ARCHIVES OF ACOUSTICS
7, 2, 155-162 (1982)

# THE ANALYSIS OF SURFACE WAVE PROPAGATION IN A CRYSTAL WITH A MONOCLINIC STRUCTURE

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The present work analyzes surface wave propagation of the Rayleigh type in a monoclinic system. The problem was considered for a TGS crystal. Surface wave propagation was examined in the following planes: (010) in the ⟨100⟩, ⟨001⟩ directions and in some chosen directions forming angles 20°, 40°, 60°, 130°, 150°, 170°, 180° corresponding to the ⟨100⟩ direction, (100) in ⟨010⟩, ⟨001⟩ directions, and (001) in ⟨100⟩ and ⟨010⟩ directions. The above analysis was made using an electronic computation technique. As a result of our calculations we have found that surface waves cannot propagate along ⟨100⟩ and ⟨001⟩ directions in the planes (001) and (100) respectively. These directions are perpendicular to the axes of symmetry and they do not lie in the (010) plane.

## 1. Introduction

Surface wave propagation of the Rayleigh type has been considered by a number of authors. However, most of these authors considered surface wave propagation only in crystals with regular, tetragonal, trygonal or hexagonal symmetry. Very few papers deal with the monoclinic system. Numerous papers consider the problem of the existence of forbidden directions for surface wave propagation in the corresponding crystals. Thus, for example, STONLEY [1] discovered several directions forbidden for the plane (001) in cubic crystals. However, his considerations took into account only exponential terms of damping. GAZIS [2] calculated the velocities of surface wave propagation for a free surface in the (001) plane of many cubic crystals. Moreover, he proved that for aluminium and copper, surface waves do not exist in the range of ⟨110⟩ directions. BUCHWALD and DAVIS [3] show that surface waves in anisotropic media are possible only if the free plane is a symmetry plane of the crystal. In a medium with cubic symmetry surface wave propagation is possible only in the planes (001) and (100). Their calculations show the ranges of a forbidden direction: ⟨100⟩ in the (001) plane of aluminium, iron and lead. In their paper [4] other authors prove that in all cubic crystals surface waves cannot propagate in the (001) plane. The criterion given (necessary but not sufficient) for surface wave propagation has the following form: $c_{11}(c_{11}-c_{44}) > (c_{12}+c_{44})^2$.

Computations of a similar nature for LiF and Cu have been published by TURSONOV [5], who showed that the direction ⟨110⟩ in the (001) plane is forbidden for surface wave propagation. The author presents the results of numerical computations for LiF, for a propagation direction forming an angle of 15° with the axis, $x_1$, of the coordinate system.

The problem of the existence of the forbidden directions for regular systems has mainly been considered. Our aim was to investigate this problem in a crystal with a monoclinic structure. It was performed for a TGS crystal.

## 2. Calculation procedure

The general surface wave problem is formulated by assuming that the equation of motion is given by

$$\varrho \frac{\partial^2 u_i}{\partial t_2} = c_{ijkl} \frac{\partial^2 u_k}{\partial x_i \partial x_l}, \tag{1}$$

where $\varrho$ is the density of the material, $u_i$ are the particle displacements and $c_{ijkl}$ is the the elastic stiffness tensor.

For example the solution of equation (1) for the (010) plane is as follows

$$u_i = \sum_{n/1}^{3} C_n a_i^{(n)} \exp[ik(l_1 x_1 + l_2 x_2 + l_3^{(n)} x_3 - vt)], \tag{2}$$

where $a_i$ is the amplitude of the wave, depending on polarization, $\exp(ikl_3^{(n)} x_3)$ is the factor assuring the properties of a surface wave, $l_3$ is the parameter which characterizes the wave decaying into the depth of the solid, and $\exp[ik(l_1 x_1 + l_2 x_2 - vt)]$ is the change of amplitude in time and space, as it is in case of bulk wave.

Substituting equation (2) into (1) the relation between $a$ and $k$ is obtained. Using the stress-free boundary conditions on $x_3 = 0$,

$$\sigma_{3j} = c_{3jkl} \frac{\partial u_k}{\partial x_l} = 0 \quad (j, k, l = 1, 2, 3), \tag{3}$$

the parameters $a_1, a_2, a_3$, the velocity of surface wave, and also the vector components of the particle displacements were obtained.

In this work an analysis of surface wave propagation for the TGS crystal in the three following planes has been made: (010), (100) and (001). In the plane (010), the propagation of the surface wave was analyzed along the ⟨100⟩ and ⟨001⟩ directions and in the directions which form angles of 20°, 40°, 60°, 120°, 150°, 170°, 180° with the ⟨100⟩ direction. In both remaining cases our calculations were made in the (001) plane in the ⟨100⟩, ⟨001⟩ directions and in the (100) plane along the ⟨010⟩, ⟨001⟩ directions.

The coordinate system assumed for surface waves is presented in Fig. 1, where $a$, $b$, $c$ are the crystallographic axes of the TGS monocrystal, $x_1$, $x_2$, $x_3$ are the axes of an orthogonal system with respect to which surface wave propagation has been considered. The above calculations were made by applying an electronic computation technique using an ODRA 1305 computer. The values of the velocity as a parameter were changed with a step of $\pm 0.4$ m/s.

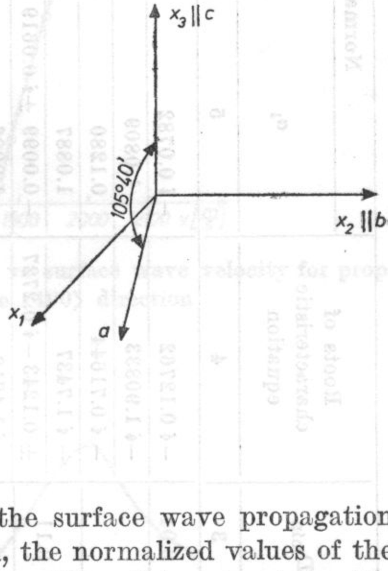


Fig. 1. Coordinate system for surface wave propagation in TGS crystal

### 3. Calculation results

Table 1 presents the values obtained for the surface wave propagation velocity, the roots of the characteristic equation, the normalized values of the eigenvector, and the values of the boundary condition determinant for the surface wave propagation directions considered in the present work. Fig. 2 presents, as example, the magnitude of the boundary condition determinant of surface propagation velocity in the (010) plane along the ⟨100⟩ direction. Fig. 3 shows the dependence of the surface wave propagation velocity on the direction in the (010) plane.

Moreover, the components of the particle displacement along the directions determined by the axes of the coordinate system have been calculated. These components in the (010) plane in the ⟨100⟩ direction are as follows:

$$u_1 = C_1[0.0782 \exp(0.127\, kx_2) - 0.429 \exp(1.0035\, kx_2) - \\ -0.0166 \exp(0.0308\, kx_2)] \sin k(x_1 - vt), \qquad (4a)$$

**Table 1.** Results of numerical calculations

| Plane | Direction of surface wave propagation | $V\left[\frac{m}{s}\right]$ | $D_{min}$ | Roots of characteristic equation | Normalized values of eigenvectors $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| (010) | ⟨100⟩ | 1875.2 | 0.04 | $-i\,0.12762$ | $-i\,0.0782$ | $-1.0035$ | $-i\,0.0308$ |
| | | | | $-i\,1.90833$ | $-1.0809$ | $i\,0.4145$ | 0.0588 |
| | | | | $-i\,0.71644$ | 0.1280 | $-i\,0.1032$ | 0.9974 |
| | 20° | 1868.0 | 0.11 | $-i\,1.7437$ | 1.0887 | $-i\,0.4315$ | 0.0316 |
| | | | | $\pm 0.1243 - i\,0.2727$ | $-0.0099 \pm i\,0.0619$ | $\pm 0.8898 - i\,0.2719$ | $0.6515 \pm i\,0.3724$ |
| | 40° | 1704.0 | 0.02 | $-i\,1.4812$ | 1.0906 | $-i\,0.4383$ | 0.0531 |
| | | | | $\pm 0.3839 - i\,0.1430$ | $-0.1777 \mp i\,0.0587$ | $\mp 0.6304 - i\,0.0089$ | $0.7583 \mp i\,0.0212$ |
| | 60° | 1664.0 | 0.12 | $-i\,1.1505$ | $-1.0506$ | $\pm 0.3238$ | 0.0323 |
| | | | | $\pm 0.5795 - i\,0.1811$ | $-0.1132 \mp i\,0.0688$ | $\mp 0.6965 - i\,0.0340$ | $0.7140 \mp i\,0.0441$ |
| | ⟨001⟩ | 1726.0 | 0.21 | $-i\,1.04933$ | $-1.0053$ | $-i\,0.2122$ | 0.1853 |
| | | | | $\pm 0.6595 - i\,0.2026$ | $0.3423 \pm i\,0.0308$ | $\mp 0.6929 - i\,0.04423$ | $0.6402 \mp i\,0.0644$ |
| (010) | 130° | 1926.0 | 0.45 | $-i\,1.6245$ | $-1.05982$ | $-i\,0.4660$ | 0.3065 |
| | | | | $\pm 0.3179 - i\,0.1567$ | $0.4620 \mp i\,0.0510$ | $\pm 0.5704 \mp i\,0.0079$ | $0.6816 + i\,0.0280$ |
| | 150° | 1999.2 | 0.46 | $-i\,0.2156$ | $-i\,0.0343$ | $-1.0582$ | $-i\,0.3443$ |
| | | | | $-i\,1.8323$ | $-1.06635$ | $-i\,0.4371$ | 0.2323 |
| | | | | $-i\,0.4772$ | 0.3993 | $-i\,0.6879$ | 1.1462 |
| | 170° | 1903.2 | 0.30 | $-i\,0.1277$ | $i\,0.0705$ | $-1.0027$ | $-i\,0.0234$ |
| | | | | $-i\,1.9258$ | $-1.0765$ | $-i\,0.4152$ | 0.1158 |
| | | | | $-i\,0.7460$ | 0.2013 | $-i\,0.0466$ | 0.9806 |
| (001) | ⟨010⟩ | 1810.0 | 0.10 | $-i\,0.2506$ | $-1.0000$ | $i\,0.0065$ | 0.0019 |
| | | | | $\pm 0.7638 - i\,0.2999$ | $0.3009 \pm i\,0.0187$ | $\pm 0.5878 + i\,0.0907$ | $0.7592 \pm i\,0.0628$ |
| (100) | ⟨010⟩ | 1868.0 | | $-i\,0.1147$ | 1.0031 | $i\,0.1027$ | 0.0655 |
| | | | | $-i\,1.5071$ | $i\,0.3699$ | $-1.0682$ | $i\,0.0661$ |
| | | | | $-i\,0.5760$ | 0.0738 | $i\,0.0264$ | 0.9976 |

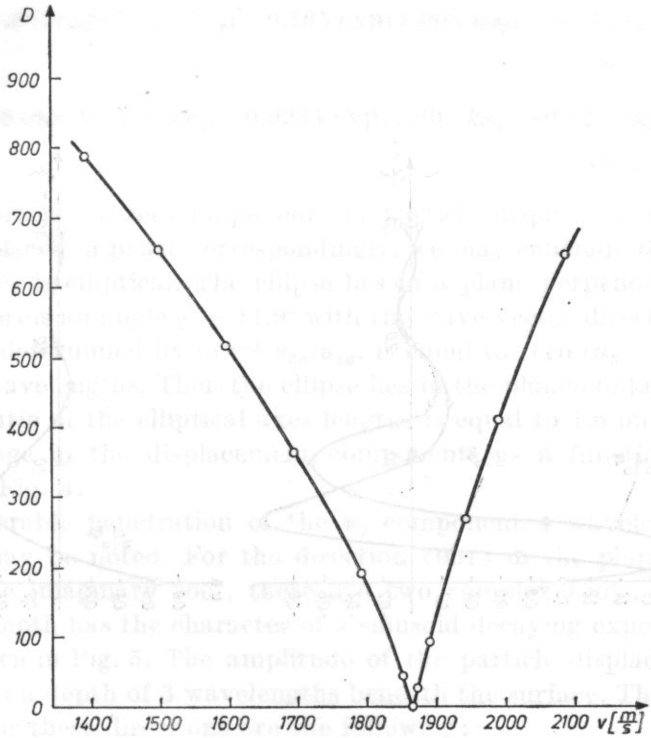


Fig. 2. Magnitude of boundary-condition determinant vs surface wave velocity for propagation in the (010) plane along the ⟨100⟩ direction

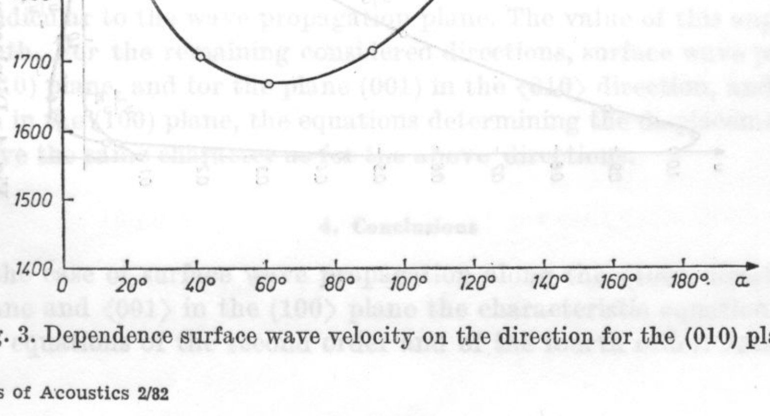


Fig. 3. Dependence surface wave velocity on the direction for the (010) plane

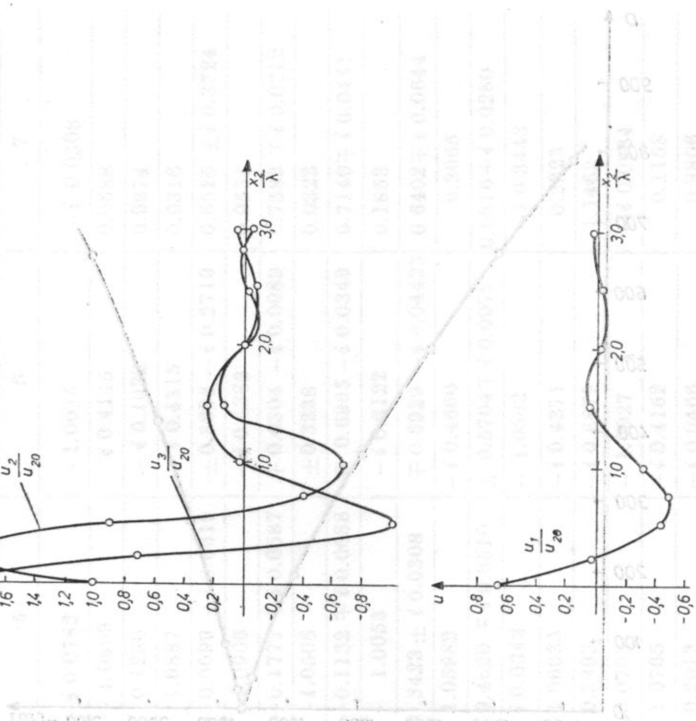


Fig. 5. Displacement components as a function of depth for the ⟨001⟩ direction in the (010) plane

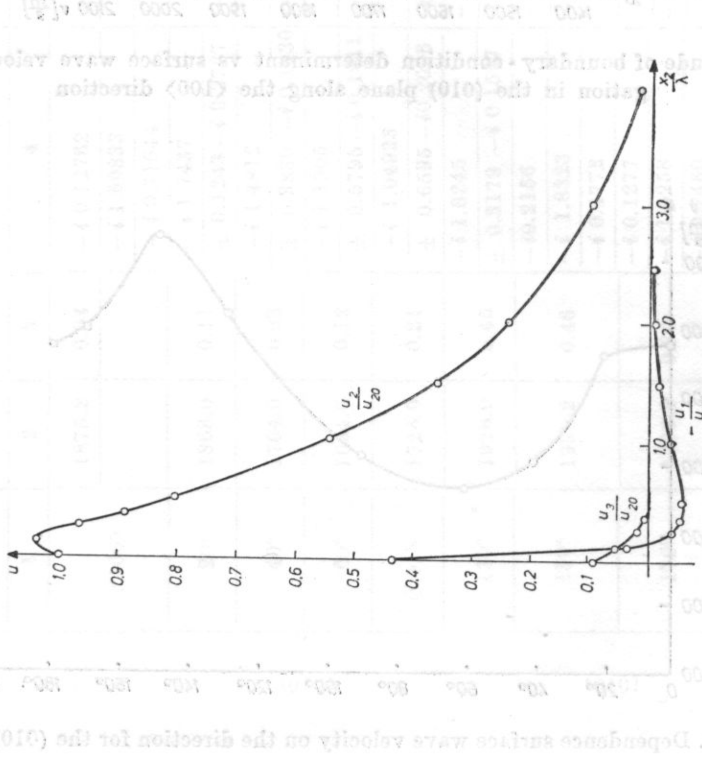


Fig. 4. Displacement components as a function of depth for the ⟨100⟩ direction in the (010) plane

$$u_2 = C_1[1.003\exp(0.127\,kx_2) - 0.165\exp(1.003\,kx_2) - 0.13\exp(0.0308\,kx_2)] \times \times \cos k(x_1 - vt), \qquad (4b)$$

$$u_3 = C_1[0.0308\exp(0.1276\,kx_2) + 0.0234\exp(1.003\,kx_2) - 0.123\exp(0.031\,kx_2)] \times \times \sin k(x_1 - vt). \qquad (4c)$$

Since there are three components of particle displacement not equal to zero, and displaced in phase correspondingly, we may conclude that the motion of the particles is elliptical. The ellipse lies in a plane perpendicular to a free surface and forms an angle $\varphi = 11.9°$ with the wave vector direction. The value of this angle, determined by $\tan^{-1} u_{30}/u_{10}$, is equal to zero ($u_3 = 0$), at a depth equal to 0.5 wavelengths. Then the ellipse lies in the plane containing the wave vector. The ratio of the elliptical axes lengths is equal to 1.6 on a free surface.

The change in the displacement components as a function of depth is presented in Fig. 4.

A considerable penetration of the $u_2$ component 4 wavelengths beneath the surface may be noted. For the direction ⟨001⟩ in the plane (010) where, except for one imaginary root, there are two complex roots, the amplitude change with depth has the character of a sinusoid decaying exponentially. This change is shown in Fig. 5. The amplitude of the particle displacement components decays at a depth of 3 wavelengths beneath the surface. The displacement components for these directions are the following:

$$u_1 = C_1\{-1.005\exp(1.049\,kx_2) + \exp(0.203\,kx_2)[2.27\cos(0.659\,kx_2) - -1.024\sin(0.659\,kx_2)]\}\cos k(x_3 - vt), \qquad (5a)$$

$$u_2 = C_1\{-0.212\exp(1.049\,kx_2) + \exp(0.203\,kx_2)[2.187\cos(0.659\,kx_2) + +4.53\sin(0.659\,kx_2)]\}\sin k(x_3 - vt), \qquad (5b)$$

$$u_3 = C_1\{0.1853\exp(1.049\,kx_2) + \exp(0.203\,kx_2)[3.811\cos(0.659\,kx_2) - -2.684\sin(0.659\,kx_2)]\}\cos k(x_3 - vt). \qquad (5c)$$

The movement of particles in this case is elliptical, as for the ⟨100⟩ direction. The plane of this ellipse forms an angle of 17° with the wave vector direction and is perpendicular to the wave propagation plane. The value of this angle changes with depth. For the remaining considered directions, surface wave propagation in the (010) plane, and for the plane (001) in the ⟨010⟩ direction, and the ⟨010⟩ direction in the (100) plane, the equations determining the displacement components have the same character as for the above directions.

### 4. Conclusions

In the case of surface wave propagation along the ⟨100⟩ direction in the (001) plane and ⟨001⟩ in the (100) plane the characteristic equation is divided into two equations of the second order and of the fourth order. Analyzing the second order equation and assuming stress-free surface boundary conditions, it has been proved that only a transverse bulk wave can propagate in the ⟨100⟩ direction in the (001) plane. This wave propagates at an angle of $\tan^{-1} l_3^1 = 1.6°$ to the free surface. The velocity which corresponds to the wave is equal to $V = 1919.9$ m/s. The calculated displacement components of the particles in this wave are as follows

$$u_1 = u_3 = 0, \quad u_2 = C\exp[ik(0.027\, x_3 + x_1 - vt)]. \tag{6}$$

While solving the equation of fourth order roots with the imaginary part not equal to zero have not been found. It is known that only these roots correspond to a surface wave which would simultaneously satisfy the boundary conditions. The boundary conditions were satisfied only in the range of real roots. Therefore, it may be assumed that only transverse bulk waves can propagate in the direction considered. This wave propagates at an angle equal to 10.6° to the free surface with a velocity $V = 2038$ m/s.

Similar results for the ⟨001⟩ direction in the (100) plane have also been obtained. Thus, in the case of the TGS crystal considered the ⟨100⟩ direction in the (001) plane, and ⟨001⟩ in the plane (100) are forbidden for surface wave propagation.

**References**

[1] R. Stonley, *The propagation of surface elastic waves in a cubic crystal*, Proc. Roy. Soc., A **232**, 44 (1955).

[2] A.C. Gazis, R. Herman, R. Vallis, *Surface elastic waves in cubic crystals*, Phys. Rev., **119**, 533 (1960).

[3] T.C. Lim, G.W. Farnell, *Search for forbidden directions of elastic surface-wave propagation in anisotropic crystals*, Journal of Applied Physics, **39**, 9, 4319 (1968).

[4] L.F. Protopopova, A.M. Fiedorchenko, *Volny Rayleigha v poluprovodnikach i yannych kristalach z kubicheskoy symetrej*, Nauch. trudy vysshich ucheb. zaved. Litevskoy CCR, Ultrazvuk, **3**, 61 (1971).

[5] A.D. Tursonov, *Obobshchennye poverchnostnye volny v kubicheskich kristalach*, Acust. Journal, **13**, 100 (1967).

*Received on December 16, 1980; revised version on November 3, 1981.*