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GENERATION OF ACOUSTO-ELECTRICAL WAVES USING A SOURCE OF TRANSVERSE VIBRATIONS

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The paper presents a theoretical analysis of the generation of transverse surface waves, using a linear source of vibrations, placed in the plane in which the generated wave propagates. On the basis of the results of the theory presented, the problem of the generation of a surface wave excited by a source of finite dimensions in the form of piezoelectric plates is considered. This method of wave generation was used for the generation of transverse surface waves on a piezoelectric ceramic and on niobiate and iodate of lithium.

1. Introduction

Transverse surface waves are generated when the velocity of bulk transverse waves close to the surface decreases. This phenomenon may be caused by a decrease in the stiffness of a material, or an increase in the density in the surface layer. Such boundary conditions exist, for example, in the case of a piezoelectric material whose stiffness depends on the electric field. Waves of this type were described by BLEUSTEIN [1] and independently by GULAEV [3], in 1968. At present, with the progress in the technology of materials whose surface layer has different properties from the properties of the other parts of the material, surface waves can useful for the investigation of the properties of these layers [11]. In the case of piezoelectric materials, the investigation of surface transverse *B. G.*, also called acousto-electrical, waves is very advanced [2, 4-6]. The source of these waves is usually an interdigital transducer to which a variable electric field is applied and which, through the piezoelectric effect, generates an elastic wave. The theoretical methods of solving the problem of the generation of acousto-electric waves are based on the calculus of variations [9] and the use of the Fourier integral [9].

In addition to the above methods for the generation of surface waves using an electric field, it is possible to excite waves by the application of a suitable source of mechanical vibration on either a piezoelectric material or a nonpiezoelectric material with a layer structure. This method of exciting transverse surface acoustic waves permits a broadening of the range of their use in the investigation of the surface layers of materials treated using modern technologies.

The subsequent parts of the paper will discuss the theoretical side of the problem, and some experimental investigations performed using known piezoelectric materials.

2. Generation of transverse surface waves using a source of transverse vibrations

The paper analyzes the following cases:

a. a linear source of transverse vibrations on the surface of wave propagation,

b. a plate source of transverse vibrations on the surface of wave propagation,

c. a plate source on a surface perpendicular to the surface of wave propagation.

The methods for the analysis of these three cases are similar. Therefore, the first case considered will be the simplest: that of a linear source, where the wave propagates on a piezoelectric material with a free surface or a surface covered by a thin metal layer with very small mass and stiffness.

Using the coordinate system shown in Fig. 1, and assuming that the surface of piezoelectric material is covered with metal layer, we take the direction X_1 as the direction of surface wave propagation; the vibration direction coincides with the X_3 axis, which is an axis of twofold symmetry for a $LiIO_3$ crystal, and the polarisation axis for a ceramic. We look for a component of the vector displacement in the X_3 direction on the surface $X_2 = 0$, caused by the linear source. For the case of a ceramic, the boundary conditions are the following:

$$T_{23}|_{X_2=0} = C_{44}^E \frac{\partial U_{3m}^{(1)}}{\partial \chi_2} = \delta(X_1) e^{-j\omega t}, \tag{1}$$

$$\Phi|_{X_2=0} = \frac{e_{15}}{\varepsilon_{11}^5} U_{3m}^{(1)} + \varphi|_{X_2=0} = 0, \tag{2}$$

where $U_{3m}^{(1)}$ is the component of particle displacement in the X_3 direction for the metal surface and $\delta(x)$ is the Dirac function.

$U_{3m}^{(1)}$ and Φ satisfy the following conditions

$$C_{44}^E \nabla^2 U_{3m}^{(1)} + e_{15} \nabla^2 \Phi = \varrho \ddot{U}_3, \tag{3}$$

$$e_{15} \nabla^2 U_{3m}^{(1)} - \varepsilon_{11}^S \nabla^2 \Phi = 0. \tag{4}$$

C_{44}^E, e_{15}, ε_{11}^S are, respectively, the elastic, piezoelectric and dielectric constants, Φ is the electric potential connected with the electric field, $E = -\text{grad}\,\Phi$ and φ is a potential satisfying the following equation:

$$\frac{\partial^2\varphi}{\partial^2 X_1^2}+\frac{\partial^2\varphi}{\partial^2 X_2^2}=0. \tag{5}$$

It is well known that the vector of displacement of a particle of the medium can be written, in general, as

$$\vec{V}=\text{grad}\,\tau+\text{rot}\,\psi. \tag{6}$$

τ and ψ are the scalar and vector potentials of the acoustic field. Since the required wave is a plane transverse wave, $\tau \equiv 0$ and ψ has only one component $\psi_y \equiv \psi(X_1, X_2)$. Expression (6) takes the form

$$U_{3m}^{(1)}=\frac{\partial\psi}{\partial X_1}. \tag{7}$$

$\psi(X_1, X_2)$ satisfies the equation

$$\frac{\partial^2\psi}{\partial X_1^2}+\frac{\partial^2\psi}{\partial X_2^2}+k_t^2\psi=0, \tag{8}$$

where $k_t = \omega/C_t$ is the wave number of the transverse wave in the material which is stiffened as a result of the piezoelectric effect

$$C_t=\sqrt{\frac{C_{44}^E}{\varrho}(1+K^2)}$$

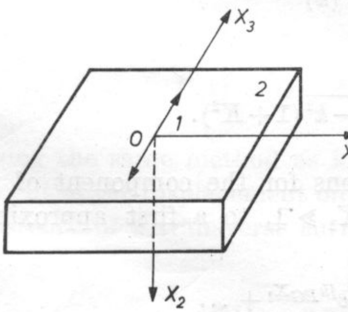

Fig. 1. A linear source with transverse vibration on the surface of wave propagation

1 – linear source, *2* – the surface of wave propagation

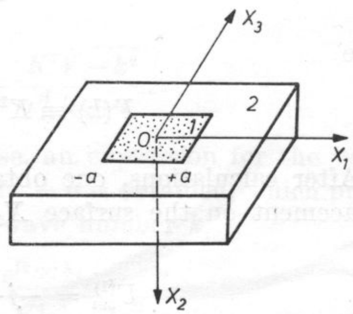

Fig. 2. A plate source with transverse vibration on the surface of wave propagation

1 – plate source, *2* – the surface of wave propagation is the velocity of transverse waves and

$$K^2 = \frac{e_{15}^2}{\varepsilon_{11}^S C_{44}^E}$$

is the coefficient of electromechanical coupling.

On the basis of expressions (5) and (8) the following expressions for φ and ψ can be written in the form of the Fourier integral:

$$\varphi(X_1, X_2) = \int_{-\infty}^{+\infty} \varphi(k) \exp\{j(\sqrt{-k^2} X_2 + kX_1 - \omega t)\} dk, \tag{9}$$

$$\psi(X_1, X_2) = \int_{-\infty}^{+\infty} \psi(k) \exp\{j(\sqrt{k_t^2 - k^2} X_2 + kX_1 - \omega t)\} dk. \tag{10}$$

From expressions (9) and (10) and the boundary conditions (1) and (2) one can obtain

$$\varphi_{(k)} = -\frac{e_{15}}{\varepsilon_{11}^S} jk\psi(k), \tag{11}$$

$$\psi(k) = \frac{1}{2\pi C_{44}^E k [K^2 \sqrt{-k^2} - \sqrt{k_t^2 - k^2}(1+K^2)]}. \tag{12}$$

Inserting (12) into (10) and (7) results in an expression for the component of vector particle displacement on the surface $X_2 = 0$,

$$U_{3m}^{(1)} = \frac{j}{2\pi C_{44}^E} \int_{-\infty}^{+\infty} \frac{e^{j(kX_1 - \omega t)}}{F(k)} dk, \tag{13}$$

where

$$F(k) \equiv K^2 \sqrt{-k^2} - \sqrt{k_t^2 - k^2}(1+K^2). \tag{14}$$

After calculations, one obtains expressions for the component of vector displacement on the surface $X_2 = 0$, for $kX_1 \gg 1$, to a first approximation

$$U_{3m}^{(1)} = -j \frac{K^2}{C_{44}^E (1+2K^2)} e^{jk_{BG}X_1} + \ldots \tag{15}$$

The wave number k corresponds in this case to the wave number of the transverse surface wave, for which the designation k_{BG} was introduced.

In the case of a nonmetallized surface the boundary conditions are the following:

$$T_{23}|_{X_2=0} = C_{44}^E \frac{\partial U_3^{(1)}}{\partial X_2} + e_{15}\frac{\partial \Phi}{\partial X_2} = \delta(X_1)e^{-j\omega t}, \tag{16}$$

$$e_{15}\frac{\partial U_3^{(1)}}{\partial X_2} - \varepsilon_{11}^S \frac{\partial \Phi}{\partial X_2}\bigg|_{X_2=0} = \varepsilon_0 \frac{\partial \Phi'}{\partial X_2}[1-\delta(X_1)], \tag{17}$$

$$\Phi|_{X_2=0} = \Phi'[1-\delta(X_1)], \tag{18}$$

where ε_0 is the dielectric constant of a vacuum, and Φ' is the electrical potential in a vacuum, satisfying Laplace's equation

$$\frac{\partial^2\Phi'}{\partial X_1^2} - \frac{\partial^2\Phi'}{\partial X_2^2} = 0, \tag{19}$$

$\Phi(X_1, X_2) \to 0$ when $X_2 \to -\infty$.

The remaining symbols are the same as before, using expressions (11) and (12), Φ' in the form

$$\Phi' = \int_{-\infty}^{+\infty} \Phi'(k)\exp\{j(-\sqrt{-k^2}X_2 + kX_1 - \omega t)\}\,dk, \tag{20}$$

and the boundary conditions (16)-(18), one obtains:

$$\psi(k) = \frac{1}{C_{44}^E 2\pi k}\,\frac{1}{\dfrac{K^2\sqrt{-k^2}}{\varepsilon_{11}^S/\varepsilon_0+1} - (1+K^2)\sqrt{k_t^2-k^2}}. \tag{21}$$

Inserting formula (21) into (10) and then into formula (7), one obtains

$$U_3^{(1)} = \frac{j}{2\pi C_{44}^E}\int_{-\infty}^{+\infty} \frac{e^{j(kX_1-\omega t)}}{F(k)}\,dk, \tag{22}$$

where

$$F(k) = (1+K^2)(\sqrt{k_t^2-k^2}) - \frac{K^2\sqrt{-k^2}}{\varepsilon_{11}^S/\varepsilon_0+1}.$$

Using the same method as in the first case, an expression for the vector component of the displacement on the surface $X_2 = 0$ is obtained, which proves the existence of a transverse surface wave of wave number k_{BG},

$$U_3^{(1)} = -\frac{jK^2(1+\varepsilon_{11}^S/\varepsilon_0)\,e^{jk_{BG}X_1}}{C_{44}^E\{(1+2K^2)(\varepsilon_{11}^S/\varepsilon_0+1)^2 + K^4\varepsilon_{11}^S/\varepsilon_0(\varepsilon_{11}^S/\varepsilon_0+2)\}}, \tag{23}$$

where

$$k_{GB} = \frac{kt}{[1-K^4/(1+K^2)^2\,1/(1+\varepsilon_{11}^S/\varepsilon_0]^{1/2}}$$

is the wave number of the transverse surface *BG* wave in the case of nonmetallized surface. It can be seen from formulae (15) and (23) that the source generates a surface wave only when the medium is piezoelectric ($K \neq 0$, $U_{31}^{(1)}$, $U_{3m} \neq 0$) The effectiveness of the radiation, as defined by the wave amplitude, depends on the coefficient of electromechanical coupling K (when K changes from 1/3 to 1, U_3 increases by a factor of 3). The effectiveness of the excitation in the case of a source placed on the surface of a metallized medium is about $\varepsilon_{11}/\varepsilon_0$ times greater than in the case of nonmetallized medium. Thus the wave amplitude on a metallized surface is approximately $\varepsilon_{11}^S/\varepsilon_0$ times greater than the wave amplitude on a nonmetallized surface. In the case of lithium iodate $LiIO_3$ the conditions are similar. The difference is that the terms $e_{14}(\partial\Phi/\partial X_1)$ and $e_{14}(\partial U_3/\partial X_2)$ occur in the boundary conditions (1), (16), and (17), and the piezoelectric constant e_{14} enters the expressions for the wave number.

3. A source in the form of vibrating plate on the surface of wave propagation

The coordinate system is shown in Fig. 2. In the case of a metallized surface, the boundary conditions are the following:

$$T_{23}|_{X_2=0} = C_{44}^E \frac{\partial U_{3m}^{(2)}}{\partial X_2} + e_{15}\frac{\partial \Phi}{\partial X_2} = \begin{cases} 0, & |X| > a, \\ e^{-j\omega t}, & |X| \leqslant a, \end{cases} \tag{24}$$

$$\Phi|_{X_2=0} = 0, \qquad \frac{e_{15}}{\varepsilon_{11}^S} U_{3m}^{(2)} + \varphi|_{X_2=0} = 0. \tag{25}$$

Using a similar calculation method, one can obtain the following formulae for the vector component of particle displacement on the surface $X_2 = 0$:

$$U_{3m}^{(2)} = -\frac{j2K^2}{C_{44}^E(1+2K^2)}\frac{\sin k_{BG}}{k_{BG}}\exp\{jk_{BG}X_1\}. \tag{26}$$

While for a nonmetallized surface one obtains:

$$U_3^{(2)} = \frac{2jK^2(1+\varepsilon_{11}^S/\varepsilon_0)\dfrac{\sin K_{BG}}{k_{BG}}\exp\{jk_{BG}X_1\}}{k_{BG}C_{44}^E\{(1+2K^2)(\varepsilon_{11}^S/\varepsilon_0+1)^2+K^2(\varepsilon_{11}^S/\varepsilon_0)(\varepsilon_{11}^S/\varepsilon_0+2)\}} + \ldots \tag{27}$$

It can be seen from formulae (26) and (27) that the conclusions for the first case are also valid for this case. The dependence of the amplitude of the transverse *BG* wave on the width of the surface of the source also occurs. Maxima and minima of the amplitude are seen, depending on the width of the source a. When

$$a = \frac{2n+1}{k_{BG}}\frac{\pi}{2}, \qquad n = 0, 1, 2, \ldots, \tag{28}$$

the amplitude is a maximum, and when

$$a = \frac{n}{k_{BG}^2} \frac{\pi}{2}. \tag{29}$$

4. A source in the form of plate vibrating on a surface perpendicular to the surface of wave propagation

The coordinate system is shown in Fig. 3. It is assumed that the *BG* wave propagates from the surface $X_1 = 0$ to the surface $X_2 = 0$ with a certain transmission coefficient N. In this case a change of the energy of the transverse wave into the energy of *BG* wave occurs close to the line at which the surface $X_2 = 0$ crosses the surface $X_1 = 0$.

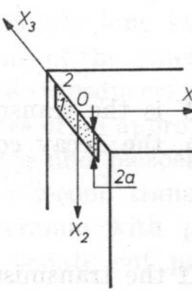

Fig. 3. A plate source on the surface perpendicular to the surface of wave propagation

1 – plate source, *2* – the surface of wave propagation

If the surface is metallized, two sources of strain are placed on the surface $X_2 = 0$, satisfying the following boundary conditions; for the first source

$$T_{23}|_{X_2=0} = C_{44}^E \frac{\partial U_{3m}^{(3)}}{\partial X_2} + e_{15} \frac{\partial \Phi}{\partial X_2} = \delta(X_1)\left(\frac{1-e^{-2a\alpha}}{\alpha}\right) e^{-j\omega t}, \tag{30}$$

where α is a coefficient of decay. It is assumed that it is equal to the decay coefficient of the *BG* wave, i.e.

$$\frac{K^2}{1+K^2} k_{BG}, \tag{31}$$

$$\Phi|_{X_2=0} = \frac{e_{15}}{\varepsilon_{11}^S} U_{3m}^{(3)} + \varphi|_{X_2=0} = 0. \tag{32}$$

For the second source

$$T_{23}|_{X_2=0} = C_{44}^E \frac{\partial U_{3m}^{(3)}}{\partial X_2} + e_{15} \frac{\partial \Phi}{\partial X_2} = \begin{cases} 0, & x > 2a,\ x < 0, \\ Ne^{-j\omega t}, & x \leqslant 2a, \end{cases} \tag{33}$$

$$\Phi|_{X_2=0} = \frac{e_{15}}{\varepsilon_{11}} U_{3m}^{(3)} + \varphi|_{X_2=0} = 0. \tag{34}$$

After calculation, the following results are obtained for the vector component of particle displacement on the surface $X_2 = 0$:

$$U_{3m}^{(3)} = -\frac{j2K^2}{C_{44}(1+2K^2)}\left[\left(\frac{1-e^{2aa}}{a}\right)p\right]e^{jk_{BG}X_1}, \tag{35}$$

where

$$p = \left|\frac{1}{jk_{BG}}(e^{jk_{BG}2a}-1)N'\right|, \tag{36}$$

and N' is the transmission coefficient in the case of metallized surface.

For a nonmetallized surface one obtains

$$U_3^{(3)} = \frac{-jK^2(1+\varepsilon_{11}^S/\varepsilon_0)e^{jk_{BG}X_1}}{C_{44}^E\{(1+2K^2)(\varepsilon_{11}^S/\varepsilon_0-1)^2+K^4(\varepsilon_{11}^S/\varepsilon_0)(\varepsilon_{11}^S/\varepsilon_0-2)\}}, \tag{37}$$

where

$$p' = \left|\frac{1}{jk_{BG}}(e^{jk_{BG}2a}-1)\right|N. \tag{38}$$

N is the transmission coefficient in the case of a nonmetallized surface, and a, the decay coefficient, is given by

$$a' = \frac{K^2}{(1+K^2)(1+\varepsilon_{11}^S/\varepsilon_0)}k_{BG}. \tag{39}$$

If the transmission coefficients N and N' are very low, the second term in formulae (35) and (37) can be neglected. Then the amplitude of the BG wave on the surface $X_2 = 0$ is inversely proportional to the decay coefficient $U_{3m}^{(3)} \sim \sim 1/a$, $U_3^{(3)} \ll 1/a'$. Thus the optimum transducer height is given by:

$$2a_{\text{opt}} = \frac{1}{a} \quad \text{(metallized)},$$

$$2a'_{\text{opt}} = \frac{1}{a} \quad \text{(nonmetallized)}.$$

For $a' \ll a$ and $a'_{\text{opt}} \gg a_{\text{opt}}$, the ratio of the amplitude interfering bulk wave to that of the BG wave is very large in the case of a nonmetallized surface, and thus the excitation effectiveness of the wave is very small.

If the transmission coefficients N and N' are large, the first terms in formulae (35) and (37) can be neglected, and the amplitude of the BG wave on the surface $X_2 = 0$ is proportional to p or p', i.e. it depends on the width of the source.

5. Experimental methods for the generation and detection of transverse waves

It follows from the theoretical investigations that the transverse surface wave in the case of a free surface penetrates in the piezoelectric material and has a velocity close to the velocity of the bulk wave, so that in practice the two waves cannot be distinguished [6]. Covering the propagation surface with a thin and weightless conductor changes the conditions of wave propagation in favour of the surface wave. A rise in the pulse transmitted by this wave and a decrease in the wave propagation velocity can be observed. This is caused by a formation on the surface of the piezoelectric material, of a layer of decreased stiffness, due to the compensation of the inner piezoelectric field. The experimental investigation of the generation and propagation of transverse surface waves is usually performed in the presence of a conducting surface, but even with a conducting surface, the separation of the surface *BG* wave from the bulk *SH* wave is a difficult but vital problem, which is not always appreciated by experimentors. This problem also occurs in the use of interdigital (IDT) transducers [8], and the impossibility of differentiating the pulses of the *BG* and *SH* waves can lead to incorrect conclusions.

As a result of a small difference in velocity, the separation of pulses of *BG* and *SH* waves can be achieved in the case of a sufficiently long sample. In the experimental phase of the present work, investigations of the generation and detection of the surface wave were performed using plate transducers placed on the sides of the sample as shown in **Fig. 3**. Acoustic pulses of an appropriate frequency were generated using an electrical pulse generator and piezoelectric transducers. The presence of the waves was detected by a second transducer connected to an oscilloscope. Samples of piezoelectric ceramic with polarisation parallel to the propagation plane, and of lithium iodate cut parallel to the z axis, were used in the investigations.

The presence of the *BG* wave can be found by exerting mechanical pressure on the surface of wave propagation or better still by covering this surface with viscous resin. The pressure causes damping of the *BG* pulse (Fig. 4) which is not observed in the bulk wave. In the case a conductive surface, the surface wave propagates in the layer below the surface, while the bulk *SH* wave propagates more deeply. This phenomenon can be used for the separation of the pure *BG* wave.

It was found possible to obtain a separated, pure transvers surface *BG* wave on the samples of piezoelectric material, with a dimension of 46 mm, using piezoelectric transducers contiguous to the sample along a line (Fig. 5). This contact can be considered as a linear wave source, and it can also be realized by a slight inclination of the lateral planes of the sample.

It can be seen from Fig. 6 that the surface wave is almost completely attenuated by the viscous resin placed on the propagation surface. However, in the case of a nonmetallized surface, only the bulk *SH* wave occurs in practice and it is not attenuated by the resin, Fig. 6.

The excitation of the *BG* wave by a transducer placed on the propagation surface is shown in Fig. 7. Moving the transmitting transducer over the surface causes the position of the pulse on the time axis of the oscilloscope to shift in direct proportion to the path length of the wave. The measured wave velocity shows that a surface (*BG*) wave occurs.

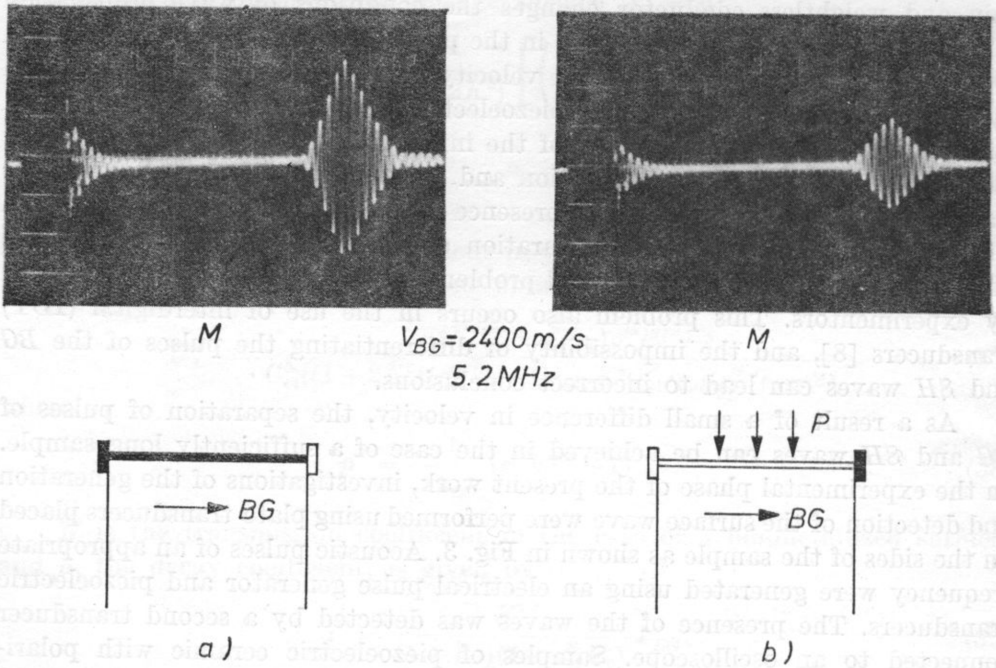

Fig. 4. The effect of pressure on the amplitude of the transverse surface wave on lithium iodate with a metallized surface

a – the pulse without loading on the surface, b – the pulse on the loaded surface

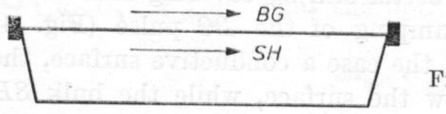

Fig. 5. The manner of exciting a pure acousto-electrical (BG) wave of a piezoelectric ceramic

Generally, separation of the transverse surface (BG) wave is easier in lithium iodate. This is connected with the properties of the surface layer. In this case, this layer seems to reach deeper and in this connection the transducer generating the wave is within the region of the surface layer.

6. Conclusions

It follows from the theoretical consideration and from the measurements performed, that it is possible to obtain pulses of pure transverse surface waves using plate transducers, despite the fact that the velocity of the surface wave is only slightly different from the velocity of the bulk wave, and superposition of the pulses can occur with short samples and wide transducers. Excitation of surface waves with plate transducers is relatively easy and can be used in the investigation of new piezoelectric materials, and of those materials whose surface layers are changed by technological processes [11]. Separation of pure surface waves is also significant in the investigation of their properties [10].

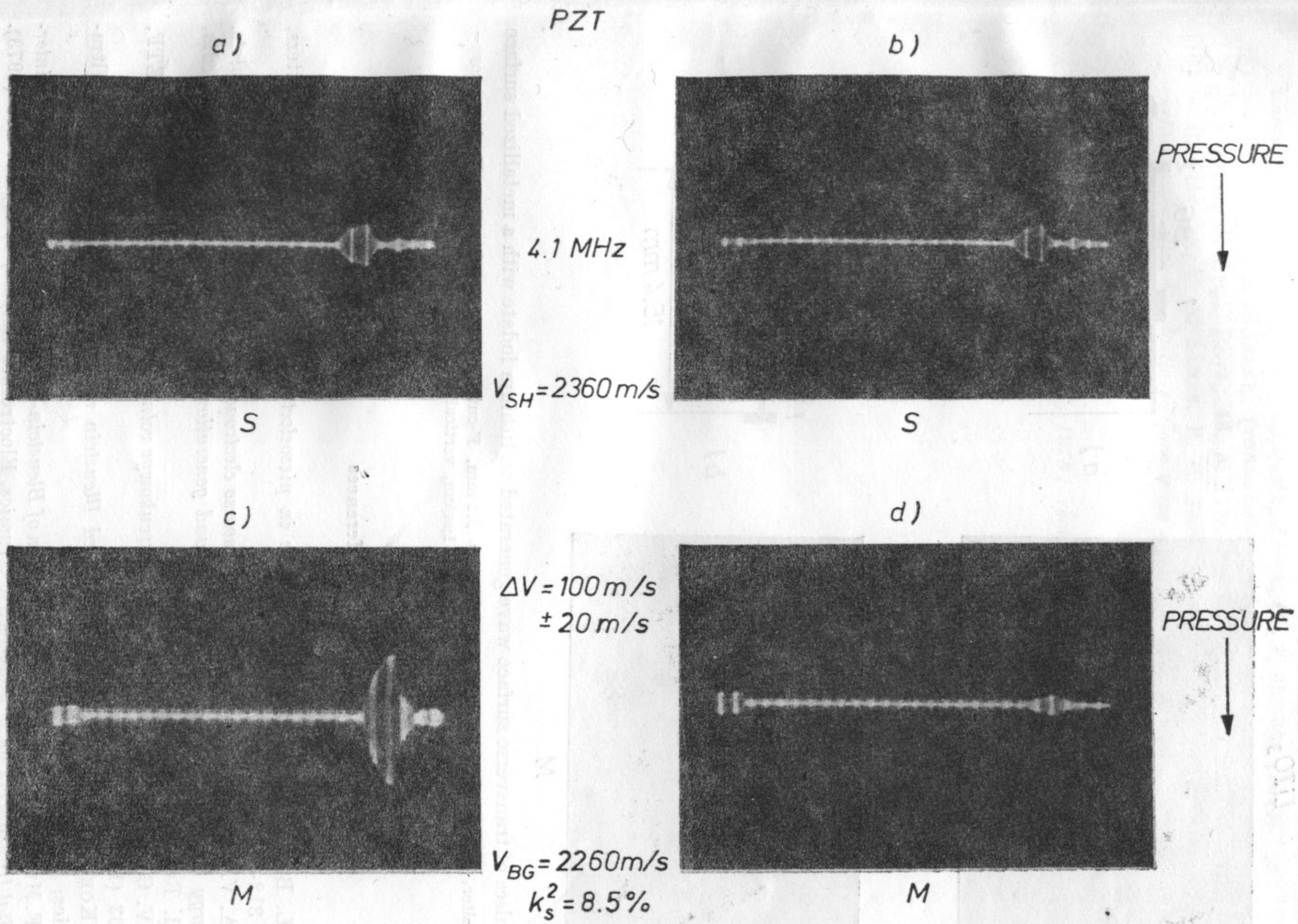

Fig. 6. Pulses of the transverse bulk wave (a-b) and acoustoelectric *BG* wave (c-d) at a frequency of 4.1 MHz excited as in Fig. 5

a – a nonmetallized ceramic surface, b – a nonmetallized surface loaded with a liquid resin, c – a metallized surface, d – a metallized surface loaded with resin indices in 1μs, voltage in 0.5×0 V/cm

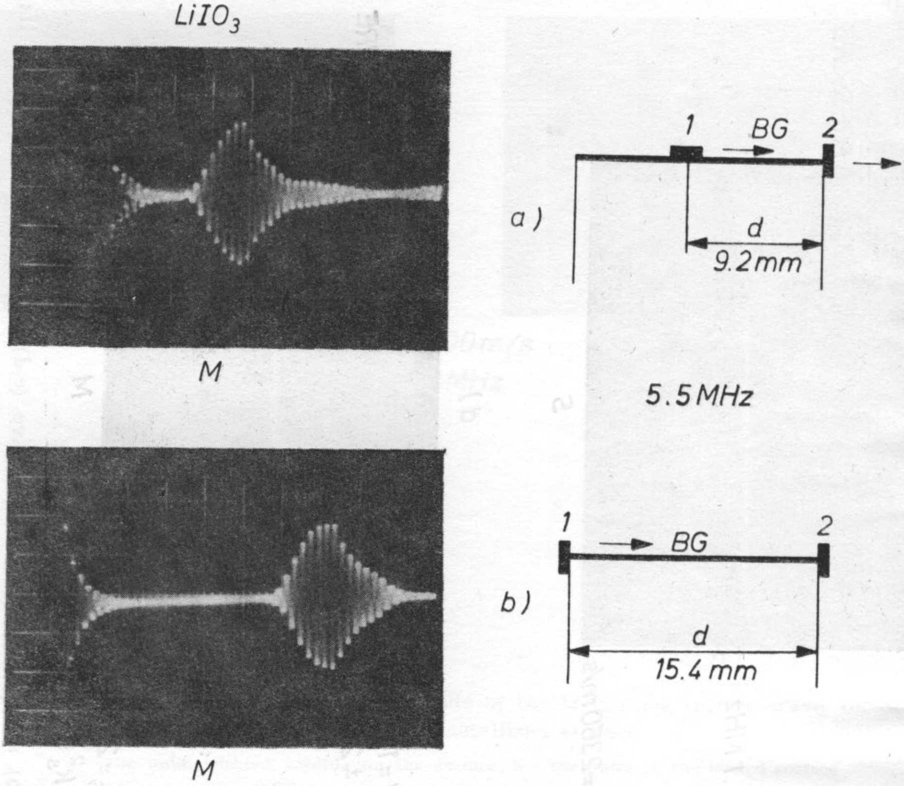

Fig. 7. Pulses of transverse surface wave generated on lithium iodate with a metallized surface

1 – transmiter, *2* – receiver, d = 4.6 mm, (3-4) d = 9.2 mm, Y-cut, propagation direction X, frequency – 5.5 MHz, horizontal scale 1μs/cm, vertical scale 0.1 V/cm

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