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PROBLEM OF THE INSTANTANEOUS SOUND FREQUENCY MEASUREMENT

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This paper presents a method for the measurement of the so - called quasi-instantaneous frequency of acoustic signals, which is an approximation of the instantaneous frequency defined on the basis of theoretical considerations. It also describes a measurement system of the $(T, f\text{-}V)$ type which was built for this purpose, at the output of which the instantaneous value of the voltage V is proportional to the value of the quasi - period T_q or the quasi - instantaneous frequency f_q of a signal fed to the input. This system permits continuous registration of variations of the quasi - instantaneous frequency in real time. The characteristics of the system $(T, f\text{-}V)$ given here and also the oscillograms of chosen test signals (with prescribed modulating functions) and real signals (e.g. those occurring in rooms) indicate the wide possibilities of its use in practice.

1. Introduction

A characteristic feature of most acoustic signals is instantaneous variations in the value of their physical parameters. These variations, which most frequently have the character of irregular fluctuations in amplitude, frequency and phase, carry fundamental information about the signal, which is important e.g. from the point of view of the formation of sound sensation. This is confirmed by the increasingly often carried out psychoacoustic investigations of these variations, based on the so - called dynamic perception, which is understood to be the perception of signals with parameters variable in time. In the range of both these investigations and also some physical research, an essential question is that of the measurement of the signal frequencies rapidly varying in time, which is characterized by its instantaneous frequency. A typical example of such signals are those frequency - modulated, for which the classically conceived frequency is not unambiguous.

In view of this, investigations were undertaken with the basic purpose of developing a conception of the measurement of this frequency rapidly variable in time and designing and constructing an appropriate system to serve for the determination of its instantaneous values.

2. Determination of the instantaneous frequency of a signal

In order to define the notion of the instantaneous frequency, the analytical signal $\Psi(t)$ can be represented in the following exponential form:

$$\Psi(t) = s(t) + j\sigma(t) = \exp[\Phi(t)] = \exp[\mathrm{Re}\,\Phi(t) + j\,\mathrm{Im}\,\Phi(t)], \tag{1}$$

where $\Psi(t)$ is a complex function whose real part is the real signal $s(t) = \mathrm{Re}\{\Psi(t)\}$, while its imaginary part $\sigma(t) = \mathrm{Im}\{\Psi(t)\}$ is the Hilbert transform of the real signal. The modulus of the analytical signal $\Psi(t)$

$$|\Psi(t)| = \sqrt{s^2(t) + \sigma^2(t)} \tag{2}$$

represents the instantaneous amplitude, which will be designated below as $A(t) = |\Psi(t)|$. The instantaneous phase of this signal is defined by the expression

$$\varphi(t) = \tan^{-1}\frac{\sigma(t)}{s(t)}. \tag{3}$$

In turn the complex instantaneous frequency is defined as the quantity $p(t)$, i.e.

$$p(t) = \alpha(t) + j\omega(t) = \frac{d}{dt}\,\Phi(t) = \frac{d}{dt}\ln\Psi(t) = \frac{1}{\Psi}\frac{d\Psi}{dt}. \tag{4}$$

The following formulation of the analytical signal

$$\Psi(t) = A(t)\exp[j\varphi(t)]$$

and the use of (4) give

$$p(t) = \frac{1}{A(t)\exp[j\varphi(t)]}\,\frac{d}{dt}\,[A(t)\exp[j\varphi(t)]] = \frac{1}{A(t)}\frac{dA(t)}{dt} + j\,\frac{d\varphi(t)}{dt}. \tag{5}$$

The real part $\alpha(t)$ of the complex instantaneous frequency $p(t)$ represents in a physical sense a relative increase in the instantaneous amplitude per unit time:

$$\alpha(t) = \frac{1}{A(t)}\frac{dA(t)}{dt}, \tag{6}$$

whereas the imaginary part of $p(t)$ has the meaning of angular frequency,

$$\omega(t) = \frac{d\varphi(t)}{dt}. \tag{7}$$

Using the definition of the complex instantaneous frequency, the analytical signal with the initial amplitude A_0 and the initial phase φ_0 can be given in the form

$$\Psi(t) = A_0 \exp(j\varphi_0) \exp\left[\int_0^t p(t)\,dt\right] = A_0 \exp\left[\int_0^t a(t)\,dt\right] \exp\left[j\left(\int_0^t \omega(t)\,dt + \varphi_0\right)\right] = A(t)\exp[j\varphi(t)], \tag{8}$$

where

$$A(t) = A_0 \exp\left[\int_0^t a(t)\,dt\right]$$

represents the instantaneous amplitude of the signal. At the initial time $t = 0$, $A(0) = A_0$. In turn

$$\varphi(t) = \mathrm{Im}\,\Phi(t) = \int_0^t \omega(t)\,dt + \varphi_0 \tag{9}$$

is the instantaneous phase of the signal. At the initial time $t = 0$, $\varphi(0) = \mathrm{Im}\,\Phi(0) = \varphi_0$.

In a particular case of a signal with an amplitude constant in time, i.e. for $A(t) = A_0$ and the frequency ω_0, which signifies that $\varphi(t) = \omega_0 t + \varphi_0$, we obtain

$$a(t) = \frac{1}{A(t)} \frac{dA(t)}{dt} = 0$$

and

$$\omega(t) = \frac{d}{dt}(\omega_0 t + \varphi_0) = \omega_0.$$

It can be seen, on the assumption that the instantaneous amplitude of the signal considered is approximately constant or varies relatively slowly with respect to the instantaneous phase of the signal, that the instantaneous frequency is defined by the real quantity $\omega(t)$:

$$\omega(t) = \mathrm{Im}\,p(t) = \frac{d\varphi(t)}{dt} \tag{10}$$

or, otherwise,

$$f(t) = \frac{1}{2\pi} \frac{d\varphi(t)}{dt}. \tag{10a}$$

It is interesting to note that in the literature [3] one may also find another definition of the instantaneous frequency, based on analysis of the number of zero crossings of the real signal investigated. In this case the instantaneous frequency is defined as the ratio of the number of zero crossings of this signal, determined over some time interval $\Delta\tau$, and the value of this interval. This ratio corresponds to the mean density of zeroes of the signal over this interval and is sometimes called the Rice frequency (f_R):

$$f_R = \lim_{T\to\infty} \frac{N}{\Delta\tau},$$

where N is the number of "positive" or "negative" zero level crossings of the signal and $\Delta\tau$ is the averaging time interval.

It should be pointed out that, depending on the width of the interval $\Delta\tau$ selected, this ratio expresses the mean frequency in this interval, and thus it does not correspond to the strict definition of the instantaneous frequency $\omega(t)$ subordinated to a given time t. E. g. for a frequency - modulated signal, even on the assumption that the interval $\Delta\tau$ is small compared with the period of the modulating frequency, the quantity $\omega(t)$ determined is nevertheless a considerable averaging of the instantaneous frequency, as it occurs for a number of periods of the carrier frequency.

The quantity $\omega(t)$ as defined by expression (10) is thus a theoretical one, as it determines the value of the instantaneous frequency at a given time t, which cannot be implemented in experimental conditions. In these conditions, in expression (10) the differential quantities should be replaced by the difference ones, i.e.

$$\omega(\Delta t) = \frac{\Delta\varphi}{\Delta t}. \tag{11}$$

In keeping with expression (11), the measure of the instantaneous frequency of the signal is the ratio of its phase changes $\Delta\varphi$, occurring over the time interval Δt, and the value of this interval. In experimental terms it is quite intricate to determine phase changes over a very short time interval Δt. Therefore, in the range of the investigations carried out, using the $(T, f-V)$ type system built (see Section 2), measurements were performed of such a time interval $\Delta t = T_q$ in which the signal phase varied by a value of 2π. On this basis, the quantity $\omega(T_q)$ was obtained, equal to

$$\omega(T_q) = \frac{2\pi}{T_q} = 2\pi f(T_q) \tag{12}$$

or

$$f(T_q) = \frac{1}{T_q}. \tag{12a}$$

It should be noted that the interval T_q, for a large number of signals (e.g. a broad class of frequency - modulated signals), varies its value in time, therefore it cannot be treated as the classically conceived vibration period. Accordingly, the practice became to call this interval a quasi - period, whereas its inverse, i.e. $1/T_q = f_q(T_q)$ was called the quasi - instantaneous frequency.

3. Construction and working principle of the system (T, f—V)

It follows from the considerations presented in Section 1 that for signals occurring in reality there are considerable difficulties in the range of the registration and measurement of changes of the instantaneous frequency. This fact encouraged the authors to undertake investigations with the intention towards the conceptual and construction development of a special system permitting the measurement of the value of the quasi-instantaneous frequency at any time of the duration of the signal. The system designed and built is characterized by that the instantaneous value of its output voltage is directly proportional to the quasi - period (T_q) of a signal (when output I is used (see Fig. 1)) or the quasi - instantaneous frequency (f_q) of the signal (when output II is used). The working principle of the system whose schematic diagram is shown in Fig. 1, is as follows. The signal analysed (see Fig. 2a), whose amplitude variation can be contained in the limits between 10 mV to 5 V, which corresponds to dynamics of about 50 dB, is fed to the amplifier - amplitude limiter (1). At the time of the zero crossing of the input signal (i.e. the transition from the

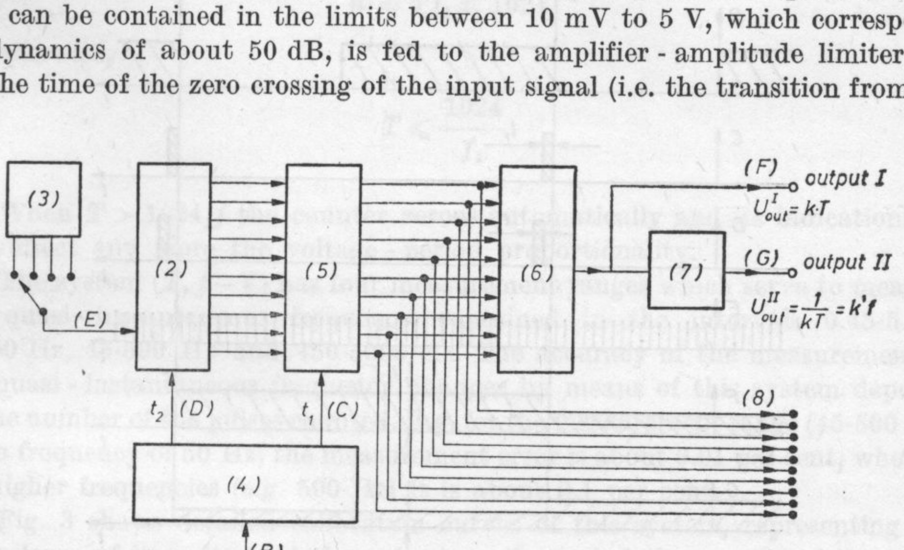

Fig. 1. A schematic diagram of the system $(T, f-V)$

1 — amplifier-amplitude limiter, 2 — binary counter, 3 — quartz generator, 4 — unit controlling the counter and memory, 5 — digital memory, 6 — digital-to-analog converter, 7 — analog divider, 8 — digital output

value "+" to "−"), the limiter gives a rising slope of a rectangular wave (see Fig. 2b). At that moment, the system which controls the counter and memory (4) generates a pulse with the duration $t_1 \cong 0.6$ μs (see Fig. 2c). This pulse causes the current state of the counter (2) to be "written" into the memory (5). Till the next zero crossing (counted with the same phase, i.e. from "+" to "−"), namely in the interval T, the pulse counter counts pulses with constant frequencies of 0.463, 4.63, 46.3 or 463 kHz (depending on the measurement range) generated by the quartz generator (3). The number of pulses counted is thus proportional to the period of the signal investigated, i.e. inversely proportional to its frequency. After the period T, the state of the counter (2) is put into the memory (5). Subsequently, by means of a digital-to-analog converter (6), the state of the counter (2) is transformed into a signal with constant voltage U_{out}, proportional to the number of the pulses memorized,

$$U_{\text{out}}^{(\text{I})} = kn = kTf_1, \tag{13}$$

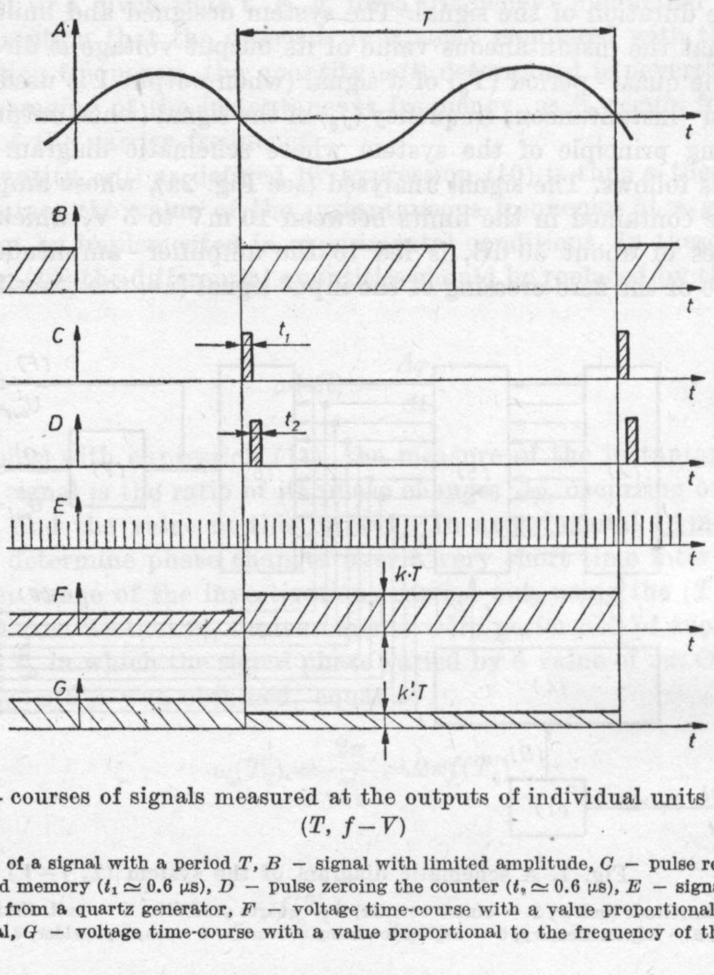

Fig. 2. Time-courses of signals measured at the outputs of individual units of the system (T, $f-V$)

A – time-course of a signal with a period T, B – signal with limited amplitude, C – pulse releasing the work of the counter and memory ($t_1 \simeq 0.6$ μs), D – pulse zeroing the counter ($t_2 \simeq 0.6$ μs), E – signal with the constant frequency f from a quartz generator, F – voltage time-course with a value proportional to the period of the input signal, G – voltage time-course with a value proportional to the frequency of the input signal where $n = Tf_1$ is the number of the pulses memorized, T is the period of the input signal, f_1 is the frequency of the pulses from the quartz generator and k is the proportionality constant.

When output II is used, the value of the signal voltage $U_{\text{out}}^{(\text{II})}$, as a result of using an analog divider (7), is

$$U_{\text{out}}^{(\text{II})} = \frac{1}{kT} = k'f. \tag{13a}$$

Thus, this gives a relation of simple proportionality between the quasi-instantaneous frequency of the signal and the value of the output voltage of the system $(T, f-V)$.

After the pulses have been "written" into the memory (corresponding to the first period of the input signal) and transformed into a voltage signal, the control system (4) generates the pulse $t_2 \cong 0.6$ μs (see Fig. 2d), which causes the counter to be zeroed. Subsequently, the process of "counting" the pulses corresponding to the second period of the input signal is resumed, lasting until the moment of another zero crossing of this signal. Since the counter used in the system $(T, f-V)$ has a limited capacity $(2^{10} = 1024)$, its correct performance requires that the following condition should be satisfied:

$$n = Tf_1 \leqslant 1024 \tag{14}$$

or otherwise,

$$T \leqslant \frac{1024}{f_1}. \tag{15}$$

When $T > 1024/f$ the counter zeroes automatically and its indications do not reflect any more the voltage - period proportionality.

The system $(T, f-V)$ has four measurement ranges which serve to measure the quasi - instantaneous frequency contained in the intervals 0.45-5 Hz, 4.5-50 Hz, 45-500 Hz and 450-5000 Hz. The accuracy of the measurement of the quasi - instantaneous frequency changes by means of this system depends on the number of the pulses counted. E.g. for the measurement range (45-500 Hz) and a frequency of 50 Hz, the measurement error is about 0.01 per cent, whereas for higher frequencies (e.g. 500 Hz) it is about 0.1 per cent.

Fig. 3 shows detailed calibration curves of this system, representing the dependence of its voltage at the output on the period (frequency) of the input signal.

The system $(T, f-V)$ works in real time, as a result of which it is possible to register continuously the changes of the quasi - instantaneous frequency of the signal investigated by means of an oscilloscope, for instance. Using the relation $U_{\text{out}}^{(\text{II})}[V] \sim f[\text{Hz}]$, it is possible, after adequate calibration of the axis

Y of the oscilloscope, to read out directly the value of the quasi-instantaneous frequency of the signal in time. Figs. 5 and 6 show as an example a few oscillograms representing the time - course (lower curve) and the corresponding changes in the quasi - instantaneous frequency (upper curve) for some chosen signals. The oscillogram - based method for the determination of the quasi - instantaneous frequency is rather time consuming in practice, and also the read out itself involves some error. Much more accurate results are obtained

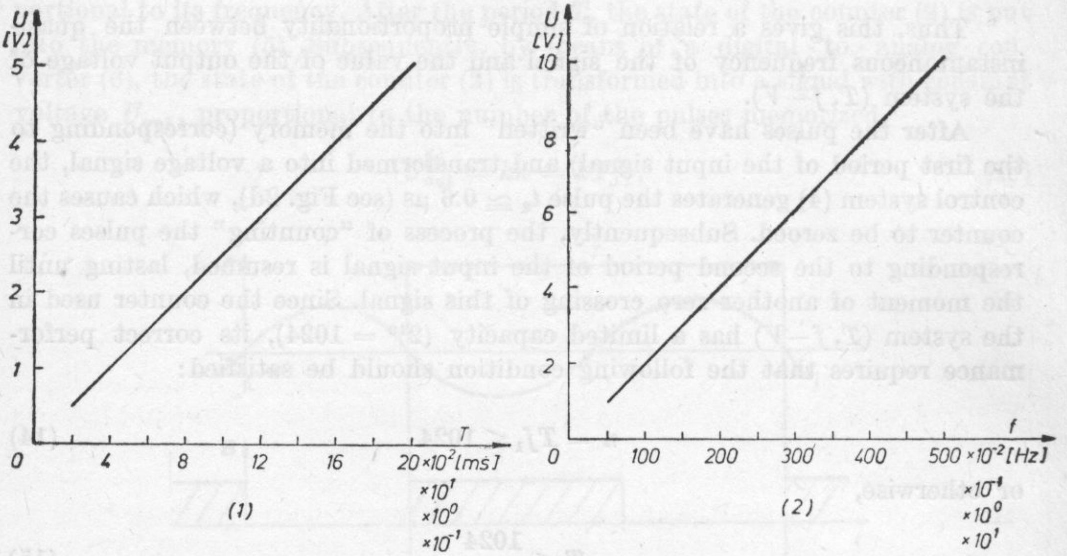

Fig. 3. Dependence of voltage on the period of the signal measured at the measurement output I (1) and the dependence of voltage on the frequency of the signal measured at the measurement output II (2) (for four measurement ranges)

using the digital method of signal processing. The principle of this method is the change of the voltage behaviour obtained at the output of the system (T, $f-V$) into discrete values, in a process of signal digitization. The series of numerical values obtained by this process constitutes input data for the computer with which the measurement results are elaborated.

Over the range of investigation, the process of the change of the analog signal obtained at the output of the system ($T, f-V$) into a digital one was carried out by means of a BK 7502 type digital event recorder.

Fig. 4 shows a schematic diagram of the full apparatus set-up by means of which it is possible to register changes in the quasi - instantaneous frequency of any signals, in particular transient behaviour. This set - up was used, for instance, to determine changes in the quasi - instantaneous frequency of transient signals in a room [1].

The apparatus set - up shown in Fig. 4 has the following working principle. An acoustic signal detected by the microphone (1) is, after amplification by the amplifier (2), fed to the input of the system $(T, f-V)$ (3). At the same time, this signal is registered by an oscilloscope with a delay line (5), which plays the role of a system delaying the moment of the release of the work of the digital event recorder (4). The oscilloscope (5) has a delay line permitting smooth motion along the time axis of a special marker by means of which it is

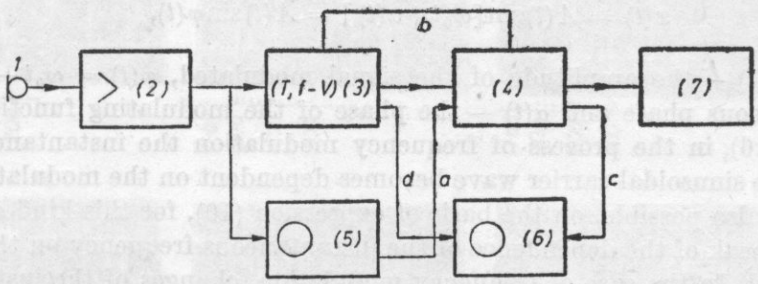

Fig. 4. A schematic diagram of the apparatus set - up for analog-digital registration of changes of the quasi - instantaneous frequency

1 – measurement microphone, *2* – microphone amplifier, *3* – system $(T, f-V)$, *4* – BK 7502 digital event recorder, *5* – oscilloscope with a delay line, *6* – control oscilloscope, *7* – Riad R20 computer, *a* – circuit releasing device 4 (Ext. Trigger), *b* – circuit controlling the sampling process, *c*, *d* – control circuits for signals at the outputs of devices *4* and *5*

possible to determine the beginning of the record of the time section of the signal of interest (e.g. the beginning of the process of sound decay in a room). At the moment when the signal registered reaches the position of the time marker on the oscilloscope display, the oscilloscope (5) generates a voltage pulse which, when fed to the digital event recorder (4), causes the release of its working cycle. From this moment on, the signal from the system $(T, f-V)$ is recorded in digital form in the memory of the device (4). The digital event recorder (4) is connected additionally to the system $(T, f-V)$ by the circuit (*b*) controlling the sampling process of the device (4). By using the oscilloscope (6), it is possible to control visually the work of the system $(T, f-V)$ (4) and to control the content of the memory of the digital event recorder (4). The data contained in the memory of the device (4) are then introduced into the operational system of a Riad R20 computer (7). From this moment on, these data can undergo any mathematical operations carried out by the computer, according to the computer programme introduced.

4. Measurement results

Fig. 5 shows as an example a few oscillograms obtained by the process of calibrating the system $(T, f-V)$ built. The lower curves in these oscillograms illustrate the time - courses of test signals which were: amplitude - modulated (1), frequency - modulated (2) and simultaneously amplitude and frequency modulated (3), successively by linear (*a*), sinusoidal (*b*) and triangular (*c*) functions. In turn, the upper curves illustrate changes of the quasi - instantaneous frequency (f_q) corresponding to the signals obtained at the output (II) of the system $(T, f-V)$. The frequency - modulated signal in Fig. 5 can be described in general by the expression

$$x(t) = A(t)\sin[\omega_0 t + \alpha(t)] = A(t)\sin\varphi(t), \tag{16}$$

where $A(t)$ – the amplitude of the signal modulated, $\varphi(t) = \omega_0 t + \alpha(t)$ – the instantaneous phase and $\alpha(t)$ – the phase of the modulating function. According to (16), in the process of frequency modulation the instantaneous phase $\varphi(t)$ of the sinusoidal carrier wave becomes dependent on the modulating signal.

It is also possible, on the basis of expression (10), for this kind of modulation, to speak of the dependence of the instantaneous frequency on the modulating signal. In the case of frequency modulation, changes of the instantaneous frequency of the modulated signal are proportional to the modulating one, i.e.

$$\omega(t) = \omega_0 + a\beta(t). \tag{17}$$

In turn the instantaneous phase of the modulated signal changes in proportion with the intergal of the modulating signal

$$\varphi(t) = \omega_0 t + a\int\beta(t)dt. \tag{18}$$

The functions $\beta(t)$, successively in linear, sinusoidal and triangular forms, correspond to the modulated signals shown in Fig. 5.2a, b, c.

As was mentioned above over the range of the investigations performed measurements were carried out over such a time interval $\Delta t = T_q$ in which the phase of the signal varied by a value of 2π. As a result of this, no exact values of the instantaneous frequency, according to (10a), but some approximation of it, which was called quasi - instantaneous frequency, was obtained (see 12a). Changes of this frequency in time, recorded at output II of the system $(T, f-V)$ for the modulated signals mentioned above, are illustrated by Fig. 5.2a, b, c. It is seen in this figure that changes of the quasi - instantaneous frequency reflect faithfully the modulating functions prescribed in the test signals; i.e. linear, sinusoidal and triangular ones. In addition, on the basis of comparison of the records given in lines *a*, *b* and *c* in Fig. 5, essential conclusions can be drawn about the accuracy of the indications of the system $(T, f-V)$ in the case of signals whose amplitude varies in time. For this purpose, Fig. 5.1a, b, c shows the time - courses of a sinusoidal signal modulated only in terms of amplitude by means of linear, sinusoidal and triangular functions.

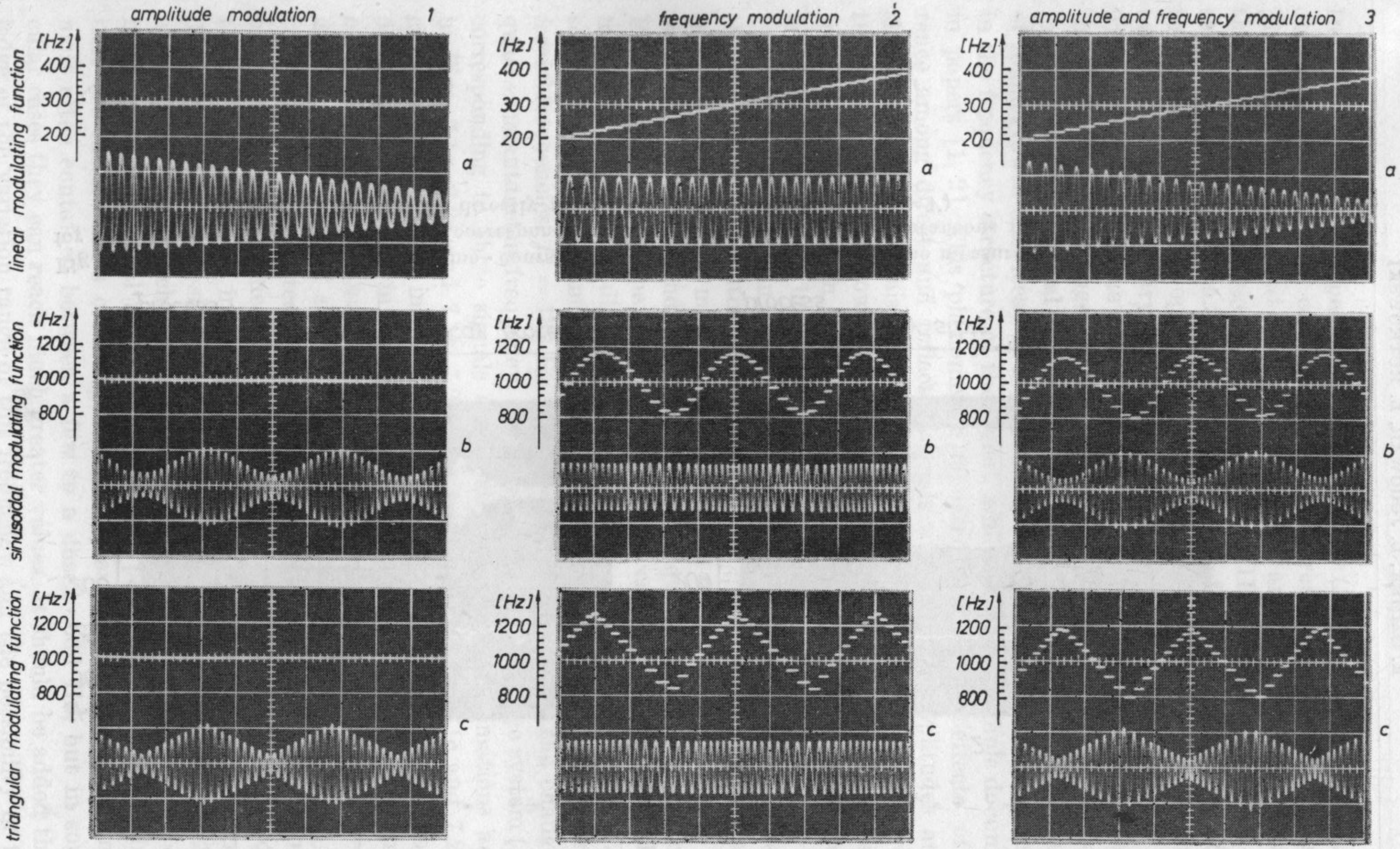

Fig. 5. Oscillograms of the time - course of test signals: amplitude - modulated (1), frequency - modulated (2), amplitude and frequency modula ted(3), for modulating functions: linear (a), sinusoidal (b) and triangular (c) (lower curves), and thec erresponding records of the quasi - instantaneous frequency (upper curves), registered directly at output II of the system (T, $f-V$)

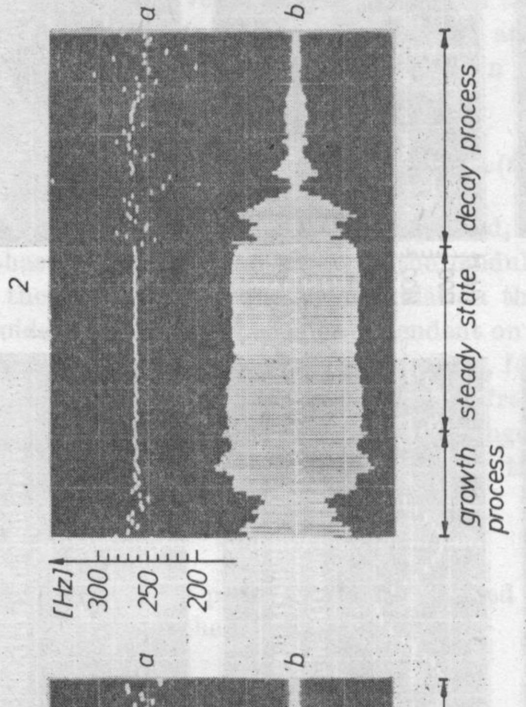

Fig. 6. Oscillograms of the signal time - course (1*b* and 2*b*) obtained at the same measurement point in a room for two different frequencies and the corresponding changes of the quasi - instantaneous frequency (1*a* and 2*a*) registered directly at output II of the system (T, $f-V$)

In such a case, at output II of the system $(T, f-V)$ constant voltage is obtained whose value corresponds to the frequency of the filling signal. The signals which are at the same amplitude and frequency modulated and the corresponding voltage changes at output II of the system $(T, f-V)$ are shown in turn in Fig. 5.3a, b, c. Comparison of Figs. 5.1 and 5.3 shows that the system $(T, f-V)$ operates well also for signals with not only frequency but also amplitude varying in time, which are most frequent in practice. These facts confirm the full usefulness of this system in investigations of the more complex changes in this frequency with fluctuating nature, occurring in a number of physical phenomena. One of these phenomena is the process of sound growth and decay in a room in which it is possible to observe changes in the frequency structure of the signal, which was given ample documentation in papers [1, 2]. This phenomenon is based on complex effects of interference among direct and reflected signals with various amplitudes and phase shifts. As a result of this, the resultant time - course of the sound pressure in the process of sound growth and decay can be regarded as one whose amplitude and phase vary in time, which, as was mentioned above, is related to a definite change in the instantaneous frequency.

Fig. 6 shows as an example two oscillograms of the signal time - course (1b and 2b) with a distinctly seen process of growth, steady state and decay, registered at the same measurement point in a room for two different frequencies and the corresponding changes in the quasi - instantaneous frequency. Fig. 7 shows in turn the results obtained in a concert hall by using the equipment whose schematic diagram is shown in Fig. 4. The oscillograms (7.1b, 7.2b) seen in Fig. 7 correspond to the time - course of signals registered for two different frequencies, $f_1 = 139$ Hz and $f_2 = 141$ Hz. In turn, the changes of the quasi - instantaneous frequency, registered at output II of the system $(T, f-V)$, corresponding to these signals and their quantitative estimations are shown in Figs. 7.1*a*, 7.2*a* and 7.1*c*, 7.2*c*. The solid line in Fig. 7.1*c* and 7.2*c* denote changes of the quasi - instantaneous frequency of the signal as averaged over 50 ms time intervals; in turn, the vertical lines represent the extreme values of these changes. The numbers given on the left (i.e. $f = 139$ Hz, $f = 141$ Hz) are the frequencies of signals transmitted into the room.

The results presented in Fig. 7 show that the range and character of changes of the quasi - instantaneous frequency depend essentially on the frequency of the signal transmitted into the room and the time interval of the signal considered. A detailed analysis of the investigations of this phenomenon [1, 2] showed a large complexity of changes in the quasi - instantaneous frequency, occurring in the transient signal in the room. It was possible to determine in general that the mean values of changes of the quasi - instantaneous frequency are most often contained between a few to a dozen - odd Hz, but in some particular cases they can reach much greater values. It should be added that knowledge of the variation range of this frequency and the possibility of their per-

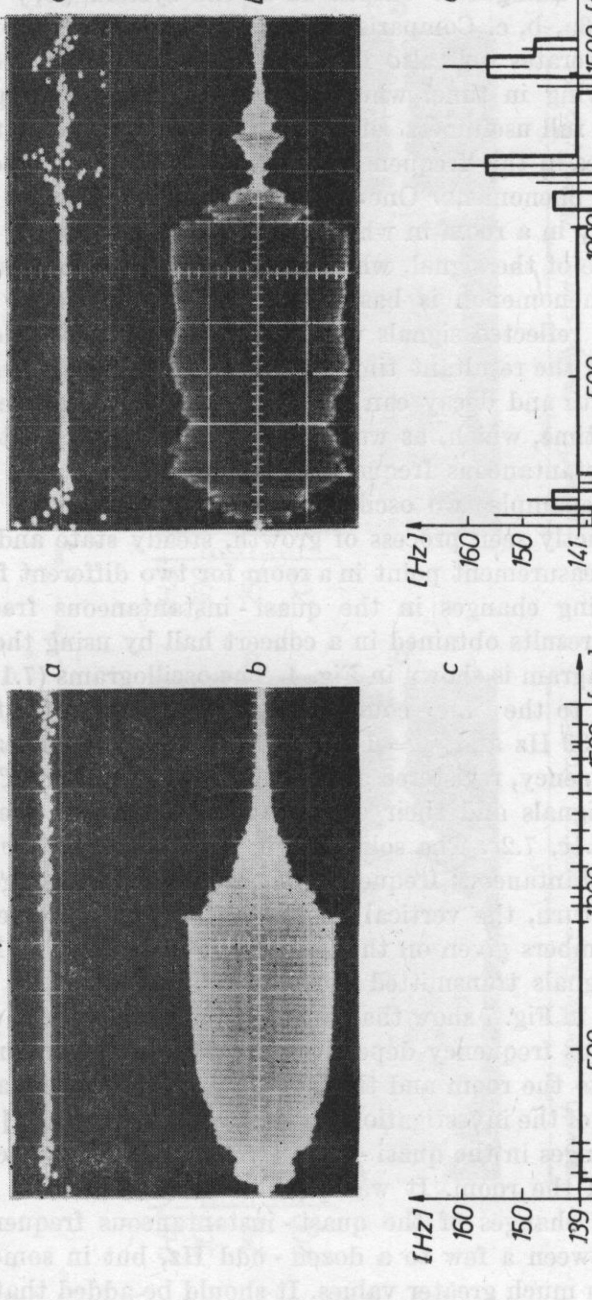

Fig. 7. Oscillograms of the signal courses (1b and 2b) registered in a concert hall for two different frequencies and the corresponding changes of the quasi-instantaneous frequency registered directly at output III of the system ($T, f-V$) (1a and 2a) and at the output of unit 7 (see Fig. 4) (1c and 2c)

ception by listeners can have in practice an essential effect on the resultant, subjective evaluation of the acoustic properties of rooms.

In conclusion, it is interesting to note that instantaneous changes (fluctuations) of frequency nearly always accompany in practice sounds of speech, music and a large number of other acoustic phenomena and processes. Thus, better knowledge of these changes and their quantitative estimation can be important elements of the investigations of the time and spectral structure of these processes and phenomena.

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