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DYNAMIC FOCUSING OF AN ULTRASONIC BEAM BY MEANS OF A PHASED ANNULAR ARRAY USING A PULSE TECHNIQUE

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An approach to compute the transient radiation resulting from an impulse velocity motion of an array of annular pistons in a rigid planar infinite baffle is presented. The approach is based on developing the expression for an impulse response function, which is the time-dependent velocity potential at a spatial point resulting from an impulse velocity of a piston. The time-dependent pressure for any piston velocity motion may then be computed by a convolution of the piston velocity with the appropriate impulse response.

Numerical results of near field time-dependent radiation from annular phased array are discussed for pulsed velocity conditions. The lateral acoustical pressure distribution at different field depths is shown. The ultrasonic beamwidth as a function of the depth for every focal zone is also presented. Obtained results were compared with corresponding dependences for steady states.

Notation

a_{nw} — the internal radius of the annular piston
a_{nz} — the external radius of the annular piston
a — the radius of the circular piston
Δb — the ultrasonic beamwidth
c — wave propagation velocity
$h(\mathbf{r}, t)$ — the impulse response
$\mathring{h}(\mathbf{r}, t)$ — the time derivative of the impulse response
N — the number of rings
n — ring number
p — sound pressure
$\mathbf{r}$ — distance from the point source
S_1 — the radiating surface
t — time

$v(\mathbf{r}, t)$ – the velocity motion of the radiating surface
x, x_1, y, y_1 – coordinates parallel to the radiating surface
z – a coordinate perpendicular to the radiating surface
ϱ – the density of the medium
τ_0 – duration of the pulse
$\Delta\tau$ – time delay in excitation
φ – acoustic potential
ω – angular frequency

1. Introduction

The main technique in ultrasonic diagnostics has recently been the pulse technique based on echosonography with B — scanning, permitting visualization of the internal body organs and thus providing information on the dimensions, localization and character of structures under investigation. One of the fundamental problems involved in this technique is the insufficient resolution which results from the finite lateral dimensions of the ultrasonic beam radiated. Namely, information is obtained as cross-sectional image of the organ of interest and consists of a limited number of lines forming the image on the monitor. An increase in the density of these lines and a narrowing of the ultrasonic beam permit identification of internal structures in greater detail and give a more precise image of them, thus making its interpretation easier.

The lateral resolution of the equipment is defined primarily by lateral dimensions of the ultrasonic beam radiated along the whole penetration depth. The method of dynamic focusing, based on the phased array principle, is the most effective method providing minimum lateral dimensions of the ultrasonic beam along the whole space of structures examined. The phased array consists of a large number of piezoelectric transducers excited with a specified time delay, permitting focusing at a desired point on the axis. Adjusting the time delay of the excitation of each transducer with delay lines, it is possible to change the curvature of the wave front radiated and thus to move the focus along the axis. Visualization is performed by means of a mechanical sector scanning of the beam.

Considering the fact that point focusing involves a sharp narrowing of the ultrasonic beam not only in the geometrical focus but also in its direct vicinity, the whole observation depth can be divided into focal zones with a geometrical focus inside each zone. Thus, switching the focus during reception time from a closer zone into an increasingly deeper zone in succession, the ultrasonic beam is sharply focussed along the whole range of the observation depth. The speed of the switching is conditioned by the propagation velocity of the waves in the area examined and depends on the width of focal zones. Fig. 1 shows a phased array of annular transducers with four focal zones and the corresponding fronts of the wave received. The use of annular transducers with axial symmetry permits the same lateral resolution to be obtained in all lateral directions for a given observation depth.

The previous theoretical analysis [5] performed for the dependence of the lateral resolution of a phased annular array on dimensions, configuration and excitation method was concerned only with steady states. In view of the predicted use of the system in gynaecology and obstetrics a frequency of 2.5 MHz ($\lambda = 0.6$) mm was chosen as the fundamental resonance frequency of the transducers and an observation area within 4 cm to 24 cm from the body surface [4] was used.

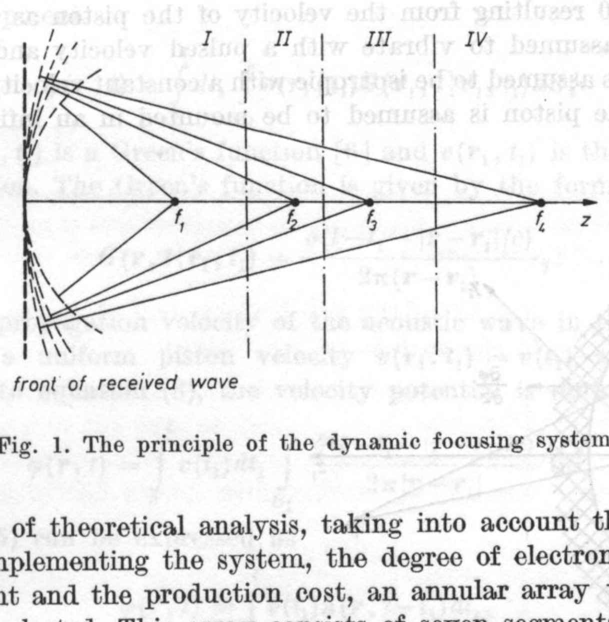

Fig. 1. The principle of the dynamic focusing system

As a result of theoretical analysis, taking into account the technological possibility of implementing the system, the degree of electronic complication of the equipment and the production cost, an annular array with a diamete of 40 mm was selected. This array consists of seven segments (six rings and a central disc) with equal surfaces, which secures a good matching with the electrical part of the equipment and effective focusing of the ultrasonic beam along the whole range of penetration depth divided into five focal zones with the geometrical focus at the distances of 4, 6, 8, 11 and 18 cm, respectively, from the radiating surface.

The aim of the present paper is to perform a theoretical analysis of the lateral resolution of a chosen phased annular array when its segments are excited by short pulses. This will permit comparison of the results obtained with the results concerning steady states and the necessary conclusions to be drawn.

2. Theoretical basis and analysis of the problem

The problem lies in calculation of the near-field transient radiation resulting from nonharmonic velocity motion of an array of annular surfaces. These surfaces are excited to vibration by signals in the form of rectangular pulsed sinusoid with time delays so selected that the beam is focused at a desired point of the field.

The theoretical method for acoustic field calculation is based on analysis of the transient radiation generated by a piston. This analysis consists in determination of the impulse response of the radiating surface at a spatial point of interest and in subsequent convolution of the piston velocity with the appropriate impulse response [8-14].

Consider the problem of determining the time-dependent pressure in half-space $z > 0$ resulting from the velocity of the piston as shown in Fig. 2. The piston is assumed to vibrate with a pulsed velocity and the medium in the half-space is assumed to be isotropic with a constant velocity of propagation. In addition, the piston is assumed to be mounted in an infinite planar rigid baffle.

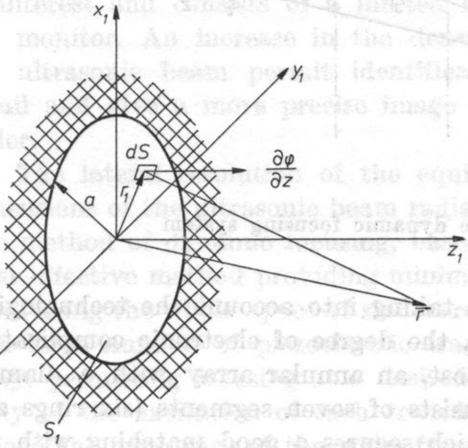

Fig. 2. A circular transducer in an infinite planar and rigid baffle

The problem of computing the pressure is formulated as a classical boundary value problem in terms of the velocity potential $\varphi(\mathbf{r}, t)$, where $\mathbf{r}$ defines the point of interest in the half-space and t is time. The pressure is obtained using the following equations:

$$p(\mathbf{r}, t) = \varrho \frac{\partial \varphi(\mathbf{r}, t)}{\partial t}, \quad v(\mathbf{r}, t) = -\nabla \varphi(\mathbf{r}, t), \tag{1}$$

where ϱ is the density of the medium.

The mathematical specification of the boundary-value problem yields the following system of equations:

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t}-\nabla^2\varphi=0, \quad \boldsymbol{r}\in A,$$
$$\frac{\partial\varphi}{\partial z}=v(\boldsymbol{r},t), \quad \boldsymbol{r}\in S, t>0, \tag{2}$$
$$\varphi(\boldsymbol{r},t)|_{t=0}=\frac{\partial}{\partial t}\varphi(\boldsymbol{r},t)|_{t=0}=0,$$

where A is the half-space $z>0$, S is the $z=0$ plane and $\boldsymbol{r}$ is a point in A and S. It should be noted that $v(\boldsymbol{r},t)=0$ for $\boldsymbol{r}$ outside the surface of the piston.

The solution of the preceding system of equations is obtained using a Green's function development

$$\varphi(\boldsymbol{r},t)=\int_0^t dt_1\int_{S_1} v(\boldsymbol{r}_1,t_1)G(\boldsymbol{r}_1,t|\boldsymbol{v}_1,t_1)dS, \tag{3}$$

where $G(\boldsymbol{r},t|\boldsymbol{r}_1,t_1)$ is a Green's function [6] and $v(\boldsymbol{r}_1,t_1)$ is the specified velocity of the piston. The Green's function is given by the formula [6]

$$G(\boldsymbol{r},t|\boldsymbol{r}_1,t_1)=\frac{\delta(t-t_1-|\boldsymbol{r}-\boldsymbol{r}_1|/c)}{2\pi(\boldsymbol{r}-\boldsymbol{r}_1)}, \tag{4}$$

where c is the propagation velocity of the acoustic wave in the medium.

Assuming a uniform piston velocity $v(\boldsymbol{r}_1,t_1)=v(t_1)$ and substituting equation (4) into equation (3), the velocity potential is obtained,

$$\varphi(\boldsymbol{r},t)=\int_0^t v(t_1)\,dt_1\int_{S_1}\frac{\delta(t-t_1-|\boldsymbol{r}-r_1|/c)}{2\pi|\boldsymbol{r}-\boldsymbol{r}_1|}\,dS. \tag{5}$$

Formula (5) can be expressed as

$$\varphi(\boldsymbol{r},t)=\int_0^t v(t_1)h(\boldsymbol{r},t-t_1)\,dt_1, \tag{6}$$

where

$$h(\boldsymbol{r},t-t_1)=\int_{S_1}\frac{\delta(t-t_1-|\boldsymbol{r}-\boldsymbol{r}_1|/c}{2\pi|\boldsymbol{r}-\boldsymbol{r}_1|}\,dS. \tag{7}$$

It can be noted that equation (6) is a familiar convolution integral and may be expressed as a convoluation of two time functions

$$\varphi(\boldsymbol{r},t)=v(t_1)*h(\boldsymbol{r},t), \tag{8}$$

where the function $h(\boldsymbol{r},t)$ is defined as the impulse response function of the piston to the spatial point of interest.

The attempts to determine analytically the impulse response of the piston undertaken by KHARKEVICH [1], OBERHETTINGER [7] and STEPANISHEN [10], who used different methods to solve this problem, gave the same result; namely, that the impulse response of the piston resulting from excitation by a Dirac function can be determined for two cases, shown in Figs. 3 and 4, in the following way:

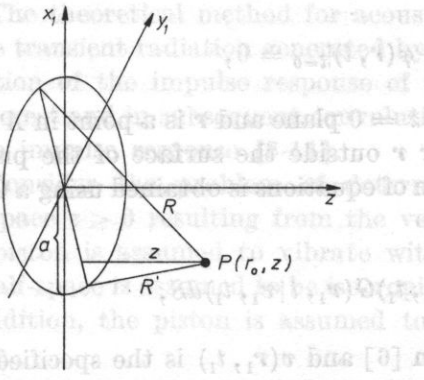

Fig. 3. The case when the observation point is within the area of the cylinder whose base is the area of the transducer

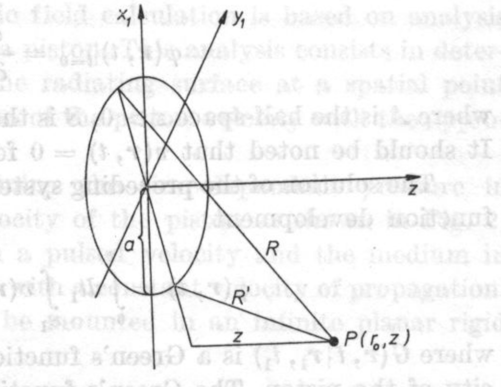

Fig. 4. The case when the observation point is outside the area of the cylinder whose base is the surface of the transducer

1. When $a > r_0$ (i.e. when the point of interest is inside the area of the cylinder whose base is the surface of the piston)

$$h(\boldsymbol{r}, t) = \begin{cases} 0, & t < \dfrac{z}{c}, \\ c, & \dfrac{z}{c} < t < \dfrac{R'}{c}, \\ \dfrac{c}{\pi} \arccos\left[\dfrac{c^2t^2 + r_0^2 - a^2 - z^2}{2r_0\sqrt{c^2t^2 - z^2}}\right], & \dfrac{R'}{c} < t < \dfrac{R}{c}, \\ 0, & t > \dfrac{R}{c}. \end{cases} \tag{9}$$

2. When $a < r_0$ (i.e. the point of interest is outside the area of the cylinder whose base is the surface of the piston)

$$h(\boldsymbol{r}, t) = \begin{cases} 0, & t < \dfrac{R'}{c}, \\ \dfrac{c}{\pi} \arccos\left[\dfrac{c^2t^2 + r_0^2 - a^2 - z^2}{2r_0\sqrt{c^2t^2 - z^2}}\right], & \dfrac{R'}{c} < t < \dfrac{R}{c}, \\ 0, & t > \dfrac{R}{c}, \end{cases} \tag{9a}$$

where $R' = \sqrt{z^2+(a-r_0)^2}$ and $R = \sqrt{z^2+(a+r_0)^2}$ are the shortest and the longest distances, respectively, from the observation point to the circumference of the piston (cf. Figs. 3 and 4).

Thus, to study the time-dependent pressure at an arbitrary near field point resulting from an impulse velocity motion of the piston, the general expression for the impulse response which is indicated in equations (9) or (9a) must be substituted into equation (6) and the resultant velocity potential substuted into equation (1). Performing the indicated substitutions and differentiation, the pressure may be evaluated from the following expression

$$p(\boldsymbol{r}, t) = \varrho \int_{-\infty}^{t} \mathring{h}(\boldsymbol{r}, t-\tau) v(\tau) d\tau, \tag{10}$$

where $\mathring{h}(\boldsymbol{r}, t)$ is the time derivative of the impulse response.

The time-dependent acoustic pressure generated by an array of annular transducers may be determined, using superposition as the sum of the pressures generated by each segment. When the system consists of a central disc with the radius a_0 and N concentric rings (Fig. 5) with the interal and external radius equal respectively to a_{nw} and a_{nz}, the summary time-dependent acoustic pressure is expressed as follows:

$$p_{\Sigma}(\boldsymbol{r}, t) = p_0(\boldsymbol{r}, t) + \sum_{N} p_n(\boldsymbol{r}, t), \tag{11}$$

where $n = 1, 2, \ldots, N$; $p_0(\boldsymbol{r}, t)$ is the time-dependent pressure resulting from the pulsed velocity of the central disk; $p_n(\boldsymbol{r}, t) = p_{nz}(\boldsymbol{r}, t) - p_{nw}(\boldsymbol{r}, t)$ is time-dependent radiation from the nth ring, $p_{nz}(\boldsymbol{r}, t)$ and $p_{nw}(\boldsymbol{r}, t)$ are time-dependent radiation from the piston with radius a_{nz} and a_{nw}, respectively.

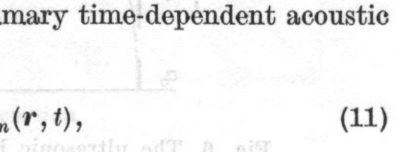

Fig. 5. The phased annular array

According to the principle of the phased array, the calculation of the field focused on axis Z at the distance f from the radiating surface requires the introduction of excitation time delays for each segment with respect to the external ring for which zero delay may be assumed.

The excitation time delay is taken into account in the following way. Assume that the segment velocity $v(t)$ is a pulsed sinusoid with a time duration τ_0 and carrier frequency equal to ω, then for the central disc

$$v_0(t) = v[H(t) - H(t-\tau_0)]\sin\omega t, \tag{12}$$

while for the nth ring of the phased array

$$v_n(t) = v[H(t+\Delta\tau_n) - H(t-\tau_0+\Delta\tau_n)]\sin\omega(t+\Delta\tau_n), \tag{13}$$

where $\Delta\tau_n = (\sqrt{f^2+a_n^2} - f)/c$ is the time delay in excitation of the nth segment (see Fig. 6).

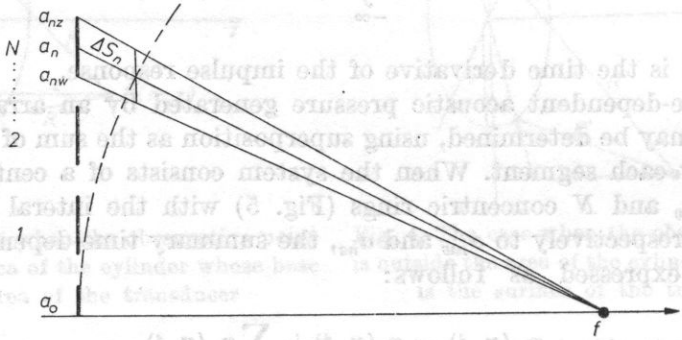

Fig. 6. The ultrasonic beam focusing by means of an annular array

Thus, according to equations (10) and (11), time-dependent pressure focused on the axis Z resulting from an impulsed velocity of annular array segments may be expressed as follows:

a) the on-axis pressure

$$\begin{aligned} p_\Sigma(z,t) = v\varrho c\Bigg\{&\left[H\left(t-\frac{z}{c}\right) - H\left(t-\frac{z}{c}-\tau_0\right)\right]\sin\omega\left(t-\frac{z}{c}\right) - \\ &-\left[H\left(t-\frac{\sqrt{a_0^2+z^2}}{c}\right) - H\left(t-\frac{\sqrt{a_0^2+z^2}}{c}-\tau_0\right)\right]\sin\omega\left(t-\frac{\sqrt{a_0^2+z^2}}{c}\right)\Bigg\} + \\ &+\sum_N \varrho c v\Bigg\{\left[H\left(t-\frac{\sqrt{a_{nw}^2+z^2}}{c}+\Delta\tau_n\right) - H\left(t-\frac{\sqrt{a_{nw}^2+z^2}}{c}-\tau_0+\Delta\tau_n\right)\right]\times \\ &\times\sin\omega\left(t-\frac{\sqrt{a_{nw}^2+z^2}}{c}+\Delta\tau_n\right) - \left[H\left(t-\frac{\sqrt{a_{nz}^2+z^2}}{c}+\Delta\tau_n\right) - \right. \\ &\left. -H\left(t-\frac{\sqrt{a_{nz}^2+z^2}}{c}-\tau_0+\Delta\tau_n\right)\right]\sin\omega\left(t-\frac{\sqrt{a_{nz}^2+z^2}}{c}+\Delta\tau_n\right)\Bigg\}, \end{aligned} \tag{14}$$

b) the pressure at an arbitrary near-field point

$$p_\Sigma(\boldsymbol{r},t) = v\varrho c\left\{\left[H\left(t-\frac{z}{c}\right)-H\left(t-\frac{z}{c}-\tau_0\right)\right]\sin\omega\left(t-\frac{z}{c}\right)-\right.$$

$$-\frac{1}{\pi}\int\limits_{t-R_0/c_0}^{t-R_0'/c}\mathring{h}_0(\boldsymbol{r},t-\tau)\left[H(\tau)-H(\tau-\tau_0)\right]\sin\omega\tau d\tau\Bigg\}+$$

$$+\sum_N\frac{v\varrho c}{\pi}\Bigg\{\int\limits_{t-R_{nw}/c}^{t-R_{nw}'/c}\mathring{h}_{nw}(\boldsymbol{r},t-\tau)\left[H(\tau+\Delta\tau_n)-H(\tau+\Delta\tau_n-\tau_0)\right]\times$$

$$\times\sin\omega(\tau+\Delta\tau_n)d\tau-\int\limits_{t-R_{nz}/c}^{t-R_{nz}'/c}\mathring{h}_{nz}(\boldsymbol{r},t-\tau)\left[H(\tau+\Delta\tau_n)-\right.$$

$$\left.-H(\tau+\Delta\tau_n-\tau_0)\right]\sin\omega(\tau+\Delta\tau_n)d\tau\Bigg\}. \tag{15}$$

Equations (14) and (15) present the basic analytical expressions to compute the near field transient radiation focused at the distance f from the radiating surface of the phased annular arrary.

9. Computing results and discussion

The calculations were performed on a Cyber (IBM) computer in Fortran language. Figs. 7-11 show the transient radiation resulting from an impulse

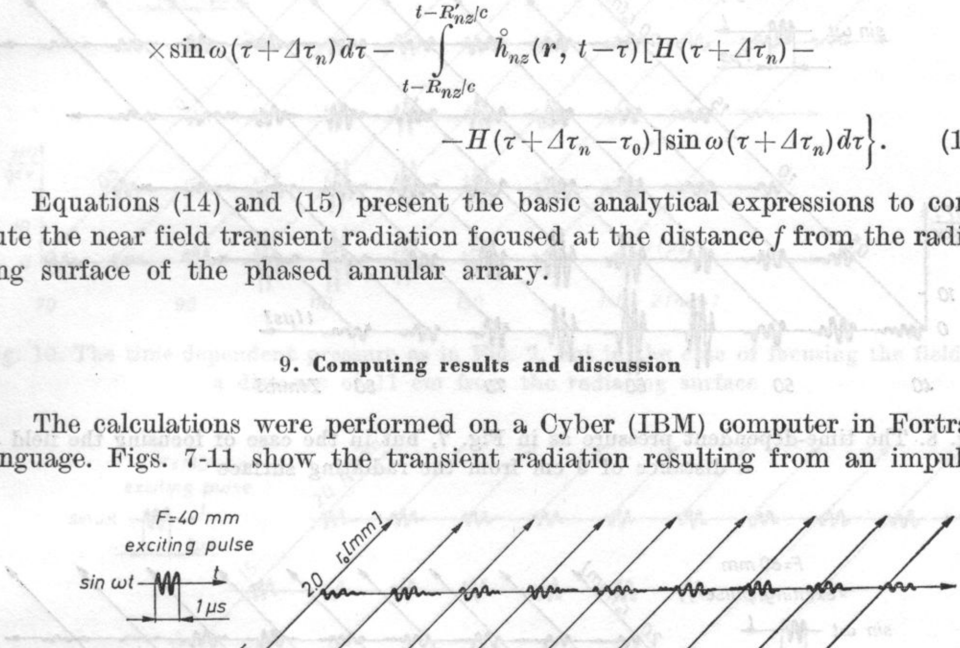

Fig. 7. The time-dependent radiation resulting from impulse velocity motion of the phased annular array in the case of focusing the field at a distance of 4 cm from the radiating surface. The segments of an array are excited by rectangular pulsed sinusoid with duration 1 μs and carrier frequency 2.5 MHz

velocity motion of the phased annular array with geometrical parameters described previously. The segments of the array are excited by rectangular pulses with duration equal to 1 μs and carrier frequency equal to 2.5 MHz. The field is focused successively in each focal zone at distnaces of 4, 6, 8, 11 and 18 cm, respectively, from the radiating surface.

On the basis of the obtained numerical data, the lateral pressure distribution at different observation depth Z, also the beam width versus observation depth for all focal zones, were determined. Fig. 12 shows the lateral pressure distribution in the focus ($F = 6$ cm) for a selected annular array excited by

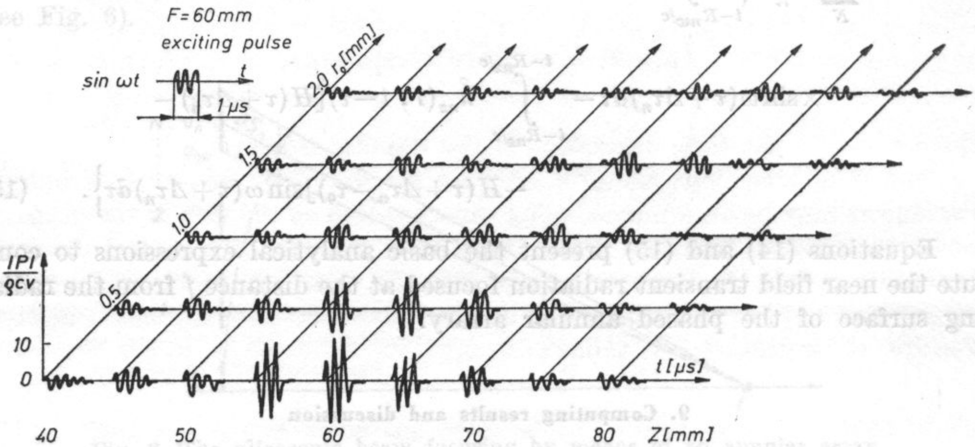

Fig. 8. The time-dependent pressure as in Fig. 7, but in the case of focusing the field at a distance of 6 cm from the radiating surface

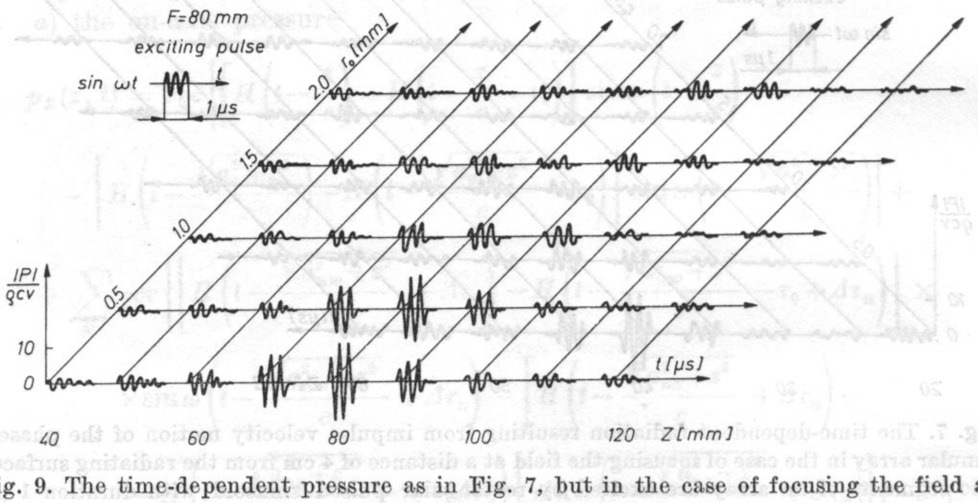

Fig. 9. The time-dependent pressure as in Fig. 7, but in the case of focusing the field at a distance of 8 cm from the radiating surface

a continuous wave (solid line) and by pulsed wave (dashed line). For comparison, the same figure shows the lateral pressure distribution calculated for a spherical transducer with the same diameter and a curvature radius of 6 cm, excited by a continuous wave [3]. It can be seen in this figure that the maximum level of the sidelobes for impulses equals – 15 dB with respect to the maximum value, for a continuous wave equals – 19 dB and for a spherical transducer is – 24 dB.

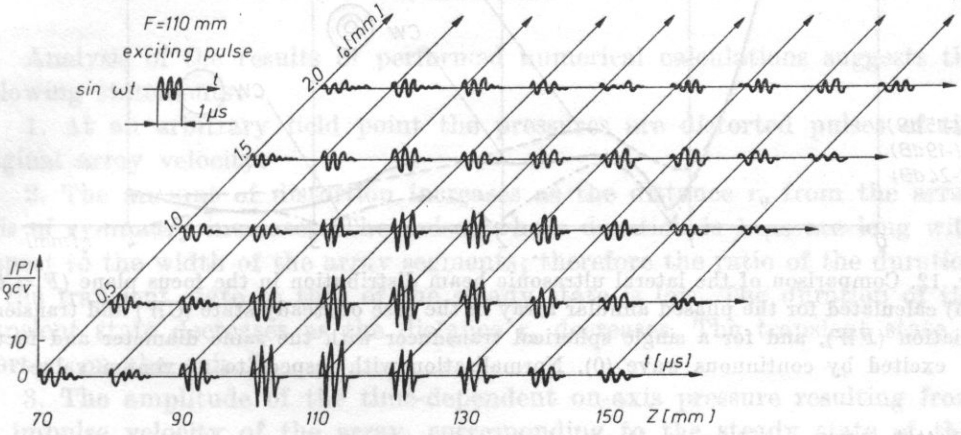

Fig. 10. The time-dependent pressure as in Fig. 7, but in the case of focusing the field at a distance of 11 cm from the radiating surface

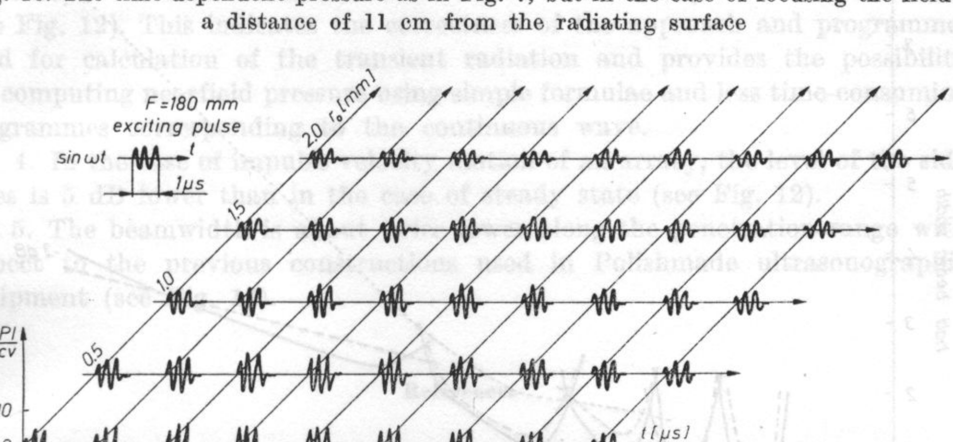

Fig. 11. The time-dependent pressure as in Fig. 7, but in the case of focusing the field at a distance of 18 cm from the radiating surface

Fig. 13 illustrates the dependence of the beamwidth generated by an annular array successively in each focal zone, on the observation depth, calculated for steady state (solid line) and for transient states (dashed line). The

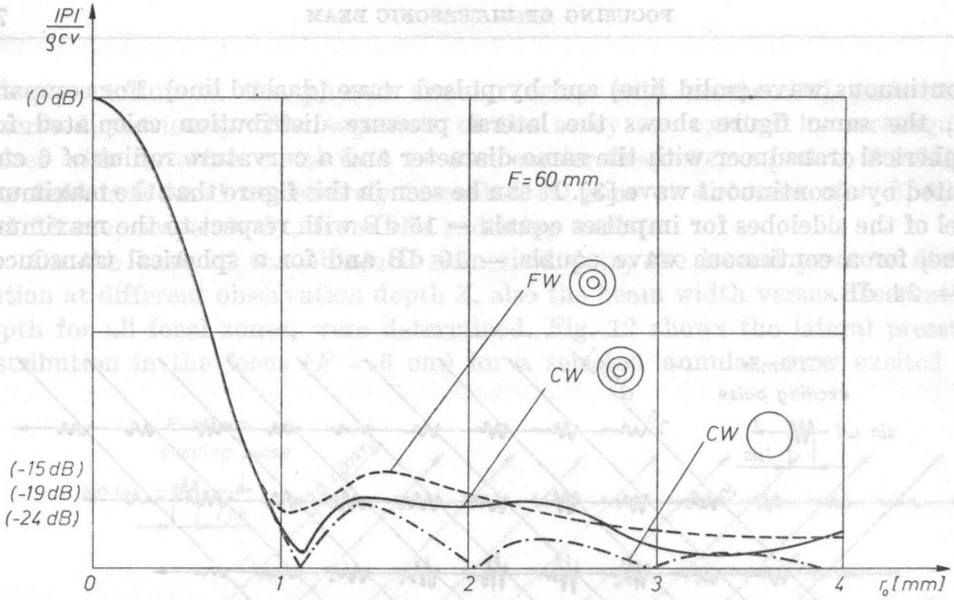

Fig. 12. Comparison of the lateral ultrasonic beam distribution in the focus plane ($F = 60$ mm) calculated for the phased annular array in the case of steady state (*CW*) and transient radiation (*FW*), and for a single spherical transducer with the same diameter and focus excited by continuous wave (0). Normalisation with respect to maximum value

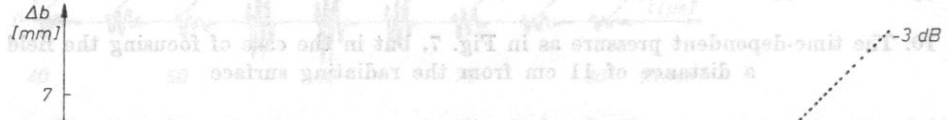

Fig. 13. The 3 dB ultrasonic beamwidth successively focussed in each focal zone, depending on the observation depth, calculated for steady state (solid line) and transient state (dashed line). This figure presents also results of measurements for a spherical transducer with 20 mm diameter and a 10 cm curvature radius (pointed line) in the case of steady state

same figure shows the results of measurements in case of a spherical transducer with a diameter of 20 mm and a curvature radius of 10 cm obtained in paper [2]. Comparison of graphs shown in Fig. 13 indicates a significant lateral resolution improvement in the case of dynamic focusing both at the beginning and at the end of the penetration range.

4. Conclusions

Analysis of the results of performed numerical calculations suggests the following statements:

1. At an arbitrary field point the pressures are distorted pulses of the original array velocity.

2. The amount of distortion increases as the distance r_0 from the array axis of symmetry increases. The pulses whose duration is 1 μs are long with respect to the width of the array segments; therefore the ratio of the duration of the transient state to that of the steady state is low. The duration of the transient state decreases as the distance r_0 decreases. The transient state is shortest on the axis Z.

3. The amplitude of the time-dependent on-axis pressure resulting from an impulse velocity of the array, corresponding to the steady state of this pressure, is the same as in the case of continuous velocity motion of the array (see Fig. 12). This indicates tne correctness of the approach and programmes used for calculation of the transient radiation and provides the possibility for computing nearfield pressure using simple formulae and less time-consuming programmes corresponding to the continuous wave.

4. In the case of impulse velocity motion of an arrary, the level of the side lobes is 5 dB lower than in the case of steady state (see Fig. 12).

5. The beamwidth is about twice lower along the penetration range with respect to the previous constructions used in Polishmade ultrasonographic equipment (see Fig. 13).

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