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THEORETICAL ANALYSIS OF INTENSITY DISTRIBUTION IN DIFFRACTION OF LIGHT BY ULTRASOUND IN THE CASE OF HOLOGRAPHIC INTERFEROMETRY*

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The theoretical background for optical holographic interferometry used in ultrasonic field investigations is presented. The double-exposure pulse technique, the time averaged technique, and also the stroboscopic real time technique are considered. Formulae for distributions of the light intensity in ultrasonic field holograms are derived for the techniques considered.

1. Introduction

Theoretical considerations of the light intensity distribution in optical holographic interferometry used for ultrasonic field examination have been performed by E. I. Kheifets [1] and P. Kwiek [2]. In this paper these results will be presented in a discussion of three methods which have been theoretically described and verified experimentally in our laboratory. These methods are:

1. pulse holographic interferometry (an ultrasonic wave as a phase object which moves with the high velocity, requires very short times of exposure for hologram recording) – usually the double-exposure technique is used.
2. holographic interferometry averaged in time.
3. stroboscopic interferometry in real time.

Let us consider the theoretical relations between the light intensity distribution and the ultrasonic wave intensity distribution for the three methods, successively.

2. Double-exposure pulse interferometry

In this case the main assumption is that the duration of the light pulse used for the exposure must be very much shorter than a period of the ultrasonic wave (for example 1/10 of a period).

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Let us consider that such a pulse of light presents a parallel beam Σ (Fig. 1) for the construction of the ultrasonic wave hologram. The beam Σ is divided into two beams: Σ_p being the object beam and Σ_0 being the reference beam. The beam Σ_p is illuminating the ultrasonic wave which represents the object

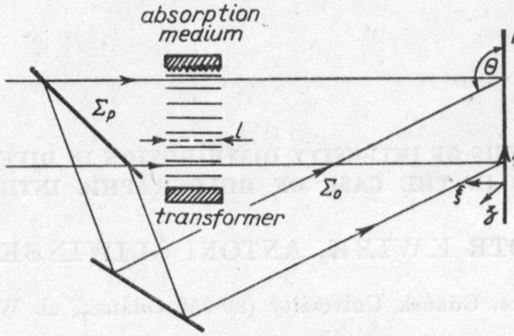

Fig. 1. Creation of an optical hologram of an ultrasonic wave

of observation. The object and the reference beams meet each other at a plane H where they interfere, creating the hologram. The beam Σ_0 makes an angle Θ with the plane H which corresponds to the plate on which the hologram can be recorded. The system of coordinates is chosen in such a way that η and ζ are coplanar to H.

Firstly, let us write the light intensity distribution for the case when the ultrasonic wave is absent from the path of the beam Σ_p, i.e. for the so called "empty" hologram. Denoting by $a_1(\eta, \zeta) = a_0 e^{-ik\zeta}$ the complex amplitude of the light wave of the beam Σ_p at the plane H, and by $F(\eta, \zeta) = F_0 e^{-ik\theta\eta}$ the amplitude of the reference beam Σ_0 one can write the expression for the illumination at the plane (η, ζ) as

$$E_1 = (a_1 + F)(a_1^* + F^*) = |a_1|^2 + |F|^2 + a_1^* F + a_1 F^*, \tag{1}$$

where * stands for the conjugate complex, a_0, F_0 are the real amplitudes of light for the object and reference beams, respectively, and k is the wave number of the light.

The hologram is recorded on the plate during an exposure time T. The energy recorded on the hologram is thus as follows:

$$W_1 = E_1 T = T|a_1|^2 + T|F|^2 + Ta_1^* F + Ta_1 F^*. \tag{2}$$

Now, let us introduce (switch on) the ultrasonic wave which changes the object beam. The thickness of the ultrasonic layer is L and the angular frequency Ω. The progressive ultrasonic wave changes the refractive index, of the medium Δn in proportion ot the acoustic pressure: $\Delta n \sim p$. In consequence the phase of the light passing through the ultrasonic wave will be changed by an amount $\Delta\psi$, and

$$\Delta\psi = \frac{2\pi L \Delta n}{\lambda} = \frac{2\pi L \delta n}{\lambda} \sin(K\eta - \Omega t) \cdot e^{+\alpha\eta}. \tag{3}$$

Thus the ultrasonic wave is treated as a phase diffraction grating (the Raman-Nath approximation). In this approximation the curvature of the light beam resulting form the gradient of the refractive index is neglected (taking this gradient into account one can consider the so-called amplitude diffraction grating as in the Lucas-Biquard theory).

In the presence of the ultrasonic wave the complex amplitude of the light beam Σ_p can be written as

$$a_2 = a_0 e^{-i[k\xi + \Delta\psi]} = a_0 \exp\left\{-i\left[k\xi + \frac{2\pi\delta n L}{\lambda}\sin(K\eta - \Omega t) e^{-\alpha\eta}\right]\right\}, \tag{4}$$

where K is the wave number of the ultrasonic wave,

α is the absorption coefficient of ultrasound in the medium,

$\delta\eta$ is the amplitude of changes in refractive index,

λ is the wavelength of the light.

When $T \ll 2\pi/\Omega$ (single pulse or stroboscopic exposure), expression (4) may be rewritten as

$$a_2 = a_0 \exp\left\{-i\left[k\xi + \frac{2\pi\delta\eta L}{\lambda}\sin K\eta e^{-\alpha\eta}\right]\right\}. \tag{5}$$

In this case the energy recorded at the plane H during the exposure time T will be equal to

$$W_2 = E_2 T = T|a_2|^2 + T|F|^2 + T a_2^* F + T a_2 F^*. \tag{6}$$

The total energy after two exposures ("empty" and "filled in") is

$$W = W_1 + W_2. \tag{7}$$

Let us assume that the recording process on the holographic plate uses the linear part of the amplitude transmission characteristic t_W of the plate (Fig. 2), i.e.,

$$t_W = t_0 - \beta(W - W_0), \tag{8}$$

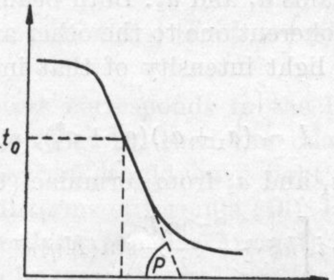

Fig. 2. Amplitude transmission characteristic of a holographic plate

where W_0 is the background energy corresponding to the amplitude transmission t_0. We can put

$$W_0 = T(|a_1|^2 + |a_2|^2), \tag{9}$$

where β is the slope of the linear part of the transmission characteristic.

In the double-exposure process the transmission function of the hologram is the following:

$$t_W = t_0 - \beta T[2|F|^2 + F^*(a_1 + a_2) + F(a_1^* + a_2^*)]. \tag{10}$$

a_2 in this expression contains all the information about the changes in refractive index of the medium which are caused by the acoustic pressure. Thus it can be seen that this information is involved in the transmission function t_W and we can speak about a hologram of the ultrasonic wave.

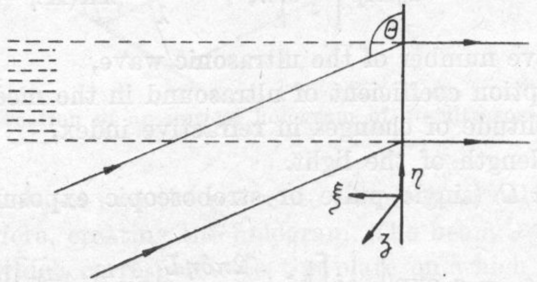

Fig. 3. Reconstruction of an optical hologram of the ultrasonic wave

Let us consider the reconstruction of this hologram, from which we shall obtain a visualization of the ultrasonic wave. The ultrasonic wave hologram of the transmission function t_W is placed in the parallel reconstruction light beam (Fig. 3) which is identical, to within to a constant factor, with the reference beam in the construction process. The output beam from the hologram can be written as:

$$bt_W = t_0 b - 2\beta' b|F|^2 + \beta' bF^*(a_1 + a_2) + \beta' bF(a_1^* + a_2^*), \tag{11}$$

where $\beta' = \beta T$ and $b = b_0 e^{-ik\theta\eta}$ is the complex amplitude of the light of the reconstruction beam at the hologram plane, and θ is the angle of incidence.

The third and fourth terms in this formula correspond to the reconstruction of images obtained with beams a_1 and a_2. Both beams are reconstructed simultaneously. The beams are coherent one to the other and produce an interference image. The distribution of light intensity of that image, to within a constant factor, can be written as

$$I \sim (a_1 + a_2)(a_1^* + a_2^*). \tag{12}$$

After substitution of a_1 and a_2 from formulae (1) and (5) one obtains

$$I = \left(a_0 e^{-ik\xi} + a_0 \exp\left\{-i\left[k\xi + \frac{2\pi\delta n L}{\lambda}\sin(K\eta)e^{-\alpha\eta}\right]\right\}\right) \times \\ \times \left\{a_0 e^{ik\xi} + a_0 \exp\left\{+i\left[k\xi + \frac{2\pi\delta n L}{\lambda}\sin(K\eta)e^{-\alpha\eta}\right]\right\}\right\}. \tag{13}$$

After some calculations and transformations using the known relations

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} \quad \text{and} \quad 1+\cos\varphi = 2\cos^2\frac{\varphi}{2}$$

formula (13) appears in the form

$$I = 4a_0^2\cos^2\left[\frac{\pi\delta n L}{\lambda}\sin(K\eta)e^{-\alpha\eta}\right]. \tag{14}$$

This distribution of light intensity in the reconstructed image from the hologram corresponds to the ultrasonic field distribution. It can be used for determination of the phase and amplitude distributions, the velocity and the attenuation of the ultrasonic wave because all of these quantities are responsible for the light distribution obtained. The formula (14) gives the basis for experimental measurements of the ultrasonic field distribution with interferometric methods. [8].

3. Holographic interferometry averaged in time

Now, we shall consider the case when the exposure time $T \gg 2\pi/\Omega$ (the exposure time is much greater than a period of the ultrasonic wave). Due to the long exposure time the light intensity distribution is averaged in time.

Assuming the same geometry as in the Fig. 1 we can write the expression for the (single-exposure) hologram as

$$E = (a+F)(a^*+F^*) = |a|^2+|F|^2+aF^*+a^*F, \tag{15}$$

where F is the complex amplitude of the reference beam, a is the complex amplitude of the object beam passing through the ultrasonic wave ($a = a_2$ from the previous case and is given by formula (4)).

The energy accumulated in the hologram during the exposure can be written as

$$W = \int_0^T E dt = \int_0^T |a|^2 dt + \int_0^T |F|^2 dt + \int_0^T aF^* dt + \int_0^T Fa^* dt. \tag{16}$$

If the recording process corresponds to the linear part of the hologram transmission characteristic (Fig. 2), then the images which arise during the reconstruction of the hologram would have light intensity distributions proportional to the individual terms of formula (16). Including the first two terms in the constant W_0 (describing the background) we shall obtain as a result of the reconstruction two images: the real one (determined by the third term) and the imaginary one (determined by the fourth term). We shall deal with the real image, only.

We can write (substituting for F and a) the expression for the light intensity distribution recorded in the hologram:

$$I \sim \int_0^T F^* a dt = \int_0^T F_0^{ik\theta\eta} a_0 \exp\left\{-i\left(k\xi + \frac{2\pi\delta n L}{\lambda}\sin(K\eta - \Omega t)e^{-\alpha\eta}\right)\right\}dt$$

$$= F_0 a_0 e^{i(k\theta\eta - k\xi)} \int_0^T \exp\left\{-i\frac{2\pi\delta n L}{\lambda}\sin(K\eta - \Omega t)e^{-\alpha\eta}\right\}dt. \quad (17)$$

Developing the expression under the integral (of type $e^{iA}\sin\varphi$) into series of Bessel functions [5], one obtains

$$\int_0^T F^* a dt = F_0 a_0 e^{i(k\theta\eta - k\xi)} \left\{\int_0^T J_0\left(\frac{2\pi\delta n L}{\lambda} e^{-\alpha\eta}\right)dt + \right.$$

$$+ \int_0^T 2\sum_{n=1}^{\infty}\left[J_{2n}\left(\frac{2\pi\delta n L}{\lambda}e^{-\alpha\eta}\right)\cos 2n(K\eta - \Omega t) + \right.$$

$$\left.\left. + iJ_{2n+1}\left(\frac{2\pi\delta n L}{\lambda}e^{-\alpha\eta}\right)\sin(2n-1)(K\eta - \Omega t)\right]\right\}. \quad (18)$$

For $T \gg 2\pi/\Omega$ the second term in { } is equal to 0.

Illuminating the hologram during the reconstruction with the reference beam we obtain the amplitude distribution of the object beam as follows

$$A \sim F\int_0^T F^* a dt = F_0^2 a_0 e^{ik\theta\eta} T J_0\left(\frac{2\pi\delta n L}{\lambda}e^{-\alpha\eta}\right)$$

$$= BJ_0\left(\frac{2\pi\,\delta n\,L}{\lambda}e^{-\alpha\eta}\right)e^{ik\theta\eta} \quad (19)$$

and the intensity distribution

$$I \sim J_0^2\left(\frac{2\pi\,\delta n\,L}{\lambda}e^{-\alpha\eta}\right). \quad (20)$$

Formula (20) does not give the possibility of phase and velocity determination, however, it is possible to determine attenuation of the ultrasonic wave α measuring the change in light intensity along the direction of propagation η.

4. Stroboscopic holographic interferometry in real time

The double-exposure pulse holography described in the first section allows comparison of only two states of the moving object being examined. However, it is possible to observe continuous changes of movement using only one hologram in the procesure called real time interferometry. The procedure depends on the interference of the object beam approaching the hologram with the beam which is previously recorded on the hologram, and which is thus simultaneously reconstructed. In our case the beam of the image of the "empty" medium (without the ultrasonic wave) recorded on the hologram interferes with the beam actually passing through the medium with the ultrasonic beam. On the base of the continuous superposition of the single hologram with the momentary image of the ultrasonic wave one can examine dynamical states of the ultrasonic field.

If in that case of real time interferometry a stroboscopic illumination is used with durations pulse much shorter than the period of the ultrasonic wave, then the light intensity distribution which arises in the interference image is described by the same formulae as for the case of double-exposure holographic interferometry.

Some experimental illustrations of the realization of the three methods described above has been presented in the paper of P. Kwiek et al. [8].

5. Conclusions

Using holographic interferometry one can visualise the space distribution of the ultrasonic wave as the distribution of the intensity of light diffracted on the ultrasonic hologram.

The theory presented here allows a quantitative comparison between measured and calculated distributions.

In the double-exposure and real time stroboscopic methods, examination of the wave fronts and phase relations is possible. In the time averaged method one can only determine the distribution of the acoustic energy density and the coefficient of attenuation of the ultrasonic wave.

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