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PROPAGATION OF ACOUSTIC WAVE ALONG A HOLLOW CYLINDER IMMERSED IN A LIQUID

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The problem of the propagation of an nonabsorbed, continuous, progressive and axially-symmetric acoustic wave along an infinite homogeneous and isotropic cylinder filled with air and immersed in an ideal liquid has been considered. The wave equations of displacement potentials have been solved. The characteristic equation has been derived for the preset boundary conditions and solved numerically for the selected data characteristic for the conditions of the biopsy performed in an ultrasonic field. It has been shown that a wave guided along a needle immersed in a liquid can propagate with the velocity only slightly smaller than the wave velocity of the surrounding liquid. The distributions of displacement, stresses and acoustic pressure of the propagating wave have been determined.

1. Introduction

The problem of the acoustic wave propagation along a hollow cylinder immersed in a liquid is an attempt to describe the wave phenomena occurring in the puncture of organs with a needle, as used in ultrasonic medical diagnostics.

A biological structure to be investigated is localized with an ultrasonic beam and the needle is subsequently entered into the body toward the biological structure through the hole in the piezoelectric transducer. It has been found that a wave propagates along the needle after it has been entered into the body. The wave after reaching the end of the needle returns, giving the image of the needle end on the oscilloscope screen.

The aim of this paper is to describe the phenomena accompanying the propagation of this wave. In the first approximation the needle was considered a solid layer [2]. It is now defined as an infinite elastic, isotropic and homogeneous hollow cylinder with the Lamé constants λ, μ and the density ϱ and surrounded by an ideal liquid. The external and internal radii of the cylinder are a and b respectively, and the velocities of longitudinal and transverse waves in the material are c_d and c_t respectively. The liquid density is ϱ_0 and the longi-

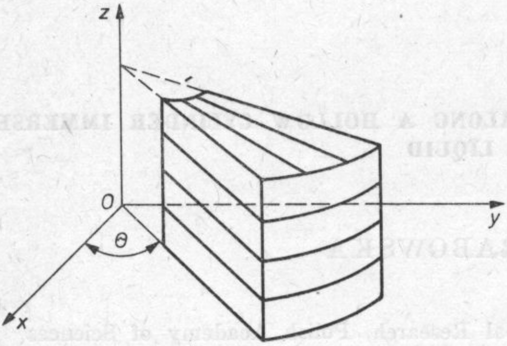

Fig. 1. A sector of circular section of the hollow cylinder

tudinal wave velocity in the liquid is c_0. The air inside the cylinder is regarded as the vacuum. This paper considers the case when a running, continuous and axially – symmetric wave propagates along the z-axis which coincides with the axis of the hollow cylinder.

2. Basic equations

In the cylindrical coordinate system displacements, strains and stresses are independent of the angle θ in the case of an axially – symmetric deformation with respect to the z-axis. The displacements in the hollow cylinder have the form

$$u_r = u_r(r, z, t), \quad u_\theta = 0, \quad u_z = u_z(r, z, t). \tag{1}$$

For harmonic vibrations the solutions of the wave equations (of displacement potentials):

$$\nabla^2 \Phi = \frac{1}{c_d^2}\frac{\partial^2 \Phi}{\partial t^2}, \quad \nabla^2 \Psi = \frac{1}{c_t^2}\frac{\partial^2 \Psi}{\partial t^2} \tag{2}$$

were assumed in the form [3]:

$$\begin{aligned} \Phi &= [A_1 J_0(k_d r) + A_2 Y_0(k_d r)]e^{i\omega t - ikz}, \\ \Psi &= [B_1 J_0(k_t r) + B_2 Y_0(k_t r)]e^{i\omega t - ikz}, \end{aligned} \tag{3}$$

where

$$k_d^2 = \frac{\omega^2}{c_d^2} - k^2, \quad k_t^2 = \frac{\omega^2}{c_t^2} - k^2, \tag{4}$$

$\omega = 2\pi f, f$ — frequency, k — propagation constant, A_1, A_2, B_1, B_2 — constants, J_n and Y_n are the Bessel and Neuman functions respectively. Displacements and stresses are expressed in the cylindrical coordinate system as follows:

$$u_r = \frac{\partial \Phi}{\partial r} - \frac{\partial^2 \Psi}{\partial r \partial z}, \quad \tau_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu \frac{\partial u_r}{\partial r},$$
$$u_z = \frac{\partial \Phi}{\partial z} - \frac{\partial^2 \Psi}{\partial^2 r} - \frac{1}{r}\frac{\partial \Psi}{\partial r}, \quad \tau_{rz} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right). \tag{5}$$

For the liquid surrounding the needle we assume the following solution of the wave equation (of displacement potential)

$$\nabla^2 \Phi_0 = \frac{1}{c_0^2}\frac{\partial^2 \Phi_0}{\partial^2 t} \tag{6}$$

in the form

$$\Phi_0 = A^0 H_0^{(2)}(k_0 r) e^{i\omega t - ikz}, \quad k_0^2 = \frac{\omega^2}{c_0^2} - k^2, \tag{7}$$

where $H_0^{(2)}(k_0 r) = J_0(k_0 r) - iY_0(k_0 r)$. Generally the Hankel function of the second kind represents a wave travelling in the direction of increasing r. For the imaginary and negative wave number k_0 it results from the asymptotic representation

$$H_0^{(2)}(k_0 r) \xrightarrow[r\to\infty]{} \sqrt{\frac{2}{k_0 r \pi}} \exp\left[-i\left(k_0 r - \frac{\pi}{4}\right)\right] \tag{8}$$

that the wave is not radiated but decays exponentially with increasing r; thus it is guided along the needle. Our present analysis considers only this case. The displacements u_r^0, u_z^0 and acoustic pressure in liquid are described by the relations

$$u_r^0 = \frac{\partial \Phi_0}{\partial r}, \quad u_z^0 = \frac{\partial \Phi_0}{\partial z}, \quad p = -\varrho_0 \frac{\partial^2 \Phi_0}{\partial t^2}. \tag{9}$$

3. Boundary conditions and charakteristic equation

The following boundary conditions should be satisfied on the surfaces of the hollow cylinder

$$\tau_{rr} = -p, \quad \tau_{rz} = 0, \ u_r = u_r^0 \quad \text{for } r = a,$$
$$\tau_{rr} = 0, \quad \tau_{rz} = 0, \quad \text{for } r = b. \tag{10}$$

After inserting relations (5) and (9) into (10) the system of five uniform equations with the unknowns A_1, A_2, B_1, B_2, A^0 is obtained

$$A_1\left[(\omega^2\varrho-2\mu k^2)J_0(k_d a)-\frac{2\mu k_d}{a}J_1(k_d a)\right]+$$
$$+A_2\left[(\omega^2\varrho-2\mu k^2)Y_0(k_d a)-\frac{2\mu k_d}{a}Y_1(k_d a)\right]+B_1\frac{2\mu ikk_t}{a}[J_1(k_t a)-k_t aJ_0(k_t a)+$$
$$+B_2\frac{2\mu ikk_t}{a}[Y_1(k_t a)-k_t aY_0(k_t a)]-\varrho_0\omega^2 A^0 H_0^{(2)}(k_0 a)=0,$$
$$2A_1 k_d ik\mu J_1(k_d a)+2A_2 k_d ik\mu Y_1(k_d a)+B_1 k_t(k^2-k_t^2)\mu J_1(k_t a)+$$
$$+B_2 k_t\mu(k^2-k_t^2)Y_1(k_t a)=0, \quad (11)$$
$$-A_1 k_d J_1(k_d a)-A_2 k_d Y_1(k_d a)+B_1 k_t kiJ_1(k_t a)+B_2 k_t kiY_1(k_t a)+$$
$$+A^0 k_0 H_1^{(2)}(k_0 a)=0,$$
$$A_1\left[(\omega^2\varrho-2\mu k^2)J_0(k_d b)-\frac{2\mu k_d}{b}J_1(k_d b)\right]+A_2\left[(\omega^2\varrho-2\mu k^2)Y_0(k_d b)-\right.$$
$$\left.-\frac{2\mu k_d}{b}Y_1(k_d b)\right]+B_1\frac{2\mu ikk_t}{b}[J_1(k_t b)-k_t bJ_0(k_t b)]+B_2\frac{2\mu ikk_t}{b}[Y_1(k_1 b)-$$
$$-k_t bY_0(k_t b)]=0,$$
$$2A_1 k_d ki\mu J_1(k_d b)+2Ak_d ki\mu Y_1(k_d b)+B_1 k_t(k^2-k_t^2)\mu J_1(k_t b)+$$
$$+B_2 k_t\mu(k^2-k_t^2)Y_1(k_t b)=0.$$

By eliminating the constants A_1, A_2, B_1, B_2 and A^0 from the system of equations (11) one obtains the characteristic equation

$$|a_{ij}|=0 \quad (i,j=1,\ldots,5), \quad (12)$$

where a_{ij} are coefficients of the constants A_1, A_2, B_2 and A^0. The sought propagation constant k occurs explicitely in (12) and also in k_d, k_t, k_0 (4), (7) and in the arguments of the Bessel and Neuman functions. In general, those complex values of k can occur in solutions (12), for which Im (k) defines attenuation of the wave along the z-axis, while imaginary values of k correspond to the unpropagating wave modes. Only the real propagation constants k were considered in the solution of the characteristic equation, which results from the assumption made that no attenuation occurs in the solid and the liquid.

The characteristic equation was solved numerically for the following data: $f=3$ MHz, $a=0.75$ mm, $b=0.5$ mm, a steel needle of the density $\varrho=7.7$ g/cm², $c_d=5.9$ km/s, $c_t=3.23$ km/s, $\lambda=1.07\cdot 10^{12}$ g/(cms²), $\mu=8.03\cdot$ $\cdot 10^{11}$ g/(cms²). Water of the density $\varrho_0=1$ g/cm³ was taken as a liquid, and $c_0=1.48$ km/s.

The previous experiments [4] showed that the velocity of this wave is close to the wave velocity in the surrounding liquid. Therefore such c was sougth that $c<c_0$, since for $c>c_0$ the wave would not be guided along the z-axis (compare (8) and (7)). The velocity of the wave propagating along the hollow cylinder, $c=1.44$ km/s, was obtained. The distributions of displacements, stresses and acoustic pressure in the needle and the liquid, calculated from (5) and (9) are shown in Figs. 2 and 3.

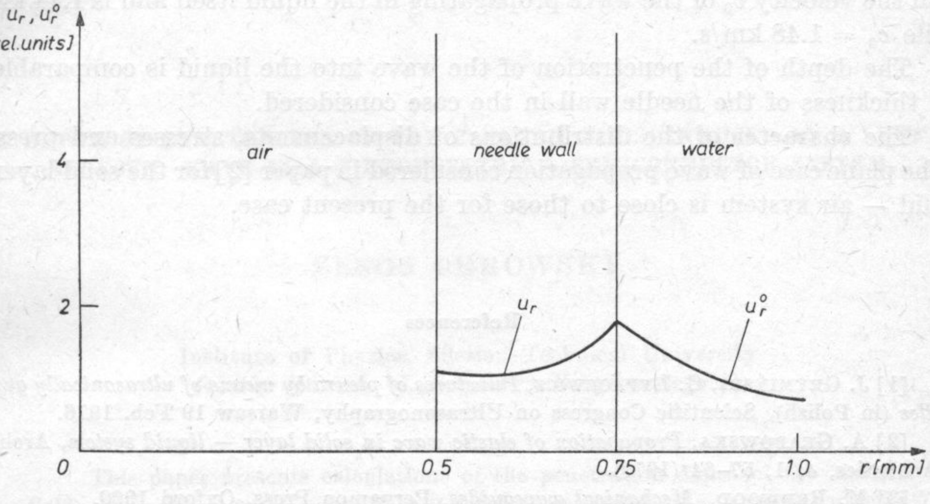

Fig. 2. The distribution of displacement in the needle and the liquid

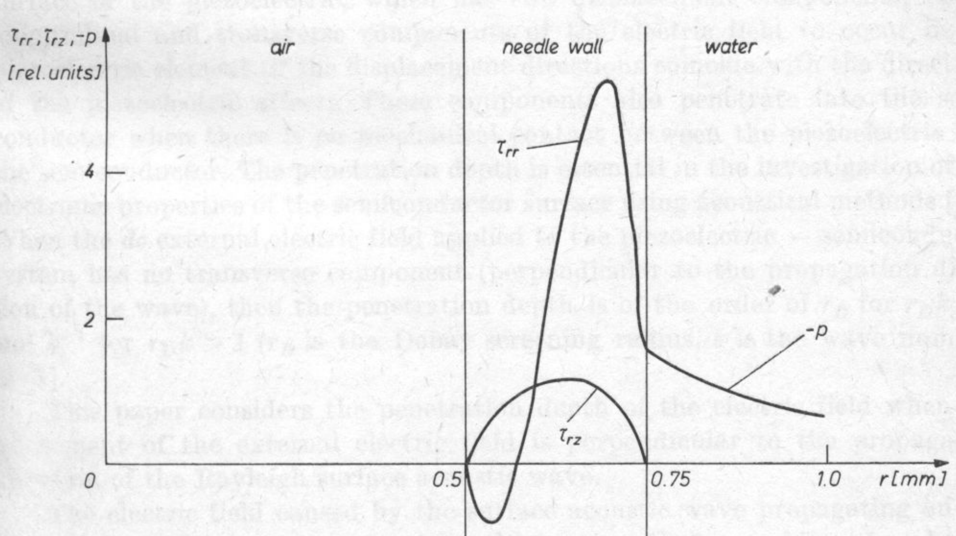

Fig. 3. The distributions of stresses in the needle and water

4. Conclusions

It has been shown that a wave can propagate along the needle immersed in a liquid. The phase velocity c of the progressive wave is then slightly smaller than the velocity c_0 of the wave propagating in the liquid itself and is 1.44 km/s, while $c_0 = 1.48$ km/s.

The depth of the penetration of the wave into the liquid is comparable to the thickness of the needle wall in the case considered.

The character of the distributions of displacements, stresses and pressure in the plane case of wave propagation considered in paper [2] for the solid layer – liquid – air system is close to those for the present case.

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