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AN ANALYSIS OF PULSED ULTRASONIC TRANSMIT-RECEIVE SYSTEMS FOR MEDICAL DIAGNOSTICS

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This paper presents a method for the calculation of the shape and size of pulses radiated and received by ultrasonic transmitting-receiving systems used in medical diagnostics. Using an equivalent electrical circuit for the transducer, the transfer functions for different working conditions of the transducer were calculated and, on the basis of these functions the acoustical and electrical behaviour was calculated. Because of the complicated nature of the mathematical relations, continuous Fourier transform (CFT) was used to describe the systems. This was then replaced by a discrete Fourier transform (DFT), thus preparing the relations for numerical calculations. The DFT was in turn calculated using the fast Fourier transform (FFT). Calculations were made for a number of practically realized cases. The paper also gives the results obtained from an analysis of the operation of a transmitting transducer for different types of acoustic matching, for a wedged transducer, and for a divided one.

Introduction

An intensively increasing interest in the ultrasonic methods used in medicine can now be observed all over the world. Medical applications require increasingly better diagnostic apparatus. This involves the problem of constructing ultrasonic transmitting-receiving systems which include an electrical transmitter, broad-band transducers, and an electrical receiver. This problem is closely connected with the problems of transient states in piezoelectric transducers.

Since a mathematical description of a transducer in a transient state is quite complicated, papers devoted to this problem do not exhaust the possibilities of definning the optimum design of acousto-electronic systems for the generation and detection of short ultrasonic pulses.

The transient state in a transducer, which results from its large Q — factor and a simultaneous lack of matching to the medium, plays an important role in the problems of signal resolution and the more exact measurement of distance.

The aim of the present work was the design of a method for calculation of the shape and size of the pulses in ultrasonic transmitting-receiving systems used in medical diagnostic apparatus, and to apply this method to a number of practically realized cases.

Analytical method

One of methods for the analysis of the performance of ultrasonic transducers is based on the use of the equivalent electrical circuits of the transducer, and the investigation, on this basis, of the relations between the input and the output [5, 8]. The transducer can then be represented as an electric four-pole network. Irrespective of the inner structure of the system, the work of a linear four-pole network in a steady state, can be characterized by two output and two input quantities, interlinked by linear relations. On the basis of these relations, the transfer function, H, of the four-pole network (which can be used subsequently for the investigation of transient states), may be determined.

For any excitation $f_{we}(t)$, under zero initial conditions, the output signal $f_{wy}(t)$ is defined by the relations [3, 15]: after Laplace transform, it has the form

$$f_{wy}(t) = \alpha^{-1}\{H(s)\cdot F_{we}(s)\},$$

where

$$F_{we}(s) = \alpha\{f_{we}(t)\}, \tag{1}$$

or, after Fourier transform,

$$f_{wy}(t) = \mathscr{F}^{-1}\{H(f)\cdot F_{we}(f)\},$$

where

$$F_{we}(f) = \mathscr{F}\{f_{we}(t)\}. \tag{2}$$

In the present paper, the Laplace transform was used for the simpler cases where the possibility of analytical calculation of the relations existed, whereas Fourier's transform, using the insertion $s = j\omega = j\cdot 2\pi f$, was used for the more complicated cases.

In the determination of the inverse Fourier transforms, the confinuous Fourier transform (CFT) was replaced by the discrete transform (DFT), using the relations describing the continuous and discrete transform and sampling theory [1, 3] (see Appendix). The DFT was subsequently calculated numerically, using the FFT algorithm (Fast Fourier Transform).

Assumptions

An electrical transmitter with an input impedance Z_n is taken to excite the transmitting transducer with the voltage E_n. This transducer radiates an acoustic force F_n into a biological medium. It was assumed that the biological medium did not introduce damping and had an acoustic impedance of $1.5 \cdot 10^6$ kg/m^{2}s (soft tissue is the most frequently investigated biological medium, for which this is the first approximation). The receiving transducer was placed in the near field of the transmitting transducer. In consequence, acoustic field problems were eliminated from the considerations. Diffraction losses were also neglected. It was assumed that this transducer receives the acoustic force F_{od} and converts it to a voltage E_{od} which is the input signal of the electric receiver which has an input impedance z_{wej}.

The analysis of the electrical systems of the transmitter and the receiver was performed using the theory of circuits, based on the designs of the ultrasonograph and ultrasonocardiograph developed in the Department of Ultrasonics IPPT PAN, with subsequent linearization. The phenomena occurring in the electrical receiver were not considered, its effect being limited to that of the first degree input impedance on the behaviour analyzed. The parameters of the tables connecting the transducers with the receiver and the transmitter were also considered.

For comparison of the sizes of the pulses generated by individual transmitting-receiving (T-R) systems, the criterion of the power of the signal received should be used. Since the comparison was performed at a fixed value of electric resistance loading the transducer, a voltage scale was used. This is particularly appropriate since the shapes of the pulses are observed on the oscilloscope screens of diagnostic devices in the forms corresponding to voltages. The time interval, after which the instantaneous value of the signal falls to 10 % of the maximum value, was assumed as pulse duration, although this criterion can be disputed in the case of very irregular pulse shapes.

Transmitting transducer

Fig. 1 shows an equivalent circuit for the transmitting transducer, vibrating in a thickness mode, with consideration of the exciting source and the shunt coil L_0. Transforms of the input quantities are connected with the transforms of the output quantities by the relation

$$\begin{bmatrix} \bar{E}_n \\ \bar{I}_n \end{bmatrix} = [A^n] \begin{bmatrix} \bar{F}_n \\ \bar{V}_n \end{bmatrix}, \tag{3}$$

where $\bar{E}_n$ and $\bar{I}_n$ are transforms of voltage and current, and $\bar{F}_n$ and $\bar{V}_n$ those of force and velocity. $[A^n]$ is a matrix describing the electric system.

On the basis of equation (3), the transfer function of the transmitting system can be determined [5],

$$\bar{E}_n = A_{11}^n \bar{F}_n + A_{12}^n \bar{V}_n, \tag{4}$$

since $\bar{F}_n/\bar{V}_n = z_B A_0$, where z_B is the impedance of the biological medium,

$$H_n(s) = \frac{V_n}{E_n} = \frac{1}{A_{11}^n z_B A_0 + A_{12}^n}. \tag{5}$$

Knowing, in turn, the transfer function, the response of the system to any excitation can be determined, using relation (1) or (2).

Two extreme cases, for which the input electric impedance of the generator is either very large or very small, were assumed as the first approximations to the operation of the transmitting transducer itself (without the compensating inductance).

In the first case the compensation of clamped capacities C_0 and $-C_0$ occurs (when the electrical terminals are open), with the matrix $[A^n]$ assuming the form

$$[A^n] = \begin{bmatrix} \frac{1}{N} & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{6}$$

where (Fig. 1)

$$\begin{aligned} N &= hC_0 = \sqrt{2f_0 C_0 \varrho_0 c_0 A_0}, \\ a_{11} &= [(m_A+1)e^{s\tau_0} - (m_A-1)e^{-s\tau_0}/M, \\ a_{12} &= z_0 A_0[(m_A+1)e^{s\tau_0} + (m_A-1)e^{-s\tau_0}]/M, \\ a_{21} &= (e^{s\tau_0} - e^{-s\tau_0})/(z_0 A_0 M), \\ a_{22} &= [(m_A+2)e^{s\tau_0} - (m_A-2)e^{-s\tau_0} - 4]/M, \\ M &= (m_A+1)e^{s\tau_0} - (m_A-1)e^{-s\tau_0} - 2, \\ m_A &= z_A/z_0. \end{aligned} \tag{7}$$

Inserting relations (6) and (7) into relation (5) we get the function

$$\begin{aligned} H_n(s) = V_0[(m_A+1)e^{s\tau_0} - (m_A-1)e^{-s\tau_0} - 2] \times \\ \times [(m_A+1)(m_B+1)e^{s\tau_0} - (m_A-1)(m_B-1)e^{-s\tau_0}]^{-1}; \\ V_0 = N/z_0 A_0, \quad m_B = z_B/z_0. \end{aligned} \tag{8}$$

If we consider the excitation $\mathbf{1}(t)$ on the electrical side, then, after using relation (1), we obtain relation (9), defining the wave of acoustic velocity:

$$\begin{aligned} V(t) = V_0\Big\{\frac{1}{m_B+1}\mathbf{1}(t) - \frac{2}{(m_A+1)(m_B+1)}\mathbf{1}(t-\tau_0) - \\ - \frac{2(m_A-1)}{(m_A+1)^2(m_B+1)}\mathbf{1}(t-2\tau_0) + \ldots\Big\}. \end{aligned} \tag{9}$$

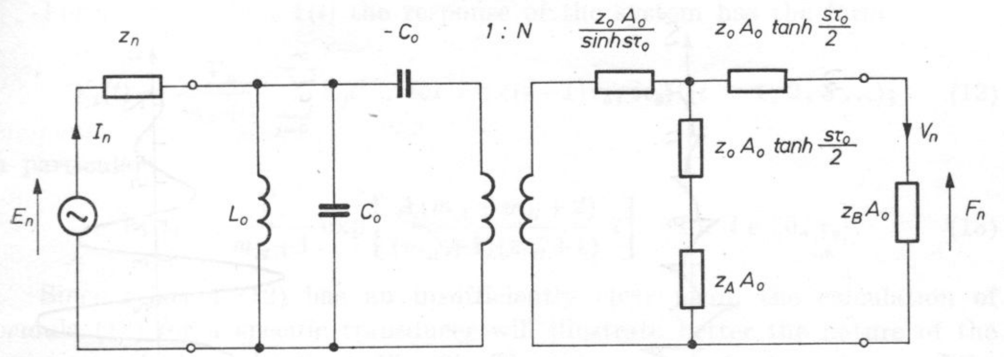

Fig. 1. An equivalent circuit for a piezoelectric transmitting transducer (after Mason) and its electric supply circuit

$z_0 = \varrho_0 c_0$, $z_A = \varrho_A c_A$, $z_B = \varrho_B c_B$ are the acoustic impedances of the transducer, backing load, and the biological medium, respectivelly; c_0 is the ultrasonic wave velocity in the transducer for an electric displacement equal to zero, A_0 is the area of the transducer, $N = hC_0 = \sqrt{2f_0 C_0 \varrho_0 c_0 A_0}$, $\tau_0 = 1/2f_0$ is the passage time taken for an ultrasonic wave to pass through the open-circuit transducer, f_0 is the frequency of the mechanical resonance, h is a piezoelectric constant, C_0 is the clamped capacity, E_n, I_n are the voltage and current supplying the transducer, z_n is the output impedance of the electrical transmitter, and F_n, V_n are the acoustic force and velocity generated by the transducer

Fig. 2a is an illustration of the result. The response of the system depends only on the purely mechanical properties of the transducer. On both faces: transducer – backing and transducer – biological medium, reflections from the mechanical impedances and partial radiation of the wave into the medium [6] occur every τ_0, i.e. subsequent waves are radiated every τ_0, which have the shape of the excitation. As a result of superposition of these waves an acoustic signal occurs.

In the second case, when the transducer is excited from a generator with very small output impedance, the matrix $[A^n]$ has the form:

$$[A^n] = \begin{bmatrix} 1 & -\dfrac{1}{sC_0} \\ sC_0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{1}{N} & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{10}$$

with the transfer function for such a transducer being thus given by relation (11),

$$\begin{aligned} H_n(s) = {} & V_0[(m_A+1)e^{s\tau_0}-(m_A-1)e^{-s\tau_0}-2]\times \\ & \times\Big\{(m_A+1)(m_B+1)e^{s\tau_0}-(m_A-1)(m_B-1)e^{-s\tau_0}- \\ & -\frac{A}{s}[(m_A+m_B+2)e^{s\tau_0}-(m_A+m_B-2)e^{s\tau_0}-4\Big\}^{-1}, \end{aligned} \tag{11}$$

where

$$A = \frac{N^2}{z_0 A_0 C_0}, \quad V_0 = \frac{N}{z_0 A_0}.$$

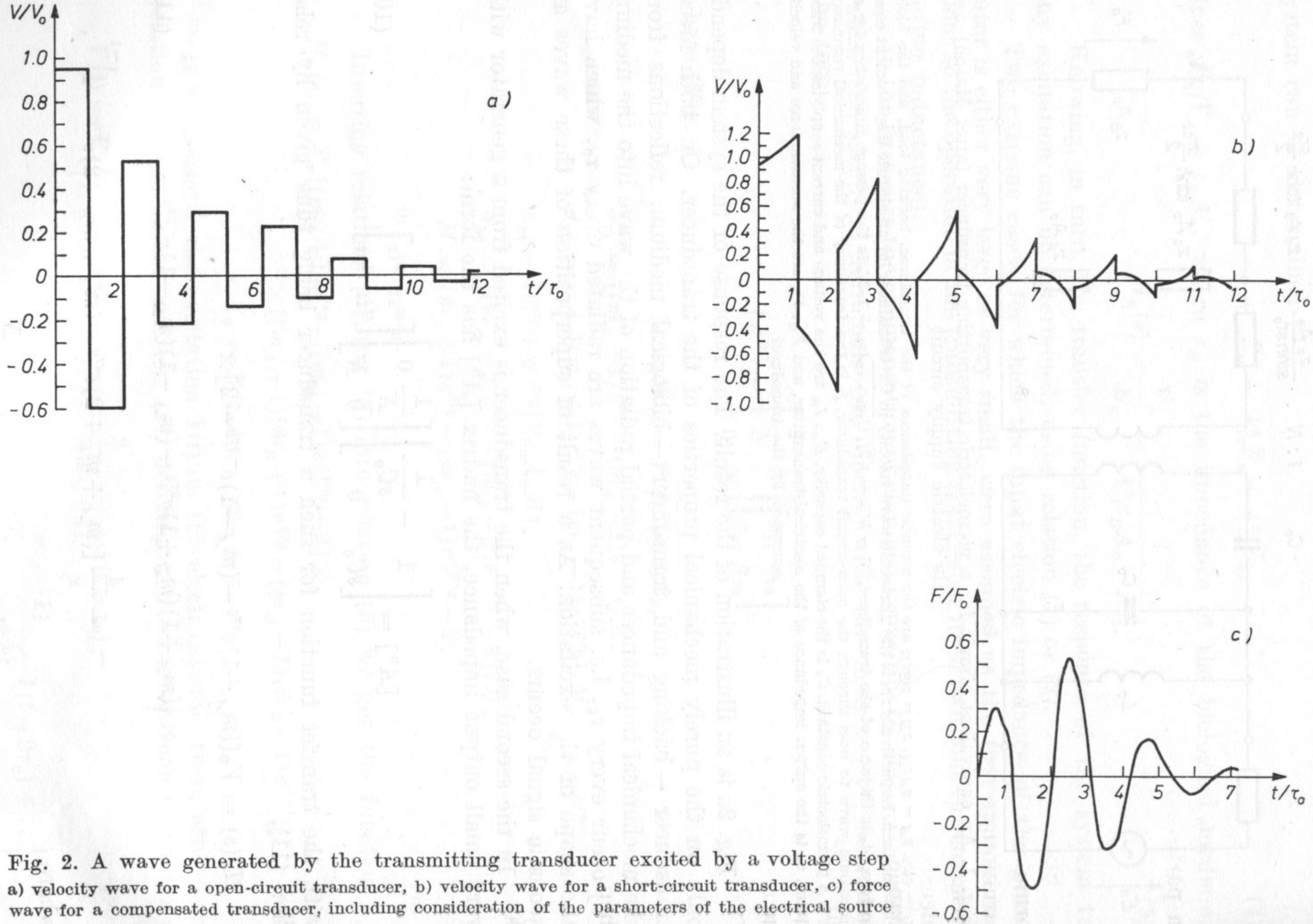

Fig. 2. A wave generated by the transmitting transducer excited by a voltage step
a) velocity wave for a open-circuit transducer, b) velocity wave for a short-circuit transducer, c) force wave for a compensated transducer, including consideration of the parameters of the electrical source

For the excitation $\mathbf{1}(t)$ the response of the system has the form

$$V_i(t) = \frac{V_0}{m_B+1} \sum_{j=0}^{\infty} b_{ij} t^j \quad \text{for } t \in \langle (i-1)\tau_0, i\tau_0) \ (i = 1, 2, 3 \ldots); \tag{12}$$

in particular

$$V_1(t) = \frac{V_0}{m_B+1} \exp\left[\frac{A(m_A+m_B+2)}{(m_A+1)(m_B+1)} t\right] \quad \text{for } t \in \langle 0, \tau_0). \tag{13}$$

Since relation (12) has an insufficiently clear form, the calculation of formula (12) for a specific transducer will illustrate better the nature of the variation, which is shown in Fig. 2b. The response of the transducer to the excitation $\mathbf{1}(t)$ is a sum of the purely mechanical response (open-circuit transducer – (9)) and of some additional function, which is responsible for the change of shape from the rectangular. Pulses are generated every τ_0 on both surfaces of the transducer, but they no longer have the shape of the excitation, but rather a shape which depends on the effect of the electrical side of the equivalent circuit of the transducer on the mechanical side. For a time $t \in \langle 0, \tau_0)$, the response of the transducer is given by relation (13). Analogously to electrical systems, the idea of a time constant can be introduced,

$$\tau' = R'C' = \frac{(m_A+1)(m_B+1)}{A(m_A+m_B+2)} = \frac{(R_A+R_0)(R_B+R_0)}{R_A+R_B+2R_0} \left(\frac{-C_0}{N^2}\right), \tag{14}$$

where $R_0 = z_0 A_0$, $R_A = z_A A_0$, and $R_B = z_B A_0$ are acoustic impedances, i.e.

$$R' = (R_A+R_0) \| (R_B+R_0)$$

(the transducer acts as such a resistance for $t \in \langle 0, \tau_0)$;

$$C' = C_0/N^2$$

(a negative capacitance transformed to the mechanical side).

Since $\tau' < 0$, the output for $t \in \langle 0, \tau_0)$ will increase exponentially, with a time constant for this increase equal to $R'C'$.

With consideration of the electrical source and the shunt coil L_0 (which compensates C_0 at the frequency f_0), we have

$$[A^n] = \begin{bmatrix} 1+\dfrac{z_n}{sL_0} & z_u \\ \dfrac{1}{sL_0} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\dfrac{1}{sC_0} \\ sC_0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{1}{N} & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{15}$$

while the transfer function of the transmitting transducer becomes:

$$H_n(s) = m_B N [(m_A+1) e^{s\tau_0} - (m_A-1) e^{-s\tau_0} - 2] \times$$
$$\times \left\{ \left(1 + \frac{z_n}{sL_0} + z_n s C_0\right) [(m_A+1)(m_B+1) e^{s\tau_0} - (m_A-1) \times \right.$$

$$\times (m_B - 1) e^{-s\tau_0}] - \frac{A}{s}\left(1 + \frac{z_n}{sL}\right)[(m_A + m_B + 2) \times$$
$$\left. \times e^{s\tau_0} - (m_A + m_B - 2) e^{-s\tau_0} - 4]\right\}^{-1}. \qquad (16)$$

Fig. 2c shows the wave of force generated by the same transducer as before, but this time including the parameters of the transmitter, according to formula (16), with the excitation, as before, of a voltage step. In this case multiple reflections depend on the impedance "seen" an the borders of the transducer. This impedance is formed by both mechanical and electrical elements, with the shape of response ultimately obtained depending on the superposition of the waves occurring as a result of multiple reflections. The calculation results shown in Fig. 2, were obtained for a transducer with the parameters: $z_0 = 27.0 \cdot 10^6$ kg/m²s, $z_A = 6.3 \cdot 10^6$ kg/m²s, $f_0 = 2.13$ MHz, $C_0 = 1730$ pF, and a diameter of 2 cm; while, for the case in Fig. 2c, $z_n = 100\ \Omega$ was assumed.

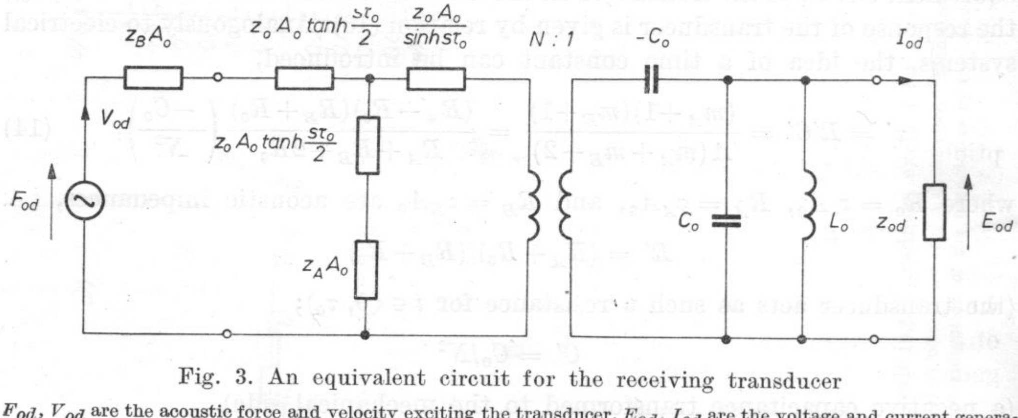

Fig. 3. An equivalent circuit for the receiving transducer

F_{od}, V_{od} are the acoustic force and velocity exciting the transducer, E_{od}, I_{od} are the voltage and current generated by the transducer, and z_{od} is the imput impedance of the electrical receiver

The circuit of the receiving transducer loaded by the electric impedance z_{od} is shown in Fig. 3. The relations describing the system are given by formulae (17) and (18):

$$\begin{bmatrix} \bar{F}_{od} \\ \bar{V}_{od} \end{bmatrix} = [A^{od}] \begin{bmatrix} \bar{E}_{od} \\ \bar{I}_{od} \end{bmatrix}, \qquad (17)$$

where

$$[A^{od}] = \begin{bmatrix} 1 & zA_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix} \begin{bmatrix} 0 & \frac{-1}{sC_0} \\ sC_0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{sL_0} & 1 \end{bmatrix}. \qquad (18)$$

The transfer function of the receiving system was obtained from (17) and (18), in the form:

$$H_{od}(s) = \frac{\bar{E}_{od}}{\bar{F}_{od}} = \frac{z_{od}}{A_{11}^{od} z_{od} + A_{12}^{od}} = \frac{N z_{od}}{z_0 A_0} [(m_A+1)e^{s\tau_0} - (m_A-1)e^{-s\tau_0} - 2] \Big\{ \Big(sC_0 z_{od} + \frac{z_{od}}{sL_0} + 1\Big) [(m_A+1)(m_B+1)e^{s\tau_0} - (m_A-1)(m_B-1)e^{-s\tau_0}] - \frac{A}{s} \Big(1 + \frac{z_{od}}{sL_0}\Big) [(m_A+m_B+2)e^{s\tau_0} - (m_A+m_B-2)e^{-s\tau_0} - 4] \Big\}^{-1}. \quad (19)$$

The transfer function of the receiving transducer $H_{od}(s)$ has the same form as the transfer function of the transmitting transducer; a similar shape for the voltage being generated by the receiving transducer, as for the shape of the force generated by the transmitting transducer (see Fig. 2c).

The transmitting-receiving system

If we consider the transmitting-receiving system described in the assumptions, then the transfer function of such a system is given by the formula

$$H(s) = \frac{\bar{E}_{od}}{\bar{E}_n} = 2H_n(s)H_{od}(s), \quad (20)$$

where $H_n(s)$ and $H_{od}(s)$ are defined by (16) and (19), respectively. On the basis of these relations, the behaviour of the receiving transducer in any T-R arrangement and for any transducer, can be calculated.

The effect of the shape of the excitation on the shape of the voltage obtained for a transducer with the parameters $z_0 = 25 \cdot 10^6$ kg/m²s, $f_0 = 2.72$ MHz, $C_0 = 1260$ pF, loaded at the back with an acoustic impedance of $z_A = 6.1 \cdot 10^6$ kg/m²s, compensated with an inductance L_0, and shunted in the transmitter with the resistance $R = 6.8$ kΩ, was analyzed for the T-R system of the ultrasonograph. All the results presented subsequently in this paper, with the exception of those for the divided transducer, are presented for a transducer with the parameters given above. The amplitude of the exciting voltage was 250 V.

A Heaviside function was assumed as the first approximation to the shape of the excitation

$$E_n(t) = -E_0 \cdot \mathbf{1}(t) \quad (21)$$

with the results being shown in Fig. 4a.

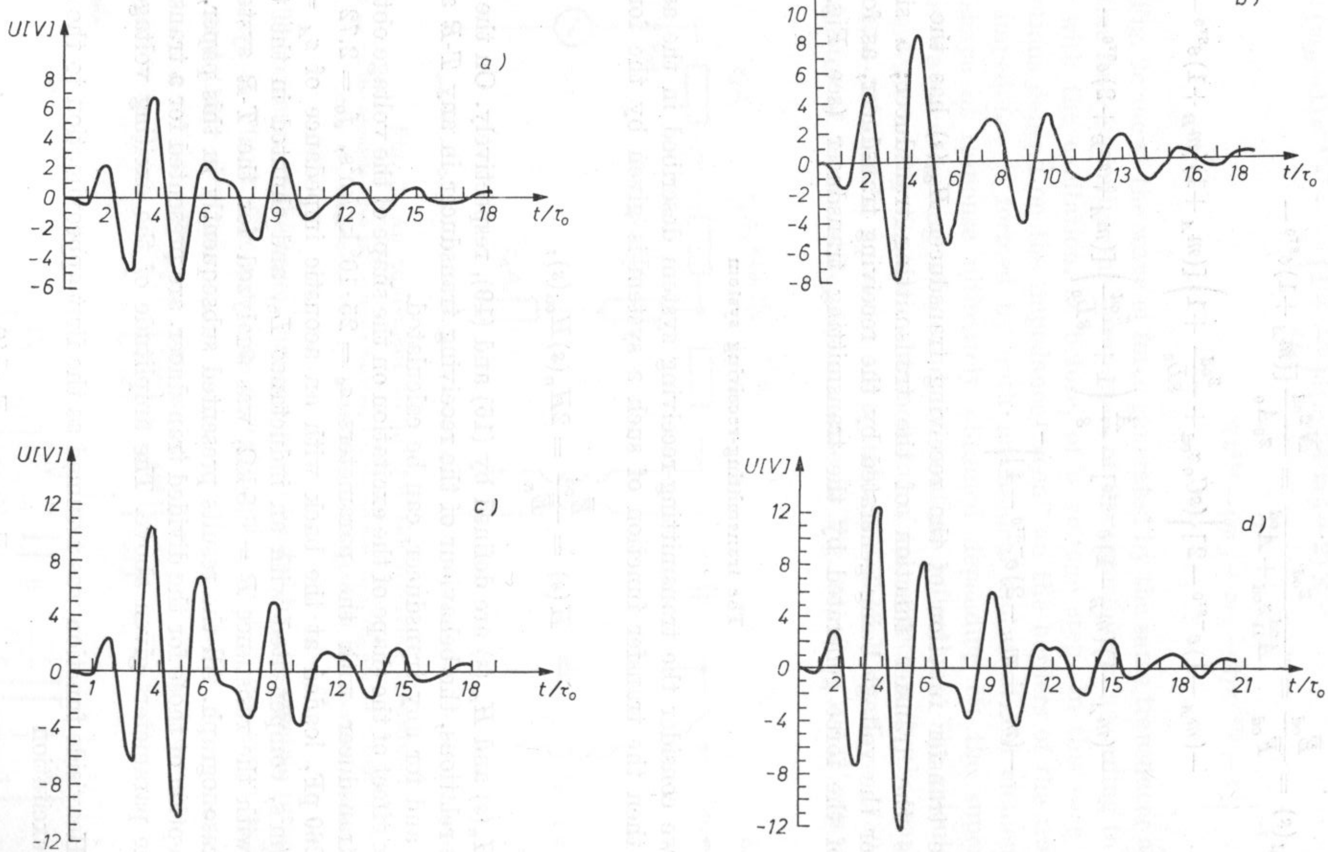

Fig. 4. Voltage received in the *T-R* system of the ultrasonograph for different excitations

a) a Heaviside time function, b) a linear increase in time to a constant value, c) a rectangular pulse of width τ_0, d) a trapezoidal pulse

As the actual excitations in the ultrasonograph have a finite rise time, an excitation closer to reality, which is described by the following function, was considered as the second excitation:

$$E_n(t) = \begin{cases} \frac{-E_0}{\tau_0} t & \text{for } t < \tau_0, \\ -E_0 & \text{for } t \geqslant \tau_0. \end{cases} \tag{22}$$

The shape of voltage obtained for this type of excitation is shown in Fig. 4b.

In the case of excitation with a rectangular pulse of width τ_0,

$$E_n(t) = -E_0[\mathbf{1}(t) - \mathbf{1}(t-\tau_0)], \tag{23}$$

the response of the system has the form shown in Fig. 4c, and that for a trapezoidal pulse of duration τ_0 and decreasing from a value E_0 to $\frac{3}{4} E_0$,

$$E_n(t) = \begin{cases} -E_0\left(-\frac{t}{4\tau_0} + 1\right) & \text{for } t < \tau_0, \\ 0 & \text{for } t \geqslant \tau_0, \end{cases} \tag{24}$$

is shown in Fig. 4d.

A number of excitation types (a rectangular pulse of duration $2\tau_0$, a double Heaviside function, a trapezoidal pulse) have also been analyzed, but this paper gives only the results which are more interesting from a practical point of view.

The shape of the pulse obtained depends on the multiple reflections occurring as a result of a lack of matching of the transducer on both electrical and acoustical sides. The excitation of the transmitting transducer causes the generation of an acoustical pulse whose shape is different from the shape of the excitation. The acoustic behaviour is the result of the superposition of successive waves generated every τ_0. In turn, the acoustical signal causes the generation of an electrical signal by the receiving transducer.

Since a number of changes occur in the instantaneous value of the acoustic behaviour, each causing the generation of a number of waves in the receiving transducer, the duration of the signal received increases even more.

In a diagnostic apparatus the signal received should have as large an amplitude and as short a duration as possible. A change in the onset time between the limits $0 < t < \tau_0$ affects the amplitude of the signal received, having, however, no larger effect on the duration. It follows from the calculations performed that the largest amplitude is obtained for excitation with a rectangular pulse of width τ_0 (see Table 1).

Illustration of the effect of the resistance shunting the head in the transmitter was analyzed for the ultrasonocardiograph system in which it is possible to adjust this resistance. A $\mathbf{1}(t)$ of amplitude of 600 V was assumed as an excitation. The results presented in Figs. 5a,b,c are for $R = 3.3\ \mathrm{k}\Omega$, $110\ \Omega$, and $20\ \Omega$, respectively. The following conclusion can be drawn on the basis of the

Table 1. Parameters of the voltage pulse received in the *T-R* system for different excitations

No.	shape of exciting voltage	maximum voltage received U_{max}[V]	duration t [s]	Fig.
1	a Heaviside time function	6.8	13.8 τ_0	4a
2	a linear increase to a constant value	8.8	13.9 τ_0	4b
3	a rectangular pulse of width τ_0	12.5	13.7 τ_0	4c
4	a trapezoidal pulse of witdh τ_0	10.9	13.0 τ_0	4d

results obtained: the durations of the transmitted and received pulses can be changed by a change in the shunting resistance. However, an optimum value of shunting resistance exists, for which the duration will decrease if a suitable sensitivity of the head is preserved (see Table 2). The shape of the pulse generated also changes. A component of higher frequency, which is difficult to explain, occurs for a low value of the shunting resistance R.

Table 2. Duration of the pulse received in the *T-R* system for different shunting resistances

No.	shunting resistance R	duration t [s]	Fig.
1	ultrasonograph 6.8 kΩ	13.8 τ_0	4a
2	ultrasonocardiograph 3.3 kΩ	17.9 τ_0	5a
3	110 Ω	10 τ_0	5b

The effect of acoustic loads

Since a large mismatch occurs between the transducer and the biological medium, the possibility of improving the operating conditions on the acoustical side was analyzed.

One solution may be the introduction of a matching layer between the transducer and the biological medium.

The requirements for matching are that the layer should have a thickness of $\lambda/4$ and an acoustic impedance equal to the geometrical mean of the impe-

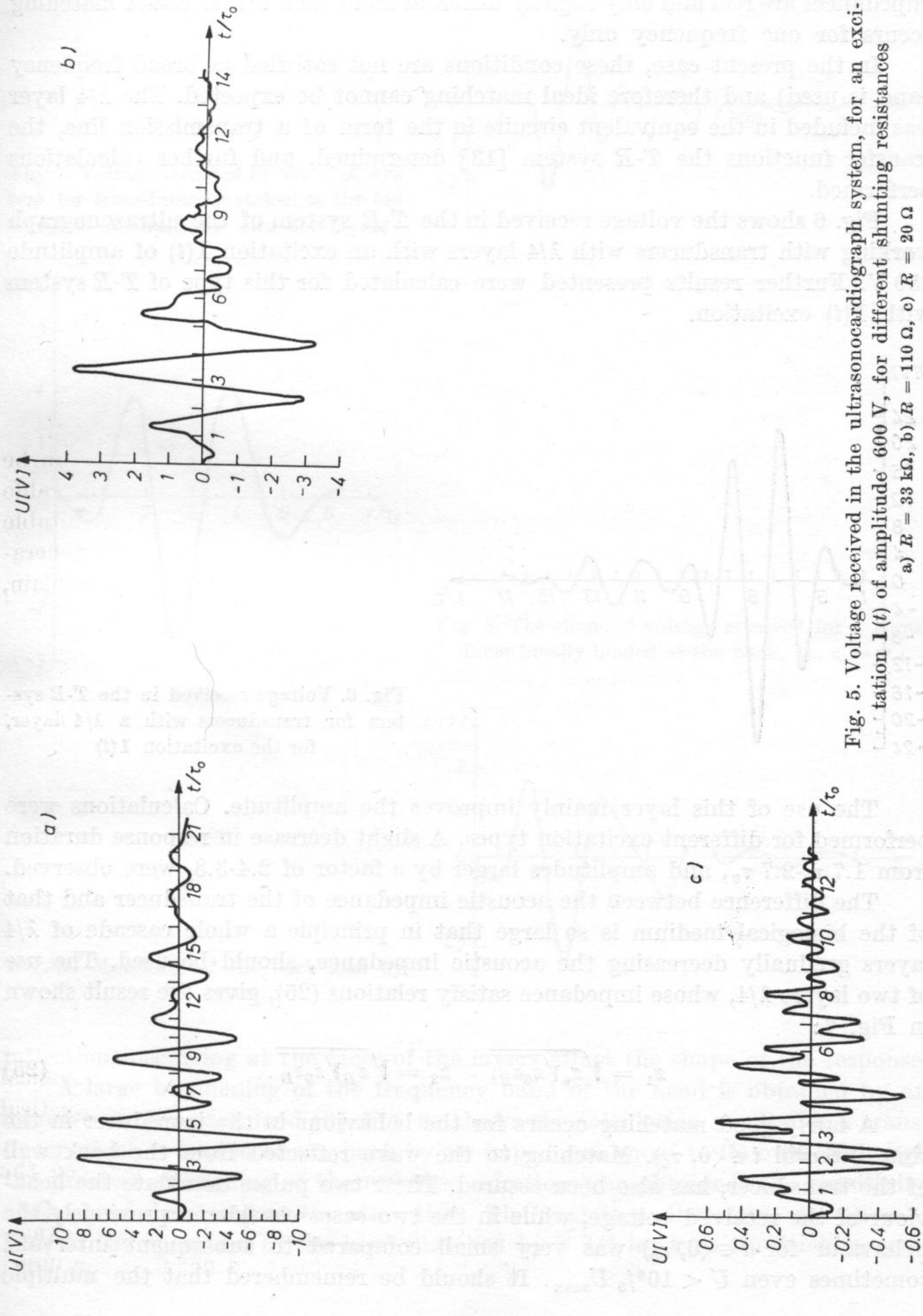

Fig. 5. Voltage received in the ultrasonocardiograph system, for an excitation $\mathbf{I}(t)$ of amplitude 600 V, for different shunting resistances a) $R = 33$ kΩ, b) $R = 110$ Ω, c) $R = 20$ Ω

dances to be matched [4, 13]. A $\lambda/4$ layer best matches those two media hwose impedances are real and only slightly different from each other. Exact matching occurs for one frequency only.

In the present case, these conditions are not satisfied (a broad frequency band is used) and therefore ideal matching cannot be expected. The $\lambda/4$ layer was included in the equivalent circuits in the form of a transmission line, the transfer functions the *T-R* system [13] determined, and further calculations performed.

Fig. 6 shows the voltage received in the *T-R* system of the ultrasonograph working with transducers with $\lambda/4$ layers with an excitation $\mathbf{1}(t)$ of amplitude 250 V. Further results presented were calculated for this type of *T-R* system with $\mathbf{1}(t)$ excitation.

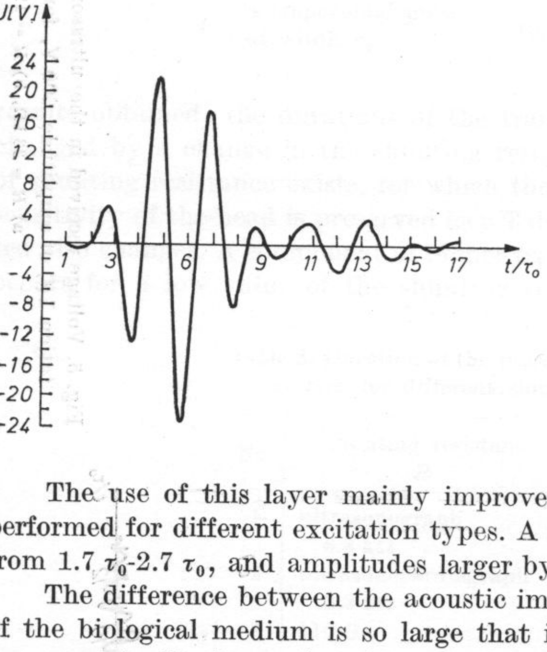

Fig. 6. Voltage received in the *T-R* system for transducers with a $\lambda/4$ layer, for the excitation $\mathbf{1}(t)$

The use of this layer mainly improves the amplitude. Calculations were performed for different excitation types. A slight decrease in response duration from 1.7 τ_0-2.7 τ_0, and amplitudes larger by a factor of 2.4-3.8, were observed.

The difference between the acoustic impedance of the transducer and that of the biological medium is so large that in principle a whole cascade of $\lambda/4$ layers gradually decreasing the acoustic impedance, should be used. The use of two layers $\lambda/4$, whose impedance satisfy relations (25), gives the result shown in Fig. 7:

$$z_1 = \sqrt{z_0\sqrt{z_0 z_B}}, \quad z_2 = \sqrt{z_B\sqrt{z_0 z_B}}. \tag{25}$$

A fairly good matching occurs for the behaviour in the transducer in the time interval $t \in \langle 0, \tau_0)$. Matching to the wave reflected from the back wall of the transducer, has also been assured. These two pulses dominate the behaviour of the received voltage, while in the two cases considered previously the behaviour for $t \in \langle 0, \tau_0)$ was very small compared to subsequent intervals, sometimes even $U < 10\%\ U_{\max}$. It should be remembered that the multiple

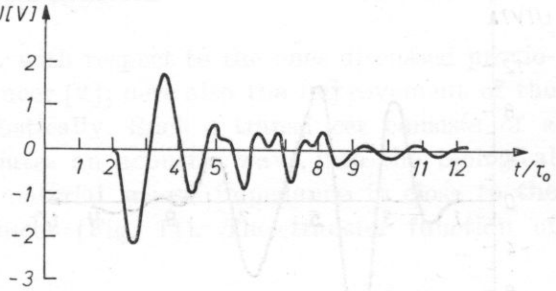

Fig. 7. Voltage received in the *T-R* system for transducers matched to the biological medium by two $\lambda/4$ layers

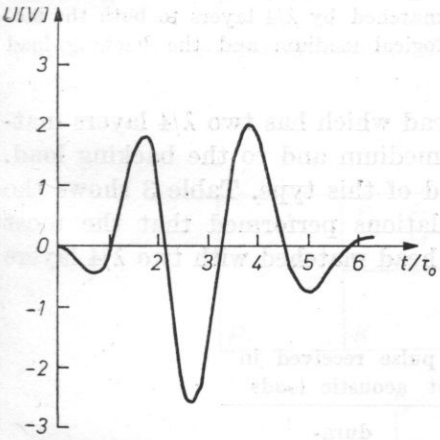

Fig. 8. The shape of voltage received for a transducer ideally loaded at the back, i.e. $z_0 = z_A$

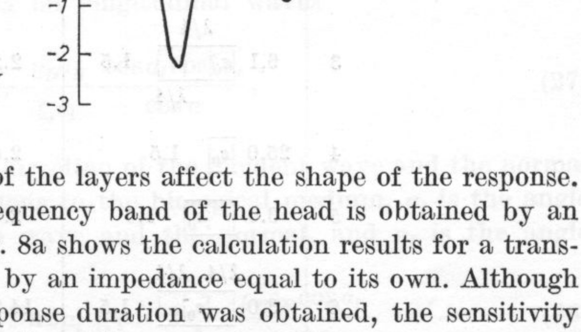

Fig. 9. Matching of the back load by a $\lambda/4$ layer

reflections occurring at the faces of the layers affect the shape of the response.

A large broadening of the frequency band of the head is obtained by an increase in the back load [9]. Fig. 8a shows the calculation results for a transducer which is loaded at the back by an impedance equal to its own. Although the greatest shortening of the response duration was obtained, the sensitivity of the head decreased at the same time. The next step, therefore, was an analysis of the operation of the head matched by a $\lambda/4$ layer to the backing load (Fig. 9).

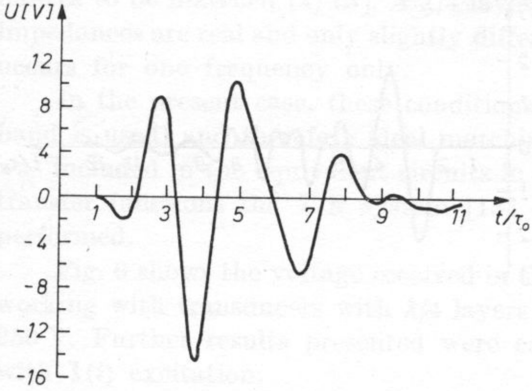

Fig. 10. Voltage received by a head matched by $\lambda/4$ layers to both the biological medium and the backing load

The result of these considerations is a head which has two $\lambda/4$ layers matching the transducer to both the biological medium and to the backing load. Fig. 10 shows the voltage received by a head of this type. Table 3 shows the results obtained. It follows from the calculations performed that the most suitable working conditions are assured by a head matched with two $\lambda/4$ layers (Table III, p. 6).

Table 3. Parameters of the pulse received in the *T-R* system for different acoustic loads

No.	acoustic load 10^6 [kg/m²·s]	U_{max} [V]	duration t [s]	Fig.
1	6.1 $\lvert z_0\rvert$ 1.5	6.8	13.8 τ_0	4a
2	6.1 $\lvert z_0\rvert$ $\lambda/4$ 1.5	23.1	11.1 τ_0	6
3	6.1 $\lvert z_0\rvert$ $\lambda/4$ $\lambda/4$ 1.5	2.2	8.1 τ_0	7
4	25.0 $\lvert z_0\rvert$ 1.5	2.6	5 τ_0	8
5	3.0 $\lvert z_0\rvert$ 1.5	2.4	10 τ_0	9
6	3.0 $\lambda/4$ $\lvert z_0\rvert$ $\lambda/4$ 1.5	14.8	6.8 τ_0	10
7	6.1 $\lvert z_0\rvert$ 33.0 \ 1.5	5.2	6.3 τ_0	13

$z_0 = 25\cdot 10^6$ kg/m²s

A wedged transducer

A completely different solution, with respect to the ones discussed previously, is the design of wedged transducer [2]; here also the improvement of the working conditions is achieved acoustically. Such a transducer consists of a conventional transducer which radiates an acoustic wave into the biological medium though a wedge made of material whose impedance is close to the acoustic impedance of the transducer (Fig. 11). The transfer function of the *T-R* system has the form

$$H_K(s) = E_{ll}H(s), \tag{26}$$

where E_{ll} is the transmittance of the echo.

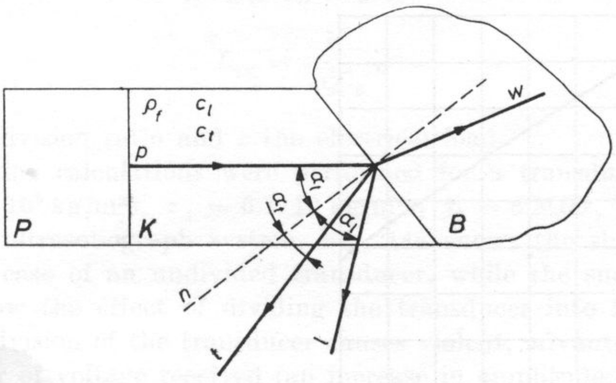

Fig. 11. A wedged transducer

P – transducer, K – matching wedge, B – biological medium (water), n – normal to the border wedge – water interface, p – the incident wave, l, t – reflected transverse and longitudinal waves, w – wave penetrating into the water (longitudinal), c_l, c_t – velocities of longitudinal and transverse waves in the wedge, ϱ_f – density of the wedge

For the *T-R* system working on longitudinal waves

$$E_{ll} = \frac{4}{N'}\frac{\varrho_B c_B}{\varrho_f c_l}\frac{\cos a_l \cos^2 a_t}{\cos a}, \tag{27}$$

where a is the angle between the direction of the incident wave and the normal to the plane of the edge, contiguous to the biological medium, a_t is the angle between the reflected transverse wave and the normal, and a_l is the angle

$$N' = \left(\frac{c_t}{c_l}\right)^2 \sin 2a_l \sin 2a_t + \cos^2 2a_t + \frac{\varrho_B c_B \cos a_l}{\varrho c_l \cos a}. \tag{28}$$

Brass was chosen as the material for the wedge.

Attenuation in brass is very small and can be neglected in this case. The transmittance of the echo calculated for a transverse wave for the media: brass – water is given in Fig. 12. The lengths of one distances of the wedge are selected so that in the receiving transducer, the transverse wave does not fall on the plane of the transducer. For the wedged transducer designed, the shape of the voltage received in the *T-R* system of the ultrasonograph, for a $\mathbf{1}(t)$ excitation was calculated, and the results are shown in Fig. 13. The duration of the response was nearly halved, with only a slight decrease in the value of the maximum voltage, compared to the conventional transducer (cf. Fig. 4a).

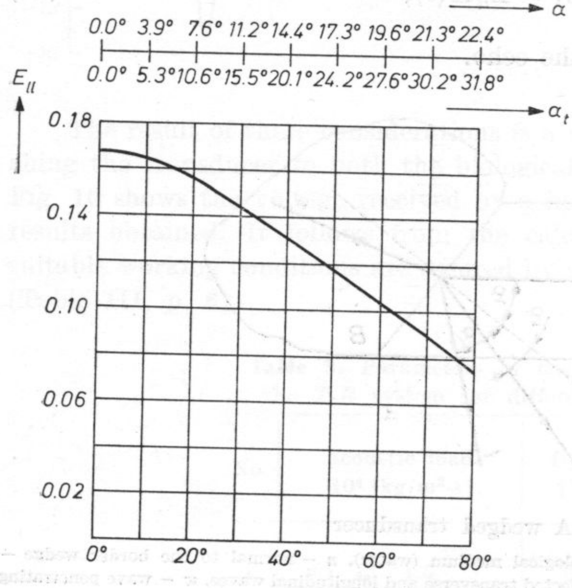

Fig. 12. Transmittance of the echo at the interface of brass – water

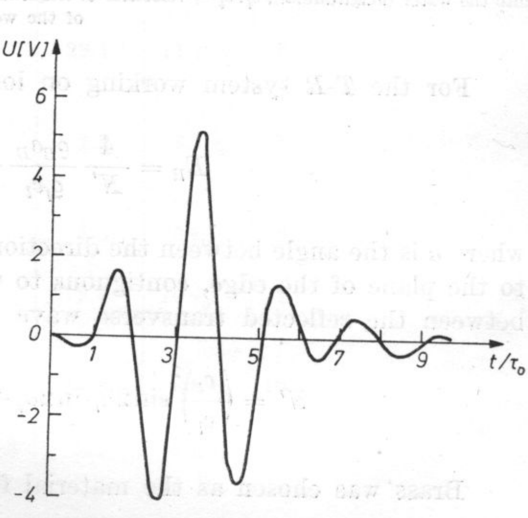

Fig. 13. The shape of voltage received in the *T-R* system working with wedge transducers

the parameters of the wedge; $z_k = 33 \cdot 10^6$ kg/m²s, the cone distances of the wedge: 55 mm and 77.5 mm ($\alpha_l = 40°$)

A divided transducer

One of the methods for matching the transducer to the electrical system at higher frequencies (the frequencies 8-12 MHz are used in ophtalmoscopy) is a division of the transducer and a serial connection of its parts. Thus an increase of the resistance of the transducer at resonance is achieved. The transmitting or receiving transfer function has the form

$$H' = \frac{N}{k}\frac{z}{z_0 A_0}[(m_A+1)e^{s\tau_0}-(m_A-1)e^{-s\tau_0}-2]\left\{\left[\left(1+\frac{z}{sL_{0K}}\right)k+\frac{zsC_0}{k}\right]\times\right.$$
$$\times[(m_A+1)(m_B+1)e^{s\tau_0}-(m_A-1)(m_B-1)e^{-s\tau_0}]-\frac{A}{s}\left(1+\frac{z}{sL_{0K}}\right)\times \quad (29)$$
$$\left.\times k[(m_A+m_B+2)e^{s\tau_0}-(m_A+m_B-2)e^{-s\tau_0}-4\right\}^{-1}.$$
$$L_{0K} = \frac{k^2}{\omega_0^2 C_0},$$

where k is the division ratio and z the electrical load.

This time, the calculations were performed for a transducer with parameters $z_0 = 25\cdot 10^6$ kg/m²s, $z_A = 6.1\cdot 10$ kg/m²s, $f_0 = 8$ MHz, $C_0 = 2930$ pF; working in the ultrasonograph system. Fig. 14a shows the shape of voltage received in the case of an undivided transducer, while the successive results (Fig. 14b-d) show the effect of dividing the transducer into 2, 3, 4, 5 parts.

An initial division of the transducer causes violent, advantageous changes in the behaviour of voltage received (an increase in amplitude and a decrease in duration). There is a certain optimum division ratio, k, which assures best matching, beyond which further division of the transducer can cause distortions of amplitude and phase to occur in the response (see Table 4). In a specific system, the division cannot be arbitrary, but such as assures a matching of resistance of the transducer and the electrical system at the resonanse frequency.

Table 4. Parameters of the pulse received in the *T-R* system in the case of a divided transducer

No.		maximum voltage received U_{max} [V]	duration t [s]	Fig.	Remarks
1	transducer undivided	2.03	$21.7\,\tau_0$	14a	
2	2	9.84	$15.4\,\tau_0$	14b	
3	3	19.1	$14.7\,\tau_0$	14c	
4	4	23.4	$11.6\,\tau_0$	14d	occurrence of distortions
5	5	23.5	$9.7\,\tau_0$	14e	

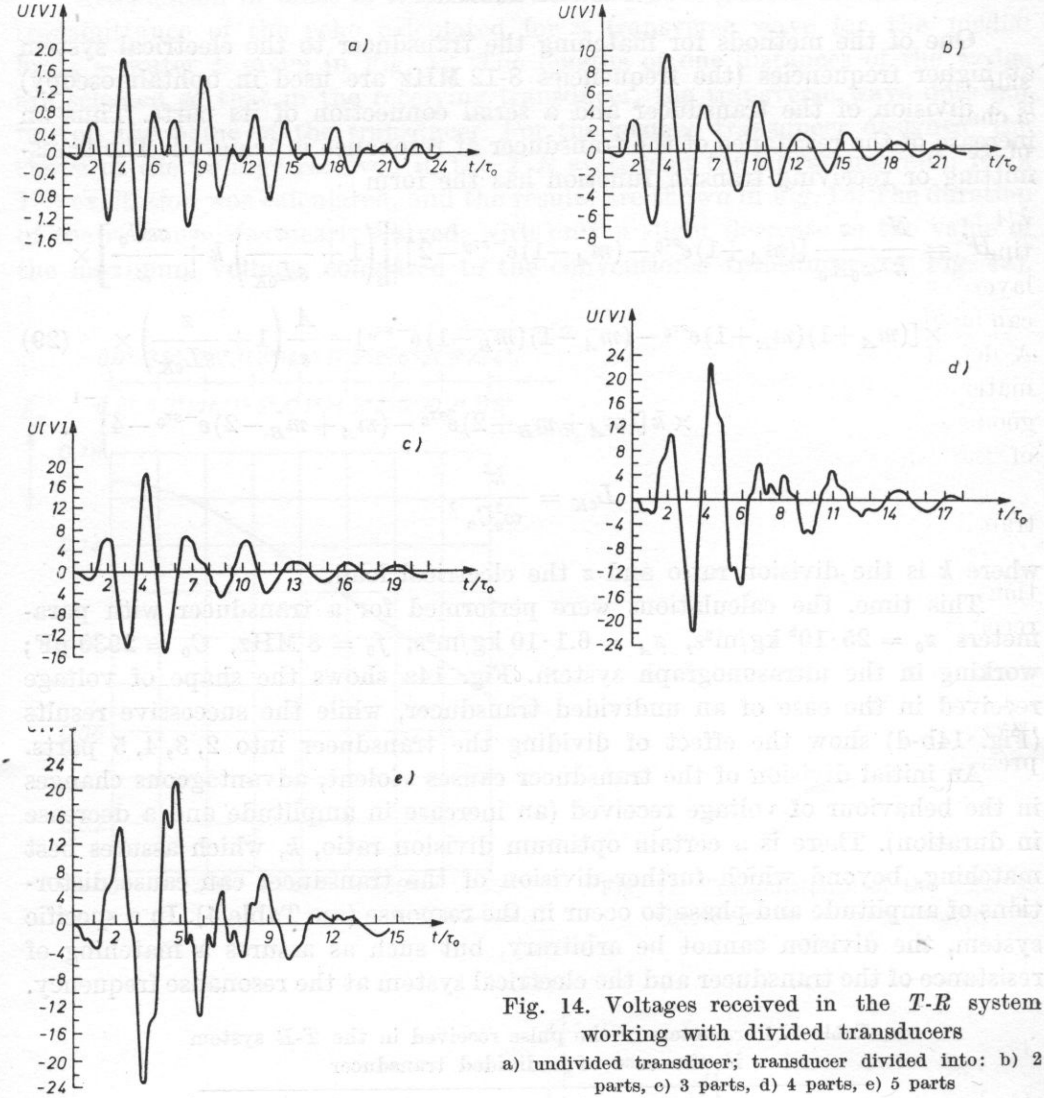

Fig. 14. Voltages received in the *T-R* system working with divided transducers

a) undivided transducer; transducer divided into: b) 2 parts, c) 3 parts, d) 4 parts, e) 5 parts

Conclusions

The method presented numerical calculation, using the FFT and circuit theory, appears to be relevant for the analysis of ultrasonic heads for different present combinations of parameters of the transducer and the *T-R* systems. Using the method described, a certain number of different desing solutions of heads were analyzed. From the results a number of conclusions can be formulated.

The multiple reflections occurring at both faces of the transducer have a decisive effect on the responses generated by and received by the head.

The selection of a suitable shape of electrical excitation on the transmitting side can improve the amplitude conditions of the whole *T-R* system. However, a change in the output impedance of the transducer not only changes the values of the maximum voltage received, but also its duration, in a broad range.

A matching of the transducer to the biological medium with just one $\lambda/4$ layer only slightly decreases the duration of the response, but at the same time increases the maximum value of voltage received. The use of two $\lambda/4$ layers advantageously decreases the duration of the pulse. Such a solution can be used particularly where we are interested in the early part of the response. A decrease in response duration can be achieved by increasing the backing matching directly, or by matching the transducer to it with a $\lambda/4$ layer. Fairly good results are obtained for a head working with two $\lambda/4$ layers on both sides of the transducer.

At higher frequencies, fairly good results can be obtained using a divided transducer.

The use of a wedged transducer considerably improves the working conditions, compared to a conventional transducer, decreasing the duration of signal received, while preserving almost the same sensitivity of the head.

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Appendix

In the numerical elaboration of the analytical method, the following relations [1, 3] were used.

The continuous Fourier transform was defined in the following manner:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt,$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} dt,$$

$$h(t) \rightleftarrows H(f).$$

The principles of discrete representation of the function and its transform are formulated by the sampling function. In the time domain it has the form

$$\bigwedge_{f>f_c} H(f) = 0 \Rightarrow \hat{h}(t) = h(nT) \sum_{n=-\infty}^{\infty} \delta(t-nT) \wedge T = \frac{1}{2f_c},$$

where $\hat{h}(t)$ is a discrete function defining a continuous function $h(t)$, n is the number of samples, $\delta(t)$ is the Dirac function. However, in the frequency domain it is defined by the formula

$$\bigwedge_{t>T_c} h(t) = 0 \Rightarrow \hat{H}(f) = H\left(\frac{n}{2T_c}\right) \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{2T_c}\right)$$

and where $\hat{H}(f)$ is a discrete function defining a continuous function $H(f)$.

Discrete Fourier transform is defined by the following relations.

If $g(kT)$ is a sampled periodic function,

$$g(kT) = g[(rN+k)T], \quad r = 0, \pm 1, \pm 2, \ldots,$$

then discrete Fourier transformation is given by

$$G\left(\frac{n}{NT}\right) = \sum_{k=0}^{N-1} g(kT)e^{-j2\pi nk/N}, \quad n = 0, +1, +2, \ldots$$

with $G(n/NT)$ also being a periodic function:

$$G\left(\frac{n}{NT}\right) = G\left[\frac{(rN+n)}{NT}\right], \quad r = 0, \pm 1, \pm 2, \ldots$$

The inverse discrete Fourier transform has the form:

$$g(kT) = \sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right)e^{j2\pi nk/N}, \quad k = 0, +1, +2, \ldots,$$

$$g(kT) \rightleftarrows G\left(\frac{n}{NT}\right).$$

If the discrete function $\hat{h}(t)$ approximates a continuous function $h(t)$, defined in the range $t \in \langle 0, L)$, then both continuous and discrete functions agree, with an exactness of a constant and

$$\hat{H}(nf_0) = LH\left(\frac{n}{NT}\right),$$

where H is the discrete Fourier transform, and $\hat{H}$ the discrete function approximating a continuous Fourier transform.