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ACOUSTIC IMPEDANCE OF AN ISOTROPIC MEDIUM FOR RAYLEIGH WAVES

ALEKSANDER OPILSKI, MARIAN URBAŃCZYK

Institute of Physics, Silesian Polytechnic (44-100 Gliwice)

The acoustic impedance of an isotropic non-piezoelectric medium has been determined for Rayleigh waves. The numerical values of this impedance are very different from the values of the impedance of the medium for a plane bulk wave.

1. Introduction

Analytical solutions of the problems of elastic surface wave propagation are known only for a limited number of half space configurations (Lamb, 1904), and for the simple layered media (Ewing at al. 1957). The problems connected with wave propagation on the surface of a bounded medium containing step discontinuities are complex in so far as it is difficult to give analytical expressions describing the behaviour of Rayleigh waves in these cases. Only the solutions for a single discontinuity (Tuan, 1974) are known. Nevertheless the problem is important in view of the wider application of systems with surface wave containing wave-guide discontinuities in such electronic devices as resonators, bandpass filters, code filters etc.

The properties of complex wave-guides can be easily examined with the use of equivalent circuits. The quantity characterizing a wave-guide medium is the acoustic impedance. The conditions of surface acoustic wave propagation of the Rayleigh type differ from those of plane bulk waves. The characteristic impedance of the medium for this type of wave differs from the analogous quantity for bulk waves.

In this paper, the acoustic impedance of Rayleigh waves in an isotropic, non-piezoelectric medium has been determined.

2. The acoustic impedance of Rayleigh waves in a plane surface layer ($x_3 \to 0$)

Let us assume that the wave propagates in the x_1 direction, in the x_1x_2 plane. The x_3 axis is directed towards the centre of the medium which is isotropic and non-piezoelectric. The component displacements of particles in the medium from the state of equilibrium will take the form [1]:

$$\begin{aligned} u_1 &= [Aike^{-\alpha x_3} - \beta Be^{-\beta x_3}]e^{i(kx_1-\omega t)}, \\ u_2 &= 0, \\ u_3 &= [-\alpha Ae^{-\alpha x_3} - Bike^{-\beta x_3}]e^{i(kx_1-\omega t)}, \end{aligned} \tag{1}$$

where α, β are decay constants,

$$\alpha^2 = k^2 - \omega^2/a_1^2, \quad \beta^2 = k^2 - \omega^2/a_2^2, \tag{2}$$

a_1 and a_2 are the velocities of the bulk longitudinal and transverse waves respectively, ω is the angular frequency, and k is the Rayleigh wave number.

Using the condition of the stress vanishing on the free surface of the medium [1],

$$\sigma_{3i} = 0, \tag{3}$$

we shall express amplitude B by amplitude A in formulae (1). The components of stress acting in the medium will have the form:

$$T_1 = \sigma_{11} + \sigma_{31}, \quad T_2 = \sigma_{22}, \quad T_3 = \sigma_{13} + \sigma_{33}. \tag{4}$$

In the case of a thin layer ($x_3 \to 0$)

$$T_1 = \sigma_{11}, \quad T_2 = \sigma_{22}, \quad T_3 = 0. \tag{5}$$

Denoting the stress tensor by the strain tensor for the isotropic medium [2],

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \tag{6}$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j, \\ 0 & \text{for } i \neq j, \end{cases}$$

and λ, μ are the Lame coefficients, we obtain:

$$\begin{aligned} T_1 &= -(2\mu+\lambda)Ak^2\left[1-\frac{2\alpha\beta}{k^2+\beta^2}\right]e^{ikx_1} + \lambda A\alpha^2\left[1-\frac{2k^2}{k^2+\beta^2}\right]e^{ikx_1}, \\ T_2 &= A\lambda\left\{\alpha^2\left[1-\frac{2k^2}{k^2+\beta^2}\right] - k^2\left[1-\frac{2\alpha\beta}{k^2+\beta^2}\right]\right\}e^{ikx_1}, \\ T_3 &= 0. \end{aligned} \tag{7}$$

The components of the velocity vector for particles in the medium are:

$$\begin{aligned} v_1 &= \left.\frac{du_1}{dt}\right|_{x_3=0} = A\omega k\left[1-\frac{2\alpha\beta}{k^2+\beta^2}\right]e^{ikx_1}, \\ v_2 &= \left.\frac{du_2}{dt}\right|_{x_3=0} = 0, \\ v_3 &= \left.\frac{du_3}{dt}\right|_{x_3=0} = Ai\omega\alpha\left[1-\frac{2k^2}{k^2+\beta^2}\right]e^{ikx_1}. \end{aligned} \tag{8}$$

Denoting the components of the acoustic impedance vector of the medium in the x_1 direction as

$$Za_i = -\frac{T_i}{v_i}, \quad i = 1, 2, 3, \tag{9}$$

we obtain

$$Za_1 = \frac{2\mu+\lambda}{v} - \frac{v\lambda[1-(v/a_1)^2]}{a_2^2\left[2-(v/a_2)^2-2\sqrt{1-\left(\frac{v}{a_1}\right)^2}\cdot\sqrt{1-\left(\frac{v}{a_2}\right)^2}\right]}, \tag{10}$$

$$Za_2 \to \infty, \quad Za_3 = 0,$$

where v is the Rayleigh wave velocity.

In the case of a bulk longitudinal wave ($v = a_1$), the impedance Za_1 takes the form

$$Za_1 = \frac{2\mu+\lambda}{a_1} = Z_{obj}, \tag{11a}$$

whereas for a bulk transverse wave ($v = a_2$) we have

$$Za_1 = \frac{2\mu+\lambda}{a_2} + \frac{\lambda}{a_2}\left[1-\left(\frac{a_2}{a_1}\right)^2\right], \tag{11b}$$

where Z_{obj} is the acoustic impedance for a bulk wave in the unbounded medium.

Formula (10) shows that value of the impedance medium for a Rayleigh wave is greater than the impedance of the same medium for a bulk wave.

Limiting the medium to a half-space has a bearing on the propagation of the bulk transverse wave ($Za_1 > Z_{obj}$), although it has no influence on the behaviour of the longitudinal wave ($Za_1 = Z_{obj}$).

3. Acoustic impedance of Rayleigh waves in an elastic, isotropic half-space

Let us widen our reasoning, taking into account the fact that a surface wave of the Rayleigh type propagates in a certain layer at the surface of the elastic half-space. In this case the stress components have the form

$$\begin{aligned} T_1 &= A\{\lambda(\xi_1-\xi_2)-2\mu(\xi_2+i\xi_3)\}e^{-ax_3+ikx_1}, \\ T_2 &= A\lambda(\xi_1-\xi_2)e^{-ax_3+ikx_1}, \\ T_3 &= A\{2\mu(\xi_1-i\xi_3)+(\xi_1-\xi_2)\}e^{-ax_3+ikx_3}, \end{aligned} \tag{12}$$

where

$$\xi_1 = \left[1-\frac{2k^2}{k^2+\beta^2}\right]a^2, \quad \xi_2 = \left[1-\frac{2a\beta}{k^2+\beta^2}\right]k^2, \quad \xi_3 = \left[1-\frac{k^2+a\beta}{k^2+\beta^2}\right]ka, \tag{13}$$

while the components of the particle velocity vector in the medium will be the following:

$$v_1 = \frac{A\omega}{k}\,\xi_2 e^{-ax_3+ikx_1},$$

$$v_2 = 0, \tag{14}$$

$$v_3 = \frac{Ai\omega}{a}\,\xi_1 e^{-ax_3+ikx_1}.$$

Applying formulae (12) and (14) we obtain a medium impedance in the x_1 direction:

$$Za_1 = \frac{\lambda+2\mu}{v} - \frac{\lambda\xi_1 - i2\mu\xi_3}{v\xi_2},$$

$$Za_2 \to \infty, \tag{15}$$

$$Za_3 = \frac{a}{\omega\xi_1}\{2\mu\xi_3 + i[(2\mu+\lambda)\,\xi_1 - \lambda\xi_2]\}.$$

The components of the acoustic impedance vector of the medium become complex when wave penetration inside the medium is considered. The imaginary part of the acoustic impedance represents those modes which do not propagate in the medium [3].

For the bulk longitudinal and transverse waves, the formulae denoting the medium impedance in the x_1 direction (Za_1) are identical to formulae (11) arrived at in section 2, whereas in the x_3, direction, the impedance for the longitudinal waves is infinitely great, while for the transverse waves it has a purely imaginary value.

In the case of CdS, the characteristic impedance for the bulk waves $Za_{obj} = 56\cdot 10^6\,\Omega$, whereas for the surface waves formulae (15) give the following values of the impedance components:

$$|Za_1| = 97\cdot 10^6\,\Omega, \quad |Za_2| = 12\cdot 10^6\,\Omega.$$

4. Conclusions

Introducing a medium acoustic impedance for surface waves is essential, since the numerical values of this impedance differ considerably from the medium impedance values for the two-dimensional bulk waves. In addition, the acoustic impedance determined for Rayleigh waves has an complex form, which points to different propagating conditions for the Rayleigh modes compared to the bulk modes in an unbounded medium.

References

[1] W. P. Mason, R. S. Thurston, *Physical acoustics, principles and methods,* New York – London 1970, vol. VI. p. 146.

[2] W. Nowacki, *Theory of elasticity* (in Polish), PWN, Warszawa 1970, p. 93.

[3] B. A. Auld, *Acoustic fields and waves in solids,* New York – London, 1973, vol. I, p. 176.

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