digital 7502 type Brüel and Kjaer recorder. The sampling frequency was changed individually for each case. A schematic diagram of the system used in the spectral analysis is shown in Fig. 8, and the hydrodynamic propeller characteristics are shown in Fig. 9.

The spectral analysis was performed numerically using a fast Fourier transformation algorithm. During the preparation of the data for spectral analysis, a cosine taper was used over 1/10 of each end of data. In addition, the spectrum was smoothed using the frequency method [16].

The spectrum of underwater acoustic disturbances produced by a ship propeller at an angular velocity $n = 10$ rps is shown in Fig. 10 for $\Delta f = 0.49$ Hz.

Knowing the acoustic pressure values for given harmonics, interrelated by the relation $f = m \cdot n \cdot z$, the propeller blade load coefficients were determined from relation (41) using formula (43). A computer programme was specially written (*Cis* 1) [16], which permitted, on the basis of the spectrum of distur-

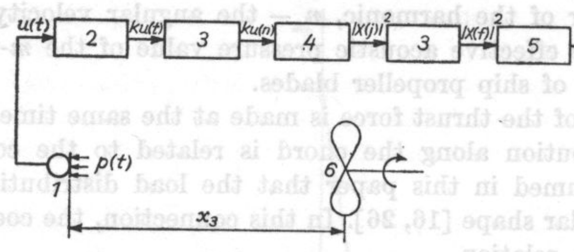

Fig. 8. A schematic diagram of the system used for spectral analysis

1 – a 8101-type Brüel and Kjaer measuring hydrophone, 2 – a 2606-type Brüel and Kjaer measuring amplifier, 3 – a 7502-type Brüel and Kjaer single pass recorder, 4 – EMC ODRA-1305, 5 – a level recorder, 6 – a ship propeller

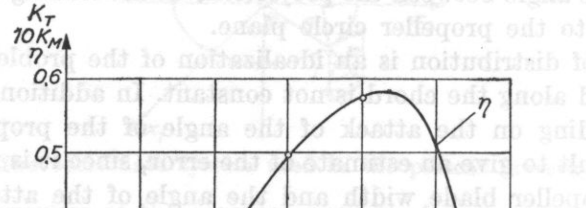

Fig. 9. Hydrodynamic characteristics of the ship propeller under investigation

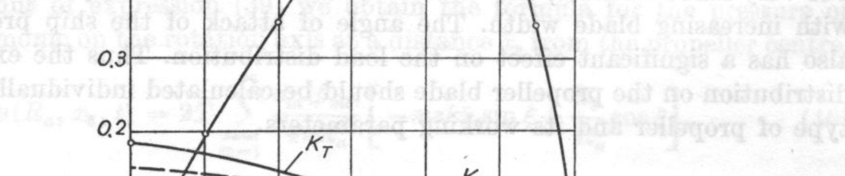

Fig. 10. The spectrum of underwater acoustic disturbances at a velocity $n = 10$ rps, $\bar{x}_3 = 8$ m

bances and working parameters of the propeller, a direct determination of the coefficients $\beta_{mz=l}$. Figure 11 shows examples of the values of these coefficients and their dependence on the value of mz.

Figure 12 shows the instantaneous acoustic pressure produced by a ship propeller at an angular velocity $n = 16.6$ rps. The acoustic pressure was record-

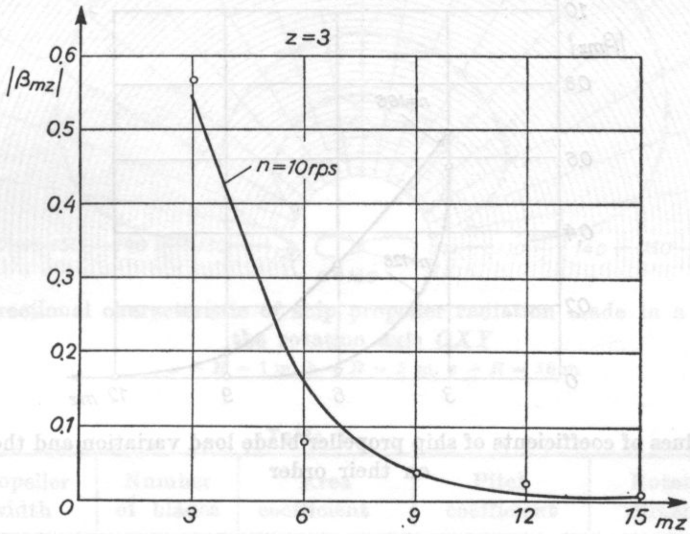

Fig. 11. The values of coefficients for the ship propeller blade load variation at $n = 10$ rps

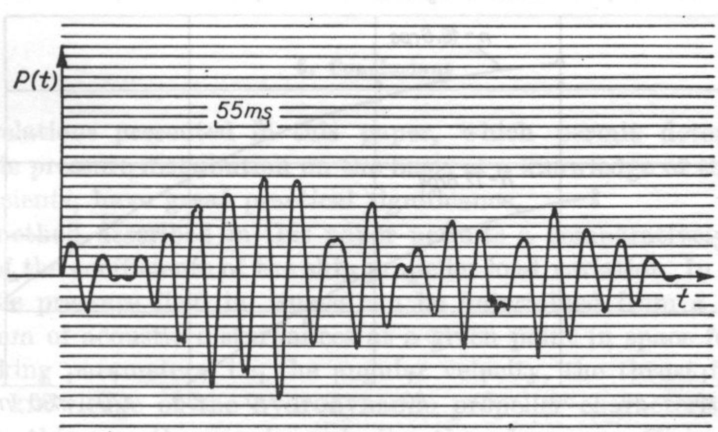

Fig. 12. Instantaneous acoustic pressure produced by a ship propeller at a distance $R = 8$ m at angular velocity $n = 16.6$ rps.

ed at a point on the propeller rotation axis, 8 metres from the propeller centre.

Figure 13 shows the coefficients of the ship propeller blade load variation for the above case.

From (36) the first four harmonics of the spectrum of underwater acoustic disturbances were calculated and compared with the values obtained from a direct measurement. The curves for these characteristics are shown in Fig. 14.

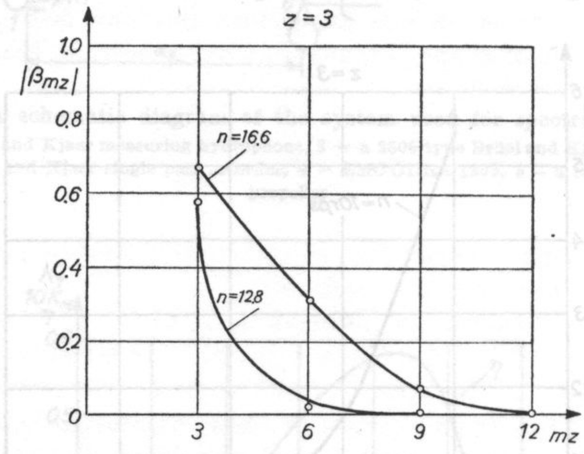

Fig. 13. The values of coefficients of ship propeller blade load variation and their dependence on their order

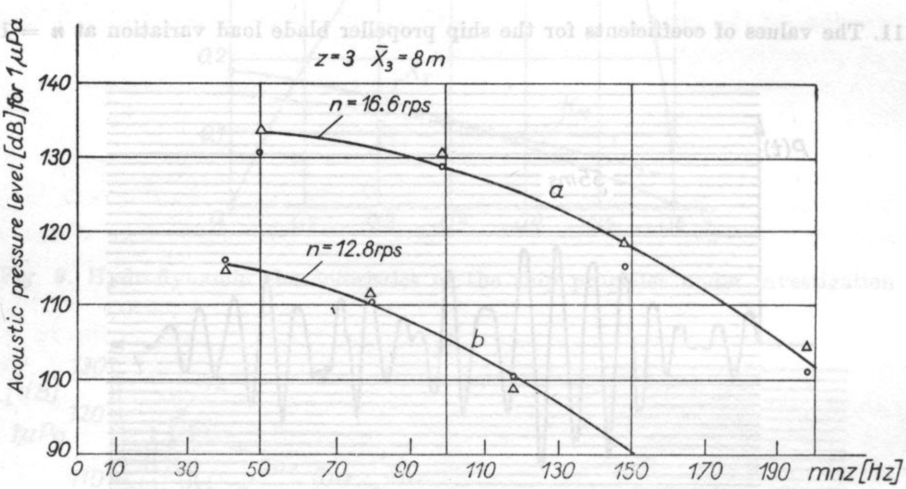

Fig. 14. Values of spectrum lines depending on their order
a – calculated from relation (36), *b* – from measurements

The directional characteristics of the sound radiated by a ship propeller were calculated for distances $R = 1$ m, 5 m, and 10 m from the propeller on the plane OXY (see Fig. 2), and are shown in Fig. 15.

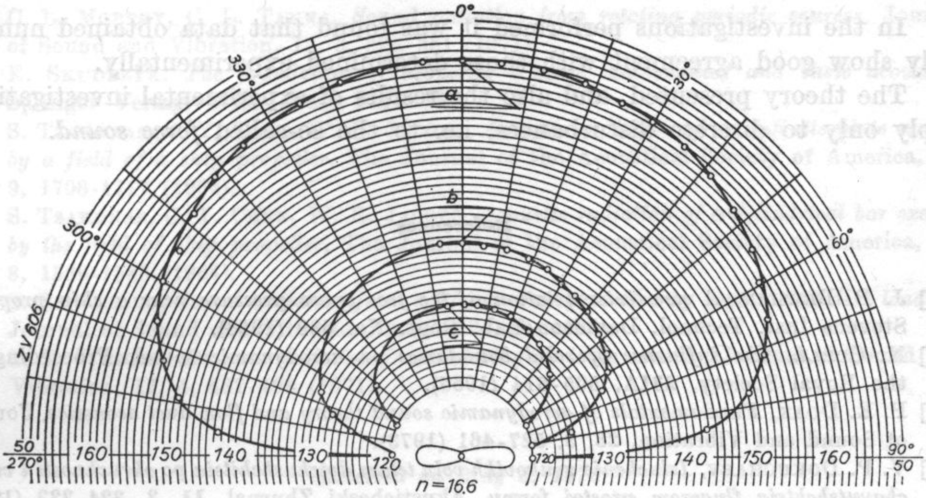

Fig. 15. A directional characteristic of ship propeller radiation made in a plane lying on the rotation axis OXY

$a - R = 1$ m, $b - R = 5$ m, $c - R = 10$ m

Table 1.

Propeller width	Number of blades	Area coefficient	Pitch coefficient	Rotation direction
D[m]	z	S/S_0	H/D	—
0.50	3	0.35	0.48	left

6. Conclusions

The relations presented in this paper, which permit determination of the acoustic pressure distribution on the basis of a knowledge of the load variation coefficients, have great practical significance.

The method described in this paper permits a comparatively easy determination of the coefficients of the ship propeller load variation. In consequence, the acoustic pressure field in space can be determined from a knowledge of the spectrum of acoustic disturbances at a given point in space for given propeller working parameters, i.e. the angular velocity, the thrust force and the torque. A knowledge of the hydrodynamic propeller characteristics can also be used. In this case the angular velocity, the advance coefficient, the thrust coefficient, and the torque coefficient must be known (see relation (36)).

From the viewpoint of the correctness of investigations an effort must be made to ensure that during the measurements the propeller works under steady-state conditions.

It is recommended that the methods of measurement are used that permit the spectra to be obtained with high frequency resolution (narrow band spectra).

In the investigations performed it was found that data obtained numerically show good agreement with those determined experimentally.

The theory presented, and also the results of experimental investigations, apply only to discrete disturbances, i.e. to the so-called *force sound.*

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Received on May 5, 1978