ARCHIVES OF ACOUSTICS
8, 4, 341-354 (1983)

THE APPLICATION OF FOURIER INTEGRAL TRANSFORMS IN A GENERAL THEORY OF DIFFRACTION

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Integral transforms have frequently been used to solve different specific problems in diffraction. There has, however, been no application of this method to the fundamental equations of diffraction theory.

The present author shows that a transition from the d'Alambert equation to the Helmholtz equation for transforms gives all pulse fields, i.e. the acoustic potential of this field as the inverse transform of the product of the transform of the time behaviour of the pulse and the potential for a harmonic wave. This method permits relatively easy calculations of the sound pulse fields, with the additional assumption that the pulse distribution on the source can be represented as the product of a position-dependent function and one which is time-dependent.

1. Introduction

It is now generally known that the use of integral transformation for arbitrary (nonharmonic) time behaviour facilitates to a large extent the solution of diffraction problems. Papers [4, 5, 7-12, 15] in which different integral transformations were used in the problems of pulse diffraction at wedges or half-planes are now classical. It was found in [6] that a Fourier integral transform changes the d'Alambert equation into the Helmholtz one and the authors were thus able to calculate the field of a pulse-excited point source.

To date, however, there has been no general formulation of this problem, i.e. the use of integral transformation in the fundamental, general problems of diffraction theory. It is to these problems that the present paper is devoted.

It is assumed that all pulses considered below do not exceed the conditions set in linear acoustics. The theory of nonlinear pulses requires a completely different approach, while this subject would go far beyond the framework of the present paper.

It is also assumed that the pulse which excites the sound source can be given in the form of the product of a function dependent on spatial variables and a function dependent on time.

2. Theory

The so-called general theory of harmonic wave diffraction is concerned with the solution of the Helmholtz equation with definite boundary conditions

$$\Delta\varphi + k^2\varphi = 0, \tag{1.1}$$

where φ is the acoustic potential and k is a wave number. In this case the time dependence factor $\exp(\pm i\omega t)$ is neglected and the acoustic potential regarded as a function of position, which can symbolically be given as

$$\varphi = \varphi(x_i), \quad i = 1, 2, 3. \tag{1.2}$$

Naturally, the physical phenomenon in the acoustic field is, according to the accepted convention, represented by one of the products

$$\varphi(x_i)\exp(i\omega t), \quad \varphi(x_i)\exp(-i\omega t). \tag{1.3}$$

Equation (1.1) is solved in volume area V limited by a closed surface S on which sound sources are distributed. The sources show the vibration amplitude

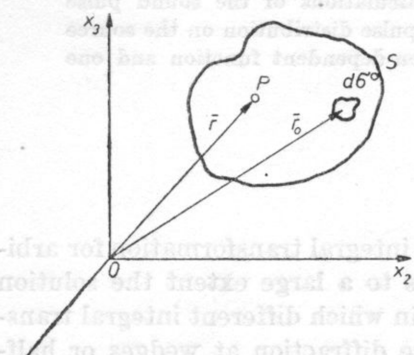

Fig. 1. The geometry of the radiating system

distribution $u_0(x_i)$, where, according to the definition of acoustic potential, φ,

$$u_0(x_i) = -\left(\frac{\partial\varphi}{\partial n}\right)_{S_0}, \tag{1.4}$$

which is shown in Fig. 1. Although the basic literature [13], [16] gives the so-called Poisson's integral formula which generalizes harmonic wave theory to arbitrary time behaviour, but this formula is very complicated and no known achievements in pulse diffraction theory have been based on it.

The present paper is concerned with nonharmonic processes; it is assumed that the acoustic potential is a function of position and time:

$$\varphi = \varphi(x_i, t), \tag{1.5}$$

i.e. it satisfies the d'Alambert wave function

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0 \tag{1.6}$$

in the volume V with the boundary conditions given above. In order to carry out Fourier integral transformation, the transform of the acoustic potential (1.5) should first be written in the form [1]

$$\Phi(x_i, \omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \varphi(x_i, t)\exp(-i\omega t)\,dt. \tag{1.7}$$

Consideration that [1]

$$\frac{\partial^2\varphi}{\partial t^2} \underset{\rightarrow}{\rightarrow} i^2\omega^2\varphi \tag{1.8}$$

gives from (1.6) the transform equation in the form

$$\Delta\Phi + \frac{\omega^2}{c^2}\Phi = 0, \tag{1.9}$$

i.e.

$$\Delta\Phi + k^2\Phi = 0. \tag{1.10}$$

(1.10) is a Helmholtz equation, but the transform $\Phi(x_i)$ must satisfy also those conditions which are set in the classical theory of harmonic waves for the potential itself. As it was mentioned in the **Introduction**, the authors of paper [6] took as the basis the fact that the acoustic potential transform satisfies the Helmholtz equation and used it in the case of a pulse-excited point source. In the present paper this problem will be considered in most general terms. Since the solution of the Helmholtz equation consists in differential and integral operations on Φ with respect to spatial variables, all solutions of the Helmholtz equation, i.e. for the harmonic wave potential, give simultaneously the expression of the Fourier transform of the potential of an arbitrary time pulse.

It can now be considered how the above statement, whose form does not seem so far to be precise enough, can be used in practice.

In the problems related to wave diffraction the following procedure can be used: the same Green function as that for harmonic waves [13], [16] is introduced for the transform $\Phi(x_i, \omega)$:

$$\Phi(x_i, \omega) = \int_S \left[G(x_i, x_i^0)\frac{\partial\Phi(x_i^0)}{\partial n} - \Phi(x_i^0)\frac{\partial G(x_i, x_i^0)}{\partial n} \right] d\sigma^0, \tag{1.11}$$

and in the case of the so-called acoustic Green function [16] satisfying the condition

$$\frac{\partial G}{\partial n} = 0 \quad \text{on the surface } S \tag{1.12}$$

this gives finally

$$\Phi(x_i, \omega) = \int_S G(x_i, x_i^0) \frac{\partial \Phi(x_i^0)}{\partial n} d\sigma^0. \tag{1.13}$$

In order to prevent their form becoming too complicated, in the above fomulae the obvious dependence on ω was not given under the integral. The symbol o refers to the sound source.

The derivative $\partial\Phi/\partial n$ can now be interpreted on the surface of the sound source. From the transformation formula (1.17)

$$\frac{\partial \Phi}{\partial n} = \frac{\partial}{\partial n} \int_{-\infty}^{\infty} \varphi(x_i, t)\exp(-i\omega t)\, dt = \int_{-\infty}^{\infty} \frac{\partial \varphi(x_i, t)}{\partial n} \exp(-i\omega t)\, dt, \tag{1.14}$$

and from the definition of the acoustic potential (1.4)

$$\frac{\partial \Phi}{\partial n} = -\int_{-\infty}^{\infty} u_0(x_i, t)\exp(-i\omega t)\, dt = -U_0(x_i, \omega). \tag{1.15}$$

It can be seen that the derivative $\partial\Phi/\partial n$ represents the Fourier transform of the vibration velocity on the sound source. It should be stressed that in this case $u_0(x_i, t)$ does not denote the vibration velocity amplitude but the time behaviour of the pulse velocity, which in a general case is also a function of position. Since the direction outside from the volume V is taken as the positive direction of the normal and the opposite direction as the positve direction of the vibration velocity, from formula (1.15), (1.13) can now be written as

$$\Phi(x_i, \omega) = \int_S G(x_i, x_i^0)\, U_0(x_i^0, \omega)\, d\sigma^0. \tag{1.16}$$

Particularly for a half-space bounded by a rigid plane on which the sound sources lie, using the notation from Fig. 1, the Green function can be given in the form

$$G(\boldsymbol{r}, \boldsymbol{r}_0) = \frac{1}{2\pi} \frac{\exp(ik|\boldsymbol{r} - \boldsymbol{r}_0|)}{|\boldsymbol{r} - \boldsymbol{r}_0|}. \tag{1.17}$$

Formula (1.16) which represents the acoustic potential tranform as a function of position and the angular frequency ω now becomes

$$\Phi(\boldsymbol{r}, \omega) = \frac{1}{2\pi} \int_{S^0} \frac{\exp(ik|\boldsymbol{r} - \boldsymbol{r}_0|)}{|\boldsymbol{r} - \boldsymbol{r}_0|} U_0(\boldsymbol{r}_0, \omega)\, d\sigma^0. \tag{1.18}$$

In practical cases it is almost always possible to represent the pulse velocity distribution as the product of the position and time functions, i.e. to assume that all points of the source are excited by the same time behaviour. In such a case

$$u_0(x_i^0, t) = u_0(x_i^0) f(t). \tag{1.19}$$

Using the Fourier transform (1.15) in $u_0(x_i, t)$ only the time function $f(t)$ is transformed, i.e.

$$\frac{\partial \Phi}{\partial n} = -u_0(x_i) \int_{-\infty}^{\infty} f(t) \exp(-i\omega t)\, dt = -u_0(x_i^0) F(\omega), \tag{1.20}$$

where $F(\omega)$ is the transform of the function representing the time behaviou of the pulse $f(t)$. Therefore, using the notation from Fig. 1,

$$U_0(\boldsymbol{r}_0, \omega) = u_0(\boldsymbol{r}_0) F(\omega). \tag{1.21}$$

Since the spatial variables are integrated in formula (1.16), this formula can be written in the form of (1.21):

$$\Phi(\boldsymbol{r}, \omega) = F(\omega) \int_{S_0} G(\boldsymbol{r}, \boldsymbol{r}_0) u_0(\boldsymbol{r}_0)\, d\sigma^0, \tag{1.22}$$

and in a specific case for a half-space

$$\Phi(\boldsymbol{r}, \omega) = \frac{F(\omega)}{2\pi} \int_{S_0} \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}_0|)}{|\boldsymbol{r}-\boldsymbol{r}_0|} u_0(\boldsymbol{r}_0)\, d\sigma^0. \tag{1.23}$$

Formulae (1.22) and (1.23) permit some very important conclusions to be drawn. Considering that the wave number $k = \omega/c$ it can be seen that the integral

$$\Phi_s(\boldsymbol{r}, \omega) = \int_{S_0} G(\boldsymbol{r}, \boldsymbol{r}_0) u_0(\boldsymbol{r}_0)\, d\sigma^0 \tag{1.24}$$

or the integral

$$\Phi_s(\boldsymbol{r}, \omega) = \frac{1}{2\pi} \int_{S_0} \frac{\exp\left(i\frac{\omega}{c}|\boldsymbol{r}-\boldsymbol{r}_0|\right)}{|\boldsymbol{r}-\boldsymbol{r}_0|} u_0(\boldsymbol{r}_0)\, d\sigma^0 \tag{1.25}$$

represent the spatial distribution of the acoustic field of a harmonic wave as determined from the amplitude distribution such as the spatial pulse distribution $u_0(\boldsymbol{r}_0)$. In addition $\Phi_s(\boldsymbol{r}, \omega)$ can be replaced with all solutions of the Huyghens integral formula (1.24) known from harmonic wave acoustics. Knowing $\Phi_s(\boldsymbol{r}, \omega)$ this value should be multiplied by the transform $F(\omega)$ of the time behaviour, achieving the acoustic potential transform of a real pulse field

$$\Phi(\boldsymbol{r}, \omega) = F(\omega) \Phi_s(\boldsymbol{r}, \omega). \tag{1.26}$$

Such a simplification is valid only when formula (1.19) can be used.

The potential of the acoustic field is determined as the inverse Fourier transform of formula (1.26), i.e. this potential is given as the integral [1]

$$\Phi(\boldsymbol{r}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) \Phi_s(\boldsymbol{r}, \omega) \exp(i\omega t) d\omega. \tag{1.27}$$

In the case when the source receives a pulse in the form of the Dirac distribution $\delta(t)$ and in view of the fact that the Fourier transform of this pulse has a value of unity [1]:

$$F_D(\omega) = 1, \tag{1.28}$$

the potential of the acoustic field is given directly in the form of the integral

$$\varphi_D(\boldsymbol{r}, t) = \int_{-\infty}^{\infty} \Phi_s(\boldsymbol{r}, \omega) \exp(i\omega t) d\omega. \tag{1.29}$$

Formula (1.29) indicates that all acoustic fields calculated for harmonic waves can be regarded as Fourier transforms for Dirac pulses while integral (1.29) transforms it into the field of these pulses.

It should be pointed out that the above train of thought is most general and can be applied to any acoustic field, also including fields of diffracted waves.

In considering the problems in wave diffraction by an obstacle two specific cases can be distinguished, i.e. an ideal rigid obstacle on which, S_p, the summary acoustic field, which is the field of incident and reflected waves, satisfies the condition [16]

$$\left(\frac{\partial \varphi_s}{\partial n}\right)_{S_p} = 0, \tag{1.30}$$

and an ideal compliant obstacle on which the summary acoustic field gives zero acoustic pressure. For harmonic waves this is equivalent to the condition [13], [16]

$$\varphi = 0 \quad \text{on } S_p. \tag{1.31}$$

In the case of pulses the relavant condition (1.31) has the form [16]

$$p = \varrho \frac{\partial \varphi}{\partial t} = 0 \quad \text{on } S_p, \tag{1.32}$$

i.e., in view of the fact that ϱ denotes here the density of the medium in rest, there is the condition

$$\frac{\partial \varphi}{\partial t} = 0 \quad \text{on } S_p. \tag{1.33}$$

In a general case there is the so-called impedance condition. Designating as z^0 the impedance of the obstacle surface by which waves are diffracted, according to the definition of acoustic impedance [13], [16],

$$\frac{p}{u} = \varrho \frac{(\partial\varphi/\partial t)_{S_0}}{(\partial\varphi/\partial n)_{S_0}} = z_p(x_i, \omega). \tag{1.34}$$

In a general case, the acoustic impedance on the diffracting surface can be a function of position and frequency, whereas the acoustic field on the surface should be such that p/u is independent of time, which would limit the class of pulses for which the diffraction problem can be solved with general impedance conditions.

Wishing to apply the concept given at the beginning of this section to diffraction problems, i.e. wishing to regard the potential of the harmonic field of diffracted waves as the Fourier transform of the pulse (after multiplication by $F(\omega)$), one must first investigate the boundary conditions for this transform. In view of the fact that the impedance conditions occur infrequently, the previous order will be retained, i.e. ideal rigid, compliant and, finnally, "impedance" surfaces will be considered in that succession. From formula (1.15) condition (1.30), after Fourier transformation, in view of the additivity of the transforms, requires that the following equation should occur for the summary acoustic field,

$$\frac{\partial\Phi_s}{\partial n} = 0 \quad \text{on } S_p \tag{1.35}$$

for an ideal rigid surface. In turn on the ideal compliant surface the derivative $\partial\varphi_s/\partial t$ undergoes Fourier transformation, i.e., in view of formula (1.8), there is the condition [1]

$$\Phi_s = 0 \quad \text{on } S_p. \tag{1.36}$$

It can be seen that in the two specific cases, infinitely rigid and infinitely compliant wave diffracting obstacles, the boundary conditions for the transform are the same as those for the acoustic potential of harmonic waves. This permits any acoustic field of diffracted harmonic waves to be treated as the field of the pulse transform Φ_s, which when multiplied by the transform of the time behaviour $F(\omega)$ gives the transform of the diffracted pulse field and the desired acoustic field in the form of integral (1.27) or integral (1.29) for the Dirac pulse.

For the impedance condition (1.34)

$$\varrho\left(\frac{\partial\varphi}{\partial t}\right)_{S_p} = z_p(x_i, \omega)\left(\frac{\partial\varphi}{\partial n}\right)_{S_p}. \tag{1.37}$$

In formula (1.37) both the acoustic potential φ and its derivatives $(\partial\varphi/\partial t)_{S_p}$ and $(\partial\varphi/\partial n)_{S_p}$ are time functions, whereas the impedance of the surface is independent of time. A direct Fourier transform of equation (1.37) gives

$$\varrho \int_{-\infty}^{\infty} \left(\frac{\partial \varphi s}{\partial t}\right)_{S_p} \exp(-i\omega t)\,dt = z_p(x_i, \omega) \int_{-\infty}^{\infty} \left(\frac{\partial \varphi s}{\partial n}\right)_{S_p} \exp(-i\omega t)\,dt. \quad (1.38)$$

In formula (1.38) the impedance, as it is independent of time, was moved before the integral sign.

From formulae (1.8) and (1.15), formula (1.38) becomes

$$\omega\varrho \Phi_s(x_i, \omega) = i z_p(x_i, \omega)\, U_{os}(x_i, \omega), \quad (1.39)$$

i.e.

$$\frac{\varrho \Phi_s(x_i, \omega)}{U_{os}(x_i, \omega)} = \frac{i}{\omega} z_p(x_i, \omega). \quad (1.40)$$

Formula (1.34) which represents the impedance condition takes in the case of a harmonic wave the form ((1.32) for $\varphi(x_i, t) = \varphi(x_i)\exp(-i\omega t)$)

$$\frac{\varrho \varphi(x_i)}{u_0(x_i)} = \frac{i}{\omega} z_p(x_i), \quad (1.41)$$

i.e. the same as (1.40).

In all cases the boundary conditions are the same for transforms as those for the acoustic potential, permitting the pulses from the diffracted acoustic fields of harmonic waves to be determined by the present acoustic field method. This approach facilitates to a large extent the solution of the problems of pulse diffraction by obstacles of different shape, and when the solution for harmonic waves is known the pulse field is obtained in the form of a single integral (1.27) which can be evaluated analytically or numerically.

3. Applications of the theory

The theory given in section 1 can be illustrated by such a large number of examples that this would exceed the range of the present paper. In principle it is enough to multiply all solutions for a harmonic acoustic field by the transform of the relevant pulse and as a result the transform of the pulse field potential can be achieved.

Since, as it was already mentioned, various integral transformations have been used to solve the problem of wave diffraction by a wedge, only a few examples will be given here to illustrate direct source radiation.

1. A system of two point sources distance d (Fig. 2) apart will be given as the first example. Both of the sources receive the same Dirac pulse δ at a time $t = \tau$. The acoustic field distribution in the far field will now be investigated. The vibration velocity amplitude in the formula of the far field potential can be

assumed to have a value of unity. The formula for a harmonic wave [13], [16] can be written directly as the formula of the transform for the Dirac pulse

$$\Phi_D(P,t) = \frac{\exp\left(-i\frac{\omega}{c}r_0\right)}{r_0}\exp\left(i\frac{\omega d}{2c}\sin\gamma\right)\cos\left(\frac{\omega d}{2c}\sin\gamma\right). \tag{2.1}$$

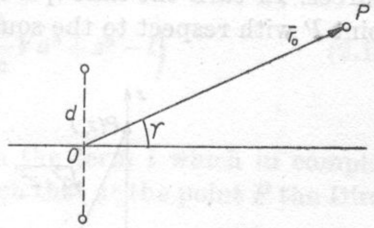

Fig. 2. A system of two point sources

The acoustic potential can be achieved as the inverse transform of [2.1), i.e. given by the integral

$$\varphi_D(P,t) = \frac{1}{r_0}\int_{-\infty}^{\infty} \exp\left[i\omega\left(\frac{d}{2c}\sin\gamma - \frac{r_0}{c}+t\right)\right]\cos\left(\frac{\omega d}{2c}\sin\gamma\right)d\omega. \tag{2.2}$$

In integration of (2.2) only the even part remains, i.e.

$$\varphi_D(P,t) = \frac{2}{r_0}\int_0^{\infty}\cos\omega\left(\frac{d}{c}\sin\gamma - \frac{r_0}{c}+t\right)d\omega + \frac{1}{r_0}\int_0^{\infty}\cos\left(\frac{\omega d}{2c}\sin\gamma\right)d\omega. \tag{2.3}$$

The use of the formula of cosine product in the subintegral expression gives [3]

$$\varphi_D(P,t) = \frac{1}{r_0}\int_0^{\infty}\cos\omega\left(\frac{d}{c}\sin\gamma - \frac{r_0}{c}+t\right)d\omega + \frac{1}{r_0}\int_0^{\infty}\cos\omega\left(\frac{r_0}{c}-t\right)d\omega. \tag{2.4}$$

The two integrals represent the Dirac distribution, according to the formula [2]

$$\int_0^{\infty}\cos(ax)\,dx = \pi\delta(a). \tag{2.5}$$

Finally,

$$\varphi_D(P,t) = \frac{\pi}{r_0}\delta\left(\frac{d}{c}\sin\gamma - \frac{r_0}{c}+t\right) + \frac{\pi}{r_0}\delta\left(\frac{r_0}{c}-t\right). \tag{2.6}$$

It can be seen that at the point P of the far field the Dirac pulse δ occurs twice, i.e. at a time t_1:

$$t_1 = \frac{r_0}{c} - \frac{d}{c}\sin\gamma \tag{2.7}$$

and at a time t_2:

$$t_2 = \frac{r_0}{c}. \tag{2.8}$$

The time t_2 is the time required for the pulse to reach the point P from the centre of the system, i.e. from the point 0 on the section d between the sources. In turn the time t_1 is caused by the asymmetry of the position of the point P with respect to the sources, and in the case when $\gamma = 0$ $t_1 = t_2 = r_0/c$.

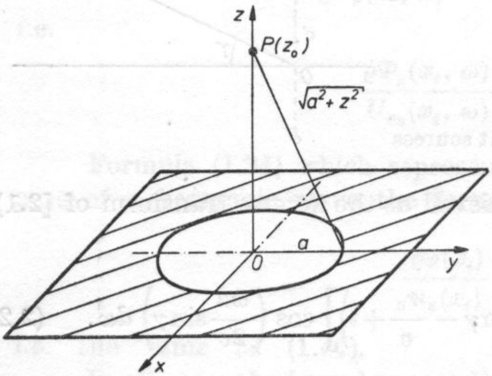

Fig. 3. The piston in the baffle

2. Another example is provided by the near field on the so-called axis of the acoustic system consisting of a rigid piston, i.e. a piston which vibrates all over its surface, placed in an infinite, rigid baffle (Fig. 3), when it receives a Dirac pulse. Assuming the vibration velocity amplitude in the formula of the harmonic solution to have a value of unity, the near field expression [13], [16] can be given in the form of the transform of the Dirac pulse

$$\Phi_D(z, \omega) = 2\varrho c \exp\left[-i\frac{\omega}{2c}(\sqrt{a^2+z^2}+z)\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right]. \tag{2.9}$$

The acoustic potential itself on the z axis thus becomes

$$\varphi(z, t) = 2\varrho c \int\limits_{-\infty}^{\infty} \exp\left[-i\frac{\omega}{2c}(\sqrt{a^2+z^2}+z)\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right]\exp(i\omega t)\,d\omega. \tag{2.10}$$

Like in the previous case, in integration only the even component remains and therefore

$$\varphi(z, t) = -2i\int\limits_{0}^{\infty} \sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}+z)-\omega t\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right]d\omega. \tag{2.11}$$

The use of the formula of cosine product on the left of (2.11) [3] gives

$$\varphi(z,t) = -i\int_0^\infty \cos\omega\left(\frac{z}{c}-t\right)d\omega + i\int_0^\infty \cos\omega\left(\frac{1}{c}(\sqrt{a^2+z^2}-t\right)d\omega. \quad (2.12)$$

Using formula (2.5) again

$$\varphi(z,t) = -i\pi\delta\left(\frac{z}{c}-t\right)+i\pi\delta\left(\frac{1}{c}\sqrt{a^2+z^2}-t\right) \quad (2.13)$$

(see formula (17) in [14]).

This result is very interesting. Apart from the term i which in complex number notation denotes phase shift, it can be seen that at the point P the Dirac pulse occurs twice, namely after the time t_1:

$$t_1 = \frac{z}{c} \quad (2.14)$$

— this is a pulse coming from the centre of the piston and after the time t_2:

$$t_2 = \frac{\sqrt{a^2+z^2}}{c}. \quad (2.15)$$

The time t_2 is the time required for a pulse from the circumference of the piston to reach the point P. The action of the piston at all points P in the near field can be reduced to the superposition of the pulse from the centre and the one from the circumference.

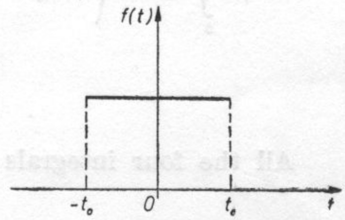

Fig. 4. The shape of the rectangular pulse

3. The final example to be considered here is a piston which is the same as that in example 2, but which receives a rectangular pulse (Fig. 4) with height of unity and active from $t = -t_0$ to $t = t_0$. The function $f(t)$ is here the distribution

$$f(t) = \begin{cases} 1, & -t_0 \leqslant t \leqslant t_0 \\ 0, & t < -t_0;\ t > t_0. \end{cases} \quad (2.16)$$

In view of the symmetry of $f(t)$ the transform $F(\omega)$ can be written in the form of the so-called cosine transform [1]

$$F(\omega)=\int_0^{\infty} f(t)\cos(\omega t)\,dt=\frac{2\sin(\omega t_0)}{\omega}. \tag{2.17}$$

From formula (1.26) the acoustic potential transform becomes

$$\Phi(z,\omega)=\frac{2\sin(\omega t_0)}{\omega}\exp\left[-i\frac{\omega}{2c}(\sqrt{a^2+z^2}+z)\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right], \tag{2.18}$$

whereas the acoustic potential itself on the z axis takes the form of the transform inverse to (2.18):

$$\varphi(z,t)=2c\int_{-\infty}^{+\infty}\frac{\sin(\omega t_0)}{\omega}\exp\left[-i\omega\left(\frac{\sqrt{a^2+z^2}}{c}-t\right)\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right]d\omega. \tag{2.19}$$

Considering the odd and even form of the subintegral function

$$\varphi(z,t)=4i\int_0^{\infty}\frac{\sin(\omega t_0)}{\omega}\sin\left[\omega\left(t-\frac{\sqrt{a^2+z^2}+z}{2c}\right)\right]\sin\left[\frac{\omega}{2c}(\sqrt{a^2+z^2}-z)\right]d\omega. \tag{2.20}$$

The use of the formula of triple sine product [3] gives

$$\varphi(z,t)=i\int_0^{\infty}\sin\omega\left(t+t_0-\frac{1}{c}\sqrt{a^2+z^2}\right)\frac{d\omega}{\omega}+$$

$$+i\int_0^{\infty}\sin\omega\left(t-t_0-\frac{z}{c}\right)\frac{d\omega}{\omega}-i\int_0^{\infty}\sin\omega\left(t-t_0+\frac{1}{c}\sqrt{a^2+z^2}\right)\frac{d\omega}{\omega}+$$

$$+i\int_0^{\infty}\sin\omega\left(t-t_0-\frac{z}{c}\right)\frac{d\omega}{\omega}. \tag{2.21}$$

All the four integrals have the form [3]

$$\int_0^{\infty}\sin_\omega(m\omega)\,d\omega=\begin{cases}\dfrac{\pi}{2}, & m>0,\\ 0, & m=0,\\ -\dfrac{\pi}{2}, & m<0.\end{cases} \tag{2.22}$$

It follows from the analysis of the value of the individual integrals that they reduce so that the gate pulse does not occur at the point P until the time $(z/c)-t_0$ and lasts till the time $(\sqrt{a^2+z^2/c})-t_0$.

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Received on 1 March, 1983