ARCHIVES OF ACOUSTICS
7, 1, 43–59 (1982)

SCATTERING OF ACOUSTIC WAVES ON FREE SURFACE PERTURBATIONS IN AN ELASTIC HALF-SPACE

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This paper considers a semi-infinite, homogeneous and linearly elastic medium with a perturbed free surface. The perturbation is material loss. Using the Green function method, the first Born aproximation is found for the field of displacements dependent harmonically on time and subsequently energy relations for solutions obtained are calculated. The character and magnitude of scattering on the perturbation are thus defined for any mode occurring in a semi-infinite, homogeneous and linearly elastic medium. In addition, the case of perturbation described by periodic functions, which is essential in practice, is analyzed.

Notation

V — the homogeneous isotropic region occupied by the linearly elastic medium,
S — some area belonging to the region V,
(I, J) — a pair defining the character and kind of wave: $I = 0$ the wave falling onto the perturbation, $I = S$ the wave scattered on the perturbation,
J — specifies modes (SH, P, SV, TR, R, L, T),
$U^{(I,J)}(x, t)$ — displacement of the point x caused by a wave defined by (I, J),
$q^{(I,J)}$ — an acoustic Poynting vector for a wave defined by (I, J),
KW_{n-m} — a directional coefficient of power transformation from the wave n into the wave m,
ϱ — the density of the medium,
c, λ, k — the velocity, length and wave vector of a wave,
e_i — the versors of a Cartesian rectangular coordinate system,
x, θ, φ — the spherical coordinates of the tracing vector x.

1. Introduction

Rayleigh waves have recently been the object of large interest. There are many methods of generation of these waves [2]. The most recent method, proposed by HUMPHRYES and ASH [4], uses the transformation of bulk waves on a system of grooves on a free surface. This method can be used for the generation of Rayleigh waves in any elastic medium, and in the hypersonic range it appears to be competitive with respect to the methods used previously. This method, confirmed by experimental works [1, 4, 13] has one theoretical elaboration, which does not exhaust the problem. At the same time there is a number of theoretical papers [9, 10-12], which discuss the transformation in "the other direction", i.e. the transformation of Rayleigh waves on different kinds of free surface perturbations. In these papers, on the basis of field theory and perturbation calculus, expressions were derived for the displacement field of scattered waves in analytical form. These expressions were confirmed by experimental works [7, 8, 13].

The aim of the present investigation is to analyze the transformation properties of such perturbations, determining the behaviour of any wave occurring in a semi-infinite, homogeneous and linearly elastic medium after its passage through the perturbation. In terms of the solution method this paper is a generalization of the considerations of Rayleigh wave scattering given in [6].

2. Waves in a half-space

Let there be

$$V_1 = \{x : x_3 \geqslant 0\}, \quad \text{and let } S_1 = \{x : x_3 = 0\}$$

be a free surface. For this elastodynamic problem the complete system of solutions for displacement fields dependent harmonically on time is formed by the following modes [3]:

(a) a transverse wave polarized parallel to the free surface (the mode SH);

(b) two modes containing a transverse wave polarized perpendicular to the free surface and a longitudinal wave: one mode describes the case of the longitudinal wave falling onto the free surface (the mode designated as P); the other mode describes the case of the transverse wave falling onto the free surface (the mode designated as SV);

(c) a wave containing a longitudinal wave decaying exponentially with increasing x_3 (the mode TR);

(d) a Rayleigh wave (the mode R).

Without decreasing generalization it is only possible to consider the waves of wave vectors of the type

$$\boldsymbol{k}^{(0)} = (k_1^{(0)}, 0, k_3^{(0)}),$$

since this condition can be satisfied by such a coordinate system that the $0X_1$ axis coincides with the projection of the mode propagation direction onto the free surface.

The displacement vectors for the particular modes take then the form:

– for the mode SH

$$u_1^{(0,SH)}(x,t) = u_3^{(0,SH)}(x,t) = 0,$$

$$u_2^{(0,SH)}(x,t) = M\cos k_\beta x_3 \exp[i(k_1^{(0)}x_1 - \omega t)], \tag{2.1}$$

where

$$k_1^{(0)} = -k_T \sin\theta_0 \cos\varphi_0, \quad k_\beta = k_T\cos\theta_0,$$

M is the amplitude;

– for the other modes

$$u_1^{(0,J)}(x,t) = iM[k_1^{(0)0}(F_1\exp(ik_a x_3) + F_2\exp(-ik_a x_3)) - k_\beta \times \times (G_1\exp(ik_\beta x_3) - G_2\exp(-ik_\beta x_3))\exp[i(k_1^{(0)}x_0 - \omega t)],$$

$$u_2^{(0,J)}(x,t) = 0, \tag{2.2}$$

$$u_3^{(0,J)}(x,t) = iM[k_a(F_1\exp(ik_a x_3) - F_2\exp(-ik_a x_3)) + k_1^{(0)}(G_1\exp(ik_\beta x_3) + G_2\exp(-ik_\beta x_3))]\exp[i(k_1^{(0)}x_1 - \omega t)].$$

The quantities F_2, G_2, $-k_a$, $-k_\beta$, $k_1^{(0)}$ and F_1, G_1, k_a, k_β, $k_1^{(0)0}$, which describe the incident and the reflected waves, respectively, occurring in the particular modes, are given by the formulae:

a) for the mode P $(J = P)$

$$F_1 = \frac{4k_1^{(0)^2}k_a k_\beta - (k_\beta^2 - k_1^{(0)^2})^2}{4k_1^{(0)^2}k_a k_\beta + (k_\beta^2 - k_1^{(0)^2})^2}\frac{1}{k_L}, \quad F_2 = \frac{1}{k_L},$$

$$G_1 = \frac{-4k_1^{(0)}k_\beta(k_\beta^2 - k_1^{(0)^2})}{4k_1^{(0)^2}k_a k_\beta + (k_\beta^2 - k_1^{(0)^2})^2}\frac{1}{k_L}, \quad G_2 = 0, \tag{2.3}$$

$$k_1^{(0)} = -k_L\sin\theta_0\cos\varphi_0, \quad k_a = k_L\cos\theta_0, \quad k_\beta = k_T\sqrt{1-(k_L^2/k_T^2)\sin^2\theta_0};$$

b) for the mode SV $(J = SV)$

$$F_1 = \frac{-4k_1^{(0)}k_\beta(k_\beta^2 - k_1^{(0)^2})}{4k_1^{(0)^2}k_a k_\beta + (k_\beta^2 - k_1^{(0)^2})^2}\frac{1}{k_T}, \quad F_2 = 0,$$

$$G_1 = \frac{4k_1^{(0)^2}k_a k_\beta - (k_\beta^2 - k_1^{(0)^2})}{4k_1^{(0)^2}k_a k_\beta + (k_\beta^2 - k_1^{(0)^2})^2}\frac{1}{k_T}, \quad G_2 = \frac{1}{k_T}, \tag{2.4}$$

$$k_1^{(0)} = -k_T\sin\theta_0\cos\varphi_0, \quad k = k_T\cos\theta_0,$$

$$k_a = k_L\sqrt{1-(k_T^2/k_L^2)\sin^2\theta_0}, \quad \theta_0 \in \langle 0, \theta_{\text{MAX}}\rangle,$$

where the angle θ_{MAX} is defined by the condition $\sin\theta_{\text{MAX}} = c_T/c_L$;

c) for the mode TR ($J = TR$) the quantities F_i, G_i, $k_1^{(0)}$, k are the same as for the mode SV, and

$$k_\alpha = -ik_L\sqrt{(k_T^2/k_L^2)\sin^2\theta_0 - 1}, \quad \theta_0 \in \langle\theta_{\text{MAX}}, \pi/2\rangle; \tag{2.5}$$

d) for the Rayleigh mode ($J = R$)

$$F_1 = \frac{1}{2ik_\alpha}, \quad F_2 = G_2 = 0, \quad G_1 = k_R(k_R^2 - k_T^2)^{-1},$$
$$k_1^{(0)} = -k_R\cos\varphi_0, \quad k_\alpha = i\sqrt{k_R^2 - k_L^2}, \quad k_\beta = i\sqrt{k_R^2 - k_T^2}. \tag{2.6}$$

In the above expressions the pair of angles (θ_0, φ_0) defines the propagation direction of the incident wave in the particular modes (Fig. 1). For the modes SH, P these are arbitrary directions in the half-space $x_3 \geqslant 0$, while the modes SV, TR impose additional restrictions on the angle θ_0, causing a case of the incident transverse wave to be assigned either to the mode SV or to the mode TR.

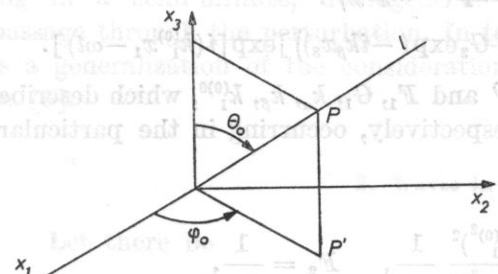

Fig. 1. The ray l represents the propagation direction for the incident waves occurring in the particular modes

The condition assumed previously for the consideration of waves with the component $k_2 = 0$ signifies that the angle φ_0 can take the values of 0 or π radians. The coupling of waves, visible in expressions (2.1)-(2.6), is a result of the existence of a free boundary plane. The coupling waves have the form of a longitudinal plane bulk wave and transverse plane bulk waves from an unbounded medium (the modes L, T) and of waves decaying exponentially with increasing distance from the boundary surface. For the boundary angles, i.e. for the rectangular incidence ($\theta_0 = 0$) and the parallel incidence ($\theta_0 = \pi/2$ and $\varphi_0 = 0$ or $\theta_0 = \pi/2$ and $\varphi_0 = 2\pi$) onto the free surface the modes show a particularly simple form, e.g. for the perpendicular incidence in the mode P only the incident and the reflected longitudinal bulk waves couple; similarly, only the incident and the reflected transverse bulk waves occur in the mode SV. It is easy to see that the variable x_2 does not occur in the expressions describing the modes and that only the mode SH has the second component of the displacement vector different from zero.

At the conclusion of this section the following notation can be introduced

$$\boldsymbol{u}^{(0,J)}(\boldsymbol{x},t) = M\boldsymbol{u}^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\omega \mid x_3)\exp(i\boldsymbol{k}_{||}^{(0)}\boldsymbol{x}_{||}-\omega t),$$
$$\boldsymbol{k}_{||}^{(0)} = (k_1^{(0)},0,0), \quad \boldsymbol{x}_{||} = (x_1, x_2, 0). \tag{2.7}$$

The vector $\boldsymbol{u}^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\,\omega \mid x_3)$, describing the behaviour of modes with increasing distance from the free surface, was purposefully separated in expression (2.7), since (as will be shown in the further considerations) it components on the free surface define the scattering on perturbations. It is easy to calculate the form of this vector for the particular modes by comparing expressions (2.1), (2.2) with (2.7).

It is easy to show that for the modes P, SV propagating perpendicular to the surface S_1 the following occurs

$$u^{(0,P)}(\mathbf{0}\omega \mid 0) = 0, \quad k_1^{(0)}u^{(0,SV)}(\mathbf{0}\omega \mid 0) = 0. \tag{2.8}$$

3. Displacement field of waves scattered on the perturbation

Let the region V_1 be so perturbed that its free surface is given in the form

$$S_2 = \{\boldsymbol{x} : x_3 = F(x_1, x_2)\},$$

where

$$F(x_1, x_2) = \begin{cases} f(x_1, x_2) & \text{in the perturbed region,} \\ 0 & \text{outside the perturbed region} \end{cases} \tag{3.1}$$

i.e. the perturbation described by the function $x_3 = f(x_1, x_2)$ is a material loss of the medium. For this function we assume, in addition, that its values are low with respect to the wavelength of the wave whose propagation is considered.

This is a new elastodynamic problem for which the modes in the previous section are only a zeroth approximation to solution. The exact solution must take into consideration the changes caused by the presence of the perturbation. To the author's knowledge the effect of the perturbation on the propagating Rayleigh wave has so far been analyzed. The most important papers on this subject were [5, 11, 8]. The aim of the present paper is to determine the scattering of the other modes (i.e. the modes SH, P, SV, TR) on such perturbations.

There are the following aspects of this new elastodynamic problem. The perturbation can be regarded as a transforming structure. The case of the incident Rayleigh wave solved so for permits the statement that the perturbation transforms part of the energy of the incident wave, causing a generation of the scattered wave into bulk and Rayleigh waves (according to the notation in Fig. 2, these processes can be observed in the systems a^*-b-a, a-b-c). What

* An inderdigital transducer and a Zn0 layer permit detection and generation of Rayleigh waves in nonpiezoelectric media [5].

remains to be calculated is the problem of bulk wave scattering (using a relevant bulk wave transducer and choosing the angle θ_0 the conditions for the generation of the modes *SH*, *P*, *SV*, *TR* can be met). The solution of this problem should define, among other things, the possibilities of the transformation: bulk waves – perturbation – Rayleigh waves (according to Fig. 2 c, b, a).

In terms of the solution method this paper is a generalization of the considerations in [6].

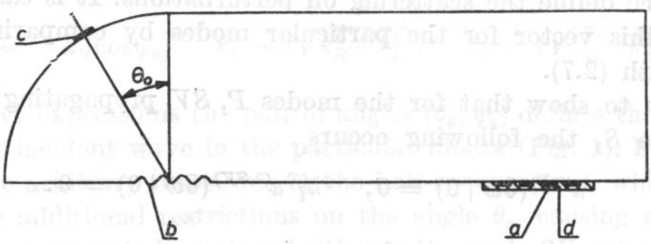

Fig. 2. An example of observation of the transforming properties for a free surface perturbation

a – interdigital transducer, *b* – perturbation, *c* – bulk wave transducer, *d* – ZnO layer

For this elastodynamic problem the first Born approximation for the displacement field of scattered waves can be expressed by Green functions with an expression of the form (expressions (2.12) and (2.13) in [6])

$$u^{(s)}(\boldsymbol{x}, t) = -(2\pi)^{-2} \sum_{\beta\gamma} \int d^2k \int_0^\infty dx_3' \exp(i\boldsymbol{k}_{||}\boldsymbol{x}_{||}) D_{\alpha\beta}(\boldsymbol{k}_{||}\omega \mid x_3 x_3') \times$$

$$\times \left\{ \int d^2x_{||} \exp(-i\boldsymbol{k}_{||}\boldsymbol{x}'_{||}) L^{(1)}_{\beta\gamma}(\boldsymbol{x}') \exp(i\boldsymbol{k}^{(0)}\boldsymbol{x}_{||}) \right\} u^{(0)}_{\gamma}(\boldsymbol{k}^{(0)}_{||}\omega \mid x_3') \exp(-i\omega t), \quad (3.2)$$

where $D_{\alpha\beta}(\boldsymbol{k}_{||}\omega \mid x_3 x_3')$ are Fourier transforms of the Green function, $L^{(1)}_{\beta\gamma}(\boldsymbol{x})$ are operators defined in the Appendix, $u^{(0)}_{\gamma}(\boldsymbol{k}^{0}_{||}\omega \mid x_3')$ can be any solution of the wave equation in a semi-infinite medium with a free surface. Since the modes selected in section 2 form a complete system, the knowledge of the solutions for the scattered waves for these modes permits the form of scattered waves to be obtained for any wave in the half-space. In paper [6] an analytical form of the solution of equation (3.2) was derived for the case when the scattered mode is the Rayleigh wave (expressions (3.48)-(3.53) in paper [6]). This solution requires the following assumptions for the incident wave:

1) the scattered wave has the form of (2.7),
2) $u_2^{(0,J)}(\boldsymbol{k}_{||}\omega \mid x_3) = 0$,
3) $k_2^{(0)} = 0$.

The modes *P*, *SV*, *TR* satisfy conditions 1)-3), which permits generalization of the solution for the scattering of the mode *R* to include the cases when the modes *P*, *SV*, *TR*, *R* are scattered.

Some dimensionless quantities can be introduced,

$$M_1(k_{\|}\omega) = \frac{k_{\|}^3(2c_T^2 - c_L^2(1+\hat{k}_1^2))}{a_L(a_T^2+k^2)2c_L^2}, \quad M_3(k_{\|}\omega) = \frac{k_{\|}}{a_T}\, 2\hat{k}_1\hat{k}_2,$$

$$M_2\,(k_{\|}\omega)\,\frac{k_{\|}(c_L^2(1+\hat{k}_1^2)-2c_T^2)}{2a_Lc_L^2}, \quad M_4(k_{\|}\omega) = \frac{-4a_La_Tk_{\|}^2+(a_T^2+k_{\|}^2)^2}{4a_La_T(a_T^2+k_{\|}^2)}, \tag{3.3}$$

$$M_5 = \left(1-\frac{c_R^2}{2c_T^2}\right) - 5\left\{\frac{c_R^4}{c_T^4}\left(1+\frac{c_T^2}{c_L^2}\right) - \frac{c_R^2}{c_T^2}\left(9-5\frac{c_T^2}{c_L^2}\right) + 8\left(1-\frac{c_T^2}{c_L^2}\right)\right\},$$

where the wave vector of the scattered wave and the coefficients of wave decay are defined in the following way

$$\boldsymbol{k}_{\|} = k_{\|}(\hat{k}_1, \hat{k}_2, 0),$$

$$a_{L,T} = \begin{cases} \sqrt{k_{\|}^2 - (\omega^2/c_{L,T}^2)} & \text{for } k_{\|} > \omega/c_{L,T} \\ -i\sqrt{(\omega^2/c_{L,T}^2) - k_{\|}^2} & \text{for } k_{\|} < \omega/c_{L,T}. \end{cases} \tag{3.4}$$

The desired first Born approximation for the case when the modes P, SV, TR, R are scattered is the sum of the longitudinal and transverse bulk waves and of the Rayleigh wave

$$\boldsymbol{u}^{(S)}(\boldsymbol{x}, t) = \boldsymbol{u}^{(S,L)}(\boldsymbol{x}, t) + \boldsymbol{u}^{(S,T)}(\boldsymbol{x}, t) + \boldsymbol{u}^{(S,R)}(\boldsymbol{x}, t), \tag{3.5}$$

where the particular scattered waves have the form

$$\boldsymbol{u}^{(S,L)}(\boldsymbol{x}, t) \sim \frac{\boldsymbol{x}}{x}\left(\frac{\omega}{c_T}\right) k_1^{(0,J)} M u_1^{(0,J)}(\boldsymbol{k}_{\|}^{(0)}\omega \mid 0)\,\hat{f}(\boldsymbol{k}_{\|}^{l} - \boldsymbol{k}_{\|}^{(0)})\,\frac{\cos\theta}{2\pi i}\,\frac{M_1(k_{\|}^{l}\omega)}{M_4(k_{\|}^{l}\omega)} \times$$

$$\times \frac{\exp i\left(\frac{\omega}{c_L}x - \omega t\right)}{x}, \tag{3.6}$$

$$\boldsymbol{u}^{(S,T)}(\boldsymbol{x}, t) \sim \frac{M_2(k_{\|}^{t}\omega)}{M_4(k_{\|}^{t}\omega)}\,[\boldsymbol{e}_1\cos\varphi + \boldsymbol{e}_2\sin\varphi - \boldsymbol{e}_3\operatorname{tg}\theta] + M_3(k_{\|}^{t}\omega) \times$$

$$\times [\boldsymbol{e}_1\sin\varphi - \boldsymbol{e}_2\cos\varphi]\,\frac{\omega}{c_T}\,k_1^{(0)} M u_1^{(0,J)}(\boldsymbol{k}_{\|}^{(0)}\omega \mid 0)\,\hat{f}(\boldsymbol{k}_{\|}^{t} - \boldsymbol{k}_{\|}^{(0)})\,\frac{\exp\left[i\left(\frac{\omega}{c_R}xi - \omega t\right)\right]}{2\pi i x}, \tag{3.7}$$

$$\boldsymbol{u}^{(S,R)}(\boldsymbol{x}, t) \sim \{[\boldsymbol{e}_1\cos\varphi + \boldsymbol{e}_2\sin\varphi][\exp(-k_R\beta_Lx_3) - (1 - 0.5c_R^2c_T^{-2})\exp(-k_R\beta_Tx_3)] +$$

$$+ \boldsymbol{e}_3 i\beta_L[\exp(-k_R\beta_Lx_3) - \exp(-k_R\beta_Tx_3)(1 - 0.5c_R^2c_T^{-2}) - 1]\}(-i)(2\pi)^{-1/2}(x_{\|})^{-1/2} \times$$

$$\times (\omega/c_R)^{3/2}\,k_1^{(0)}[M u_1^{(0,J)}(\boldsymbol{k}_{\|}^{(0)}\omega \mid 0)\,\hat{f}(\boldsymbol{k}^{R} - \boldsymbol{k}_{\|}^{(0)})]\exp\{i[(\pi/4) + (\omega/c_R) - \omega t]\}. \tag{3.8}$$

It should be noted that the following notation was used here

$$\boldsymbol{x} = x(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta) = x_1\boldsymbol{e}_1 + x_2\boldsymbol{e}_2 + x_3\boldsymbol{e}_3,$$

$$\boldsymbol{k}_{\|}^{l,t} = \frac{\omega}{c_{L,T}}(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ 0), \tag{3.9}$$

$$\boldsymbol{k}_{\|}^{R} = \frac{\omega}{c_R}(\cos\varphi,\ \sin\varphi,\ 0), \qquad \beta_{L,T} = \sqrt{1 - \frac{c_R^2}{c_{L,T}^2}}\,.$$

It can be seen from the above notation that the bulk waves are spherical; the Rayleigh wave, radial; and the amplitudes of the scattered waves also depend on the direction for which the scattering is analyzed. The amplitudes of the waves generated consist of several factors:

a) $u_1^{(0,J)}(\boldsymbol{k}_{\|}^{(0)}\omega \mid 0)$* – and, therefore, according to the considerations in the previous section, the amplitudes of the scattered waves depend on the kind of mode and are proportional to the amplitude of the incident wave; while for the incident bulk waves a change in the incidence angle also affects the magnitude of the scattered waves;

b) $f(\boldsymbol{k}_{\|} - \boldsymbol{k}_{\|}^{(0)})$ – the scattered waves are also defined by the shape of the perturbation;

c) $M_i(\boldsymbol{k}_{\|}\omega)$ – these dimensionless functions, together with a Fourier transform of the perturbation shape function, define the angular characteristic for the scattered waves.

As was stressed above, solutions (3.6)-(3.8) are asymptotic in character.

The mode SH does not satisfy the assumptions made in paper [6] and requires additional calculation from expression (3.2). In this part of the paper, consideration will be limited to the giving of the final results (the most important stages of these tedious calculations are given in the Appendix). Some dimensionless quantities can be introduced here

$$M_1^{(SH)}(k_{\|}\omega) = \frac{i\hat{k}_1\hat{k}_2k_{\|}^3}{a_L(a_T^2 + k_{\|}^2)}, \qquad M_2^{(SH)}(k_{\|}\omega) = \frac{i\hat{k}_1\hat{k}_2k_{\|}}{2a_L},$$

$$M_3^{(SH)}(k_{\|}\omega) = \frac{(\hat{k}_1^2 - \hat{k}_2^2)k_{\|}}{a_T}. \tag{3.10}$$

When the quantities M_1, M_2, M_3 are replaced with the quantities

$$M_1^{(SH)},\ M_2^{(SH)},\ M_3^{(SH)} \quad \text{and} \quad u_1^{(0,J)}(\boldsymbol{k}_{\|}^{(0)}\omega \mid 0) \quad \text{with } u_2^{(0,SH)}(\boldsymbol{k}^{(0)}\omega \mid 0),$$

expressions (3.6)-(3.8) in this new form also describe the scattered waves when the mode SH is scattered. Thus, in a medium with a perturbed free surface

* This result can be compared to that in [12], where the form of waves scattered (in the case of the incident Rayleigh save) is defined by additional stresses occurring in the free surface.

the mode SH, as also the other modes in the half-space, becomes scattered so that the displacement field of the scattered waves is the sum of spherical (longitudinal and transverse) bulk waves and a radial Rayleigh wave. It is interesting to note that for points of the medium on the $0X_1$ axis only the amplitude of the scattered transverse wave is different from zero. This signifies that in the case of solving an analogous twodimensional problem in the scattered field of the mode SH only the transverse bulk wave occurs. This is a specific property of the mode SH which makes it different from the other modes in the semi-infinite, isotropic and linearly elastic medium.

The scattering of each mode of the complete system of modes described in section 2 is thus known. An arbitrary wave occurring in a half-space can be represented as a linear combination of modes of the complete mode system

$$\boldsymbol{u}^{(0)}(\boldsymbol{x},t) = m_J \boldsymbol{u}^{(0,J)}(\boldsymbol{x},t), \quad J = SH, P, SV, TR, R, \tag{3.11}$$

where m_y are expansion coefficients. Accordingly, the first Born approximation for the displacement field of scattered waves can be expressed by the formula

$$\begin{aligned}\boldsymbol{u}^{(S)}(\boldsymbol{x},t) &= m_J u^{(0,J)}(k_{||}^{(0)}\omega \mid 0)\left\{\frac{\boldsymbol{x}}{x}\frac{\omega}{c_L}k_1^{(0)}\hat{f}(k_{||}^{l}-\boldsymbol{k}_{||}^{(0)})\frac{\cos\theta M_1^{(J)}(k_{||}^{l}\omega)}{2\pi i M_4^{(J)}(k_{||}^{l}\omega)}\times\right.\\ &\times\frac{\exp\left[i\left(\frac{\omega}{c_T}x-\omega t\right)\right]}{x}+\frac{\omega}{c_T}k_1^{(0)}\hat{f}(\boldsymbol{k}_{||}^{t}-\boldsymbol{k}_{||}^{(0)})\left[\frac{M_2^{(J)}(k_{||}^{l}\omega)}{M_4^{(J)}(k_{||}^{l}\omega)}[\boldsymbol{e}_1\cos\varphi+\boldsymbol{e}_2\sin\varphi-\boldsymbol{e}_3\operatorname{tg}\theta]+\right.\\ &\left.+M_3^{(J)}(k_{||}^{t}\omega)[\boldsymbol{e}_1\sin\varphi-\boldsymbol{e}_2\cos\varphi]\right]\times\frac{\exp\left[i\left(\frac{\omega}{c_T}x-\omega t\right)\right]}{2\pi i x}+\\ &+\left(\frac{\omega}{c_R}\right)^{3/2}k_1^{(0)}\hat{f}(k_{||}^{R}-\boldsymbol{k}_{||}^{(0)})M_5M_1^{(0)}(k_{||}^{R}\omega)\frac{2\exp(i\pi/4)}{i\sqrt{2\pi}}\times\\ &\times[\boldsymbol{e}_1\cos\varphi+\boldsymbol{e}_2\sin\varphi][\exp(-k_\beta\beta_L x_3)-\left(1-\frac{c_R^2}{2c_T^2}\right)\exp(-k_R\beta_T x_3)]+\boldsymbol{e}_3 i\beta_L\times\\ &\times\left[\exp(-k_R\beta_L x_3)-\left(1-\frac{c_R^2}{2c_T^2}\right)^{-1}\exp(-k_R\beta_T x_3)\right]\left.\frac{\exp\left[i\left(\frac{\omega}{c_R}x-\omega t\right)\right]}{\sqrt{x_{||}}}\right\},\end{aligned} \tag{3.12}$$

where

$$u^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\omega \mid 0) = \begin{cases} u_1^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\omega \mid 0), & J = P, SV, TR, R,\\ u_2^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\omega \mid 0), & J = SH,\end{cases} \tag{3.13}$$

$$M_1^{(J)}(k^{(0)}\omega) = \begin{cases} M_i(k_{||}^{(0)}\omega), & J = P, SV, TR, R,\\ M_i^{(SH)}(k_{||}^{(0)}\omega), & J = SH.\end{cases} \tag{3.14}$$

Expression (3.12) is valid for $k^{(S)}x \gg 1$.

4. Energy transformation on a perturbation

4.1. An acoustic Poynting vector. In considering the energy relations of waves it is convenient to use the acoustic Poynting vector $\boldsymbol{q}$. This vector is an analogue of the Poynting vector in electromagnetic theory and describes the wave power crossing a unit area. Using complex notation for the displacements $\boldsymbol{u}$ and the stress tensors T and with a harmonic dependence of waves on time, the acoustic Poynting vector defined by the relation

$$q_i' = -\frac{1}{2}\dot{u}_j^* T_{ij}, \tag{4.1}$$

is also a complex quantity, and its real part represents the time averaged power flux density for a given wave. The desired quantity is thus

$$q_i = Re\left\{-\frac{1}{2}\dot{u}_j^* T_{ij}\right\}. \tag{4.2}$$

4.2. An acoustic Poynting vector for waves incident to a perturbation. The use of definition (4.2) and of the solutions of the elastodynamic problems set in section 2 gives for the particular modes

$$q_1^{(0,SH)}\big|_{x_3=0} = \frac{1}{2}M^2\varrho\omega c_T^2 k_1^{(0)},$$

$$q_1^{(0,P)}\big|_{x_3=0} = \frac{1}{2}M^2\varrho\omega c_L^2 k_L\left\{\frac{32k_1^{(0)^3}(k_\alpha k_\beta)^2(k_\beta^2-k_\alpha^2)}{k_L[(k_\beta^2-k_1^{(0)^2})^2+4k_1^{(0)^2}k_\alpha k_\beta]^2}\right\},$$

$$q_1^{(0,SV)}\big|_{x_3=0} = \frac{1}{2}M^2\varrho\omega c_T^2 k_T\left\{\frac{16k_1^{(0)}k_\beta^2(k_\beta^2-k_1^{(0)^2})^2(k_\beta^2-k_\alpha^2)}{k_T[(k_\beta^2-k_1^{(0)^2})^2+4k_1^{(0)^2}k_\alpha k_\beta]^2}\right\},$$

$$q_1^{(0,TR)}\big|_{x_3=0} = \frac{1}{2}M^2\varrho\omega c_L^2 k_L\left\{\frac{16k_1^{(0)}k_\beta^2(k_\beta^2-k_1^{(0)^2})^2(k_\beta^2-k_\alpha^2)}{k_T[(k^2-k_1^{(0)^2})^4+(-4k_1^{(0)^2}k_\alpha k_\beta)^2]}\right\},$$

$$q_1^{(0,R)}\big|_{x_3=0} = \frac{1}{2}M^2\varrho\omega c_T^2 k_R\{2\xi_{LL}+2\xi_{LT}+2\xi_{TT}\},$$

$$q_1^{(0,J)}\big|_{x_3=0} = q_3^{(0,J)}\big|_{x_3=0} = 0, \quad J = SH, P, SV, TR, R,$$

where

$$\xi_{LL} = 2\frac{c_R^2}{2c_T^2}\left(1-\frac{c_T^2}{c_L^2}\right),$$

$$\xi_{LT} = -\left(1-\frac{c_R^2}{2c_T^2}\right)\left(2+\frac{c_R^2}{2c_T^2}-\frac{c_R^2}{c_L^2}\right)-\left(1-\frac{c_R^2}{c_L^2}\right)\left(2-\frac{c_R^2}{2c_T^2}\right)\left(1-\frac{c_R^2}{2c_T^2}\right)^{-1}, \tag{4.3}$$

$$\xi_{TT} = \left(1-\frac{c_R^2}{2c_T^2}\right)^{-1}\left[\left(1-\frac{c_R^2}{2c_T^2}\right)^3+\left(1-\frac{c_R^2}{c_T^2}\right)\right].$$

The (dimensionless) expressions in the braces result from the presence of coupling waves in the modes. It can be seen that, as expected, on the free surface energy is transfered only in the direction x_1. It is interesting to note that for the rectangular incidence with respect to the free surface in the modes SH, P, SV, no energy transport occurs in the direction x_1, either. For this reason, the time averaged acoustic Poynting vector can be calculated beforehand for the incident waves in the particular modes. For differentiation, this vector was marked with an additional dash (′).

$$\boldsymbol{q}'^{(0,P)} = \frac{1}{2} M^2 \varrho\omega c_L^2 \boldsymbol{k}_L, \quad \boldsymbol{q}'^{(0,J)} = \frac{1}{2} M^2 \varrho\omega c_T^2 \boldsymbol{k}_T, \qquad J = SH, SV, TR. \tag{4.4}$$

The physical sense of equations (4.4) is simple. They represent the power density for the incident waves in the particular modes; and at the same time represent the power density to be supplied for these modes to be generated. For the latter reason, it should be assumed that

$$\boldsymbol{q}'^{(0,R)} = \int_0^\infty \boldsymbol{q}^{(0,R)} dx_3 = \frac{1}{2} M^2 \varrho\omega\, c_T^2 \frac{\boldsymbol{k}_R}{k_R} \left\{ \frac{\xi_{LL}}{2\beta_L} + \frac{\xi_{LT}}{\beta_L+\beta_T} + \frac{\xi_{TT}}{2\beta_T} \right\}. \tag{4.5}$$

4.3. An acoustic Poynting vector for waves scattered on a perturbation. The use of definition (4.2) and of solutions (3.6)-(3.8) for large $k^{(S)} x$ (or $k_{||} x_{||}$) gives

$$\boldsymbol{q}^{(S,L)}(\boldsymbol{x}, t) = \frac{\boldsymbol{x}}{x^3} \frac{\varrho\omega^4 k_1^{(0)2}}{8\pi c_L^2} M^2 \,|\, u^{(0,J)}(\boldsymbol{k}_{||}^{(0)} \omega \,|\, 0) \hat{f}(\boldsymbol{k}_{||}^l - \boldsymbol{k}_{||}^{(0)}) \,|^2 \left| \frac{M_1^{(J)}(k_{||}^l \omega)}{M_4^{(J)}(k_{||}^l \omega)} \right|^2 \cos\theta^2, \tag{4.6}$$

$$\boldsymbol{q}^{(S,T)} = \frac{\boldsymbol{x}}{x^3} \frac{\varrho\omega^4 k_1^{(0)2}}{8\pi c_T} \,|\, M u^{(0,J)}(\boldsymbol{k}_{||}^{(0)} \omega \,|\, 0) \hat{f}(k_{||}^t - \boldsymbol{k}_{||}^{(0)}) \,|^2 + \\ + \left\{ \left| \frac{M_2^{(0,J)}(k_{||}^t)^2}{M_4^{(J)}(k_{||}^t)} \right| (1 + \mathrm{tg}^2\theta) + |\, M_3^{(J)} \,|^2 \right\}, \tag{4.7}$$

$$\boldsymbol{q}^{(S,R)} = (\boldsymbol{e}_1 \cos\varphi + \boldsymbol{e}_2 \sin\varphi)(\pi c_R^4 x_{||})^{-1} \, 2\omega^5 c_T^2 \,|\, M u^{(0,J)}(\boldsymbol{k}_{||}^{(0)} \omega \,|\, 0) \hat{f}(k_{||}^R - \boldsymbol{k}_{||}^{(0)}) \,|^2 \times \\ \times |\, M_5 M_1^{(J)}(k_{||}^R \omega) \,|^2 \{ \xi_{LL} \exp(-2k_R \beta_L x_3) + \xi_{LT} \exp[-k_R(\beta_L + \beta_T) x_3] + \\ + \xi_{TT} \exp(-2k_R \beta_T x_3) \}. \tag{4.8}$$

Since a twodimensional Fourier transform is inversely proportional to a squared wave vector, the Poynting vector for bulk waves is proportional to ω^2, and for a radial Rayleigh wave it is proportional to ω^3, it is interesting to note that the relation

$$\boldsymbol{q}^{(S)}(\boldsymbol{x}, t) = \boldsymbol{q}^{(S,L)}(\boldsymbol{x}, t) + \boldsymbol{q}^{(S,T)}(\boldsymbol{x}, t) + \boldsymbol{q}^{(S,R)}(\boldsymbol{x}, t) \tag{4.9}$$

does not occur, since the acoustic Poynting vector also includes cross quantities describing the interference of these three waves. In practice, however, it is the quantities defined by expressions (4.6)-(4.8) that are significant, since they define the elements of the scattering matrix for a perturbation regarded as a transducer.

4.4. Power transformation on surface roughnesses. What is often essential in practice is not the absolute value of power transformed but what part of the energy of the incident wave is radiated in the form of scattered wave or waves. It is most simple to define the directional coefficient of power transformation from the wave n into the wave m as

$$\boldsymbol{KW}_{n-m} := \frac{\boldsymbol{q}^{(S,M)}}{|\boldsymbol{q}^{(0,N)}|}. \tag{4.10}$$

When, however $\boldsymbol{q}^{(0,N)}$ is replaced with $\boldsymbol{q}'^{(0,N)}$, for the reasons given in section 4.2., the directional coefficient of power transformation defines the value and direction of energy transport in the wave for the points $\boldsymbol{x}V_1$. These values are given with respect to the energy necessary for the mode n to be generated in a semi-infinite medium. Thus, definition (4.10) should finally take the form

$$\boldsymbol{KW}_{n-m} := \frac{\boldsymbol{q}^{(S,M)}}{|\boldsymbol{q}'^{(0,N)}|}. \tag{4.11}$$

In specific interesting cases of the transformation of or into Rayleigh waves, the acoustic directional power transformation coefficient is

$$\boldsymbol{KW}_{J-R} = \frac{(\boldsymbol{e}_1\cos\varphi + \boldsymbol{e}_2\sin\varphi)\; 4c_T^2\omega^3}{x_{||}\; c_R^4 c_J \pi} \left| u^{(0,J)}(\boldsymbol{k}_{||}^{(0)}\omega \mid 0)\hat{f}(\boldsymbol{k}_{||}^{l} - \boldsymbol{k}_{||}^{(0)}) \times \right.$$
$$\left. \times (M_1^{(J)}(k_{||}R_\omega) M_5 \right|^2 \{\xi_{LL}\exp(-2\beta_L k_R x_3) + \xi_{LT}\exp[-(\beta_L+\beta_T)k_R x_3] +$$
$$+ \xi_{TT}\exp(-2\beta_T k_R x_3)\}, \tag{4.12}$$

where $J = SH, SV, TR, P$,

$$\boldsymbol{KW}_{R-L} = \frac{\boldsymbol{x}}{x^3}\, \frac{k_1^{(0)^2}\omega^3}{4c_L^2 c_T^2 \pi^2} \left| \hat{f}(\boldsymbol{k}_{||}^{l} - \boldsymbol{k}_{||}^{(0)}) \frac{M_1^{(R)}(k_{||}^{l}\omega)}{M_1^{(R)}(k_{||}^{l}\omega)} \right|^2 \cos^2\theta \left(\frac{\xi_{LL}}{2\beta_L} + \right.$$
$$\left. + \frac{\xi_{LT}}{\beta_L+\beta_T} + \frac{\xi_{TT}}{2\beta_T} \right)^{-1}, \tag{4.13}$$

$$\boldsymbol{KW}_{R-T} = \frac{\boldsymbol{x}}{x^3}\, \frac{k_1^{(0)^2}\omega^3}{4c_T^3\pi^2}\; |\hat{f}(\boldsymbol{k}_{||}^{t} - \boldsymbol{k}_{||}^{(0)})|^2 \left\{ \left| \frac{M_2^{(R)}(k_{||}^{t}\omega)}{M_4^{(R)}(k_{||}^{t}\omega)} \right|^2 \times \right.$$
$$\left. \times (1+\mathrm{tg}^2\theta) + |M_3^{(R)}(k_{||}^{t}\omega)|^2 \right\} \left\{ \frac{\xi_{LL}}{2\beta_L} + \frac{\xi_{LT}}{\beta_L+\beta_T} + \frac{\xi_{TT}}{2\beta_T} \right\}, \tag{4.14}$$

$$KW_{R-R} = \frac{(e_1\cos\varphi + e_2\sin\varphi)\,4\omega^4}{x_{||}c_R^4\pi}\,|\,f(k_{||}^R - k_{||}^R)\,M_1^{(R)}(k_{||}^R\omega)\,M_5\,|^2 \times$$
$$\times\left(\frac{\xi_{LL}}{2\beta_L} + \frac{\xi_{LT}}{\beta_L+\beta_T} + \frac{\xi_{TT}}{2\beta_T}\right)^{-1} \xi_{LL}\exp(-2\beta_L k_R x_3) + \xi_{LT}\exp[-(\beta_L+\beta_T)k_R x_3] +$$
$$+\,\xi_{TT}\exp(-2\beta_T k_R x_3)\,. \qquad (4.15)$$

The expressions given above permit the perturbation of the free surface to be regarded as a transducer with five inputs, corresponding to five modes from the unperturbed medium, and three outputs, corresponding to three kinds of scattered waves. The elements of the scattering matrix of such a transducer (giving the magnitude of power obtained at the outputs after supplying an arbitrary mode to the input) are equal to the directional power transformation coefficients calculated in section 4.4. This transducer involves the following transformations:

1) Rayleigh waves into bulk waves (proportional to ω),
2) bulk waves into Rayleigh waves (proportional to ω^{-1}),
3) Rayleigh waves into Rayleigh waves (proportional to ω^0),
4) bulk waves into bulk waves (proportional to ω^0).

The first two transformations are particularly interesting in practice. Transducers for which only these two transformations are considered are called surface-structure transducers [1]. It is interesting to note that when a specific structure is considered (Fig. 1), the quantities $x_{||}$, x are constant and only one mode of the modes SV and TR exists. Such a transducer has thus four inputs and three outputs, and the elements of the scattering matrix are proportional to $\pm\,\omega$. For a twodimensional problem, in view of the lack of scattering of the mode SH into a Rayleigh wave, the matrix of the surface-structure transducer has only three inputs and three outputs. This agrees with the results of paper [1].

5. The effect of the shape of a perturbation on the magnitude of power transformed

The directional power transformation coefficient is proportional to a Fourier transform of the perturbation shape function. This is a general conclusion from expressions (4.12)-(4.15). Very interesting results can, however, be obtained from analysis of a certain class of the perturbation shape function, defined as

$$f(x_1, x_2) = f(x_1) = f_0(x_1) + f_0(x_1+2L) + \ldots + f_0(x_1+2Lm), \qquad (5.1)$$

where $f_0(x_1)$ describes a perturbation over a rectangular area of dimensions $2L\times L_2$. Thus, $f(x_1)$ corresponds to a periodic system of grooves of arbitrary shape, parallel to the $0X_2$ axis. In practice, such a character can be observed in surface-structure transducers, bulk wave resonators.

The values of the Fourier transform $\hat{f}(k_{\|}^{(S)}-k_{\|}^{(0)'})$ can now be analyzed for the points x belonging to the plane $x_1 0 X_2$ (in the spherical coordinate system, this signifies a restriction of the angles φ to a value of zero or π radians)

$$\hat{f}(k_{\|}^{(S)}-k_{\|}^{(0)}) = \sum_{n=0}^{m} L_2 \exp[i(k_1^{(S)}-k_1^{(0)})2Ln]\hat{f}_0(k_1^{(S)}-k_1^{(0)})$$

$$= L_2 \exp[i(k_1^{(S)}-k_1^{(0)})Lm]\frac{\sin(k_1^{(S)}-k_1^{(0)})Lm}{\sin k_1^{(S)}-k_1^{(0)}L}\hat{f}_0(k_1^{(S)}-k_1^{(0)}). \quad (5.2)$$

Consideration of this result in expressions (3.12) and (4.6)-(4.8) leads to the conclusion that the resultant scattered wave is the sum of waves scattered on subsequent grooves; and the scattered power is the power of the resultant scattered wave. The latter statement also signifies that the power radiated on a system of grooves can be expressed with the power radiated on a single groove, modulated by the expression

$$W = \frac{\sin(k_1^{(S)}-k_1^{(0)})Lm}{\sin(k_1^{(S)}-k_1^{(0)})L}. \quad (5.3)$$

This expression has a maximum when the following condition is satisfied,

$$(k_1^{(S)}-k_1^{(0)})L = l\pi, \quad l = 0, \pm 1, \pm 2, \ldots \quad (5.4)$$

These conditions can be written otherwise as

a) for the scattering of Rayleigh waves into bulk waves

$$k_1^{(S)} = k_n \sin\theta\cos\varphi, \quad k_1^{(0)} = -k_R\cos\varphi_0, \quad n = L, T,$$

$$L = \frac{\pi l c_R}{\omega}\left(\frac{c_R}{c_n}\sin\theta\cos\varphi+\cos\varphi_0\right)^{-1}; \quad (5.5)$$

b) for the scattering of the modes SH, P, SV, TR, into Rayleigh waves

$$L = \frac{\pi l c_R}{\omega}\left(\frac{c_R}{c_n}\sin\theta_0\cos\varphi_0+\cos\varphi\right)^{-1}, \quad (5.6)$$

where ω is the frequency of the incident waves, and l is such that for given angles the period L is positive. At the same time, for a stable L expressions (5.5) and (5.6) define the angles at which the value of W is maximum. This is a result of the interference of waves generated on subsequent grooves. For perturbations of a periodic, large number of grooves, these are, in practice, the only directions in which energy is radiated. These conditions provide a design for the construction of transforming structures for which the power transformation coefficient is as large as possible. It is interesting to note that there is a minimum L

$$L_{\min} = \lambda_R\left(1+\frac{\lambda_R}{\lambda_n}\right)^{-1}. \quad (5.7)$$

This signifies that for $L < L_{\min}$ no energy transformation occurs in the surface-structure transducer. This condition is used in devices where this transformation is undesired [9].

6. Conclusion

This paper considered the scattering of acoustic waves (propagating in a semi-infinite medium) on perturbations of a free surface under the assumptions that 1) the function describing the shape of a perturbation takes low values with respect to the wavelength of a propagating wave, and that 2) the ratio of power transformed to the power of the incident wave is considerably lower than unity. This perturbation transforms part of the incident energy, causing generation of a scattered wave in the form of a radial Rayleigh wave and spherical bulk waves: a longitudinal wave and a transverse one. It is interesting to note that this solution is asymptotic. The amplitudes of waves generated depend on the kind of mode, the incidence angle of the mode (for bulk modes), the shape of the perturbation and the amplitude of the incident wave. For a perturbation being a periodic function, relative to one of the coordinates, the scattering of waves of stable frequency shows additional properties: 1) radiation only occurs in some directions, 2) for a period less than some $L_{\min}$, there is no scattering of Rayleigh waves into bulk waves or of bulk waves into Rayleigh waves.

In practice, these perturbations can be regarded as transducers in which the following transformations occur: 1) Rayleigh waves into bulk waves, 2) bulk waves into Rayleigh waves, 3) Rayleigh waves into Rayleigh waves, 4) bulk waves into bulk waves. In these transformations the power ratio of waves generated to the incident waves is proportional, respectively, to the powers of frequency — 1) + the first, 2) — the first, 3) the zeroth. Additional properties of structures with periodic perturbations permit the designing of optimum transformation.

Acknowledgement. This paper was written in the course of the author's doctoral studies at the Institute of Fundamental Technological Research. The author wishes to express his gratitude to prof. I. Malecki and Prof. A. Opilski for their advice.

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Received on October 3, 1979 ; revised version on June 10, 1981.

Appendix

Integral (3.2) can be calculated for the case of the scattering of the mode SH. According to expression (2.1), only the second component of the displacement vector is different from zero, and the desired solution takes thus the form

$$u_a(x, t) = \exp(-i\omega t) \sum_{\beta} \int d^2 k_{||} (2\pi)^{-2} \int_0^{\infty} dx_3' \exp(ik_{||} x_{||}) D_{a\beta}(k_{||}\omega | \times$$
$$\times x_3 x_3') \int d^2 x_{||}' \exp(-ik_{||} x_{||}) L_{\beta 2}^{(1)}(x') \exp(ik_{||}^{(0)} x_{||}') u_2^{(0,SH)}(k_{||}^{(0)}\omega \mid 0). \quad \text{(A1)}$$

The operators $L_{\beta\gamma}^{(1)}(x)$ are defined as

$$L_{\beta\gamma}^{(1)}(x) = \frac{1}{\varrho} \sum_{au} \frac{\partial c_{\beta a \gamma u}^{(1)} \partial}{\partial x_a \partial x_u} + \frac{1}{\varrho} \sum_{au} c_{\beta a \gamma u}^{(1)} \frac{\partial^2}{\partial x_a \partial x_u}, \quad \text{(A2)}$$

and the elastic constants are defined as

$$c_{\beta a \gamma u}(x) = -f(x_1, x_2) c_{\beta a \gamma u}, \quad \text{(A3)}$$

where $x_3 = f(x_1, x_2)$ is a function of perturbation shape. The necessary operators $L^{(1)}_{\beta 2}(x)$ are, respectively,

$$L^{(1)}_{12}(\boldsymbol{x}) = -c_T^2\delta(x_3)\frac{\partial f(x_1, x_2)}{\partial x_2}\frac{\partial}{\partial x_2},$$

$$L^{(1)}_{22}(\boldsymbol{x}) = -c_T^2\delta(x_3)\left\{\frac{\partial f}{\partial x_1}\frac{\partial}{\partial x_1}+f\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)\right\}-c_T^2\delta'(x_3)f\frac{\partial}{\partial x_3}, \qquad \text{(A4)}$$

$$L^{(1)}_{32}(\boldsymbol{x}) = -c_T^2\delta(x_3)\frac{\partial f}{\partial x_2}\frac{\partial}{\partial x_3}.$$

Using (A4), equation (A1) becomes finally

$$u_a^{(S)}(\boldsymbol{x}, t) = \exp(-i\omega t)\int d^2k_{||}(2\pi)^{-2}ik_1^{(0)}\hat{f}(\boldsymbol{k}_{||}-\boldsymbol{k}_{||}^{(0)})\{k_2D_{a1}+k_1D_{a2}\}\exp(i\boldsymbol{k}_{||}\boldsymbol{x}_{||}). \qquad \text{(A5)}$$

Substitution of the explicit form of the Green function for a semiinfinite region gives the displacement vector for scattered waves in the form

$$\boldsymbol{u}^{(S)}(\boldsymbol{x}, t)\exp(i\omega t) = \int d^2k_{||}\frac{u^{(l)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega)}{4\pi^2r_+(k_{||}\omega)}\left[\boldsymbol{e}_1k_1+\boldsymbol{e}_2k_2+\boldsymbol{e}_3i\frac{a_L}{k_{||}}\right]\times$$
$$\times\exp(-a_Lx_3+i\boldsymbol{k}_{||}\boldsymbol{x}_{||})+\int d^2k_{||}\frac{u^{(ta)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega)}{4\pi^2r_+(k_{||}\omega)}\left[\boldsymbol{e}_1k_1+\boldsymbol{e}_2k_2+\boldsymbol{e}_3i\frac{k_{||}}{a_L}\right]\times$$
$$\times\exp(-a_Tx_3+i\boldsymbol{k}_{||}\boldsymbol{x}_{||})+\int d^2k_{||}(2\pi)^{-2}u^{(tb)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega)[\boldsymbol{e}_1k_2-\boldsymbol{e}_2k_1]\times$$
$$\times\exp(-a_Tx_3+i\boldsymbol{k}_{||}\boldsymbol{x}_{||}), \qquad \text{(A6)}$$

where

$$r_+(\boldsymbol{k}_{||}\omega) = \frac{4a_Tc_T^2\boldsymbol{k}^2+(\omega^2-2c_T\boldsymbol{k}_{||}^2)(a_T^2-\boldsymbol{k}_{||}^2)}{4a_Ta_L(\omega^2-2c_T^2\boldsymbol{k}_{||}^2)},$$

$$u^{(l)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega) = Mu_2^{(0,SH)}(\boldsymbol{k}_{||}^{(0)}\omega\mid 0)\hat{f}(\boldsymbol{k}_{||}-\boldsymbol{k}_{||}^{(0)})\frac{ik_1^{(0)}\hat{k}_1k_2k_{||}^3}{a_L(a_T^2+k_{||}^2)},$$

$$u^{(ta)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega) = Mu_2^{(0,SH)}(\boldsymbol{k}_{||}^{(0)}\omega\mid 0)\hat{f}(\boldsymbol{k}_{||}-\boldsymbol{k}_{||}^{(0)})\frac{ik_1^{(0)}\hat{k}_1\hat{k}_2k_{||}}{2a_L},$$

$$u^{(tb)}(\boldsymbol{k}_{||}\boldsymbol{k}_{||}^{(0)}\omega) = Mu_2^{(0,SH)}(\boldsymbol{k}_{||}^{(0)}\omega\mid 0)\hat{f}(\boldsymbol{k}_{||}-\boldsymbol{k}_{||}^{(0)})\frac{ik_1k_{||}(\hat{k}_1^2-\hat{k}_2^2)}{a_L}.$$

The integrals occurring in expression (A6) are those of types considered in [6]. An asymptotic form of the solution of these integrals has already been given in the main text of the paper.