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MONTE CARLO CALCULATIONS OF ULTRASOUND SCATTERING BY BLOOD

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The ultrasound scattering by blood depends on the flow velocity profile, as can be shown by frequency analysis of the *c.w.* Doppler signals. Also, the amplitude of the Doppler signals received fluctuates relatively strongly. This fact cannot be explained completely with the random distribution of single erythrocytes or with noise sources in the signal processing unit. These effects can be understood assuming erythrocyte aggregations in blood.

Using the Monte Carlo method we calculate differential scattering cross-sections of blood under the assumption of empirical distributions of aggregates. We will present auto-correlation functions for the ultrasound scattering by erythrocyte-aggregates, taking into account geometrical effects.

1. Introduction

We know from the frequency analysis of continuous wave Doppler signals that ultrasound scattering by blood depends on the flow profile. Both time-interval-histograms (TIH) and fast-Fourier-transformations (FFT) of Doppler signals from blood flow in arterial vessels show characteristic "holes" in the frequency spectrum which cannot be explained completely by the flow profile. Therefore, we have to assume that the scattering cross-section of blood changes according to the shear rate in the flow profile.

On the other hand, there is a relatively strong fluctuation in the scattered signals from blood. These fluctuations differ from patient to patient and cannot be explained by noise from the measurement system alone.

Therefore, we have used the ECG-triggered averaging technique, in on-line with a computer, for more than 4 years [4]. By this method we have achieved an essential improvement, both in the Doppler pulsation curves and in their reproducibility [8].

The Monte Carlo calculations of ultrasound scattering were carried out to improve understanding of these influences. Theories on the ultrasound scattering of statistically distributed inhomogenities have been given in the works of RAYLEIGH [11], CHERNOW [3], FOLDY [5], TWERSKY [13], MORSE and INGARD [9], and applied to biological tissues by NICHOLAS [10] and WAAG [14]. The ultrasound scattering of blood has been calculated by BRODY and MEINEL [2], SHUNG *et al.* [12], ANGELSON [1] and HANSS and BOYNARD [6].

Nevertheless, a whole series of requirements must be met in order to obtain an analytical solution. The most essential requirements are:

1. The validity of the Born approximation, i.e. that the disturbance of the incident wave by the particles remains slight. The kinematic theory of wave diffraction is then valid.

2. The radius of the scatterers should be small compared to the wavelength.

3. The mean distance between the scatterers should be large compared to the wavelength. For other cases an empirical autocorrelation function of the spatial distribution of the scatterers is introduced. In many cases the autocorrelation function found by LIEBERMANN [7] when investigating inhomogenities in the ocean, is used. This autocorrelation function is transfered to the calculation of the ultrasound scattering of biological tissues.

On the one hand, the mean distance of scatterers in human blood at a level of 45 per cent in a normal hematocrite is small compared to the wavelength. On the other hand, small changes in the autocorrelation function have important influence on the results.

4. The dimensions of the scattering region should be large in comparison to both the wavelength and the correlation length of the ultrasound wave.

If we compare these results quantitatively we find that

— the results differ to varying degrees;

— they also differ from the experimental values given by SHUNG *et al.* [12];

— the influences of geometric interferences are not taken into account;

— there is either no or incorrect information on the signal fluctuation.

2. Monte Carlo model 1

The spatial co-ordinates x_i, y_i and z_i are calculated by the means of three random numbers, and thus the position of the scatterer in the scattering region is ascertained. In order to compare our results with those in the aforementioned literature, a spherical scattering region is adopted too. Therefore, a conversion to polar co-ordinates follows (Fig. 1):

$$r_i = R\sqrt[3]{x_i}; \varphi_i = 2\pi y_i; \quad \vartheta_i = \pi z_i, \tag{1}$$

when R is the spherical radius of the scattering region.

We assume the incident wave in the direction of the unit vector $\mathbf{s}_0$, and the scattered wave in the direction $\mathbf{s}$ beneath the scattering angle θ, to be even.

Furthermore it is assumed that the receiver is situated in the farfield of the scattering region (Frauenhofer's diffraction). The scattered wave now under-

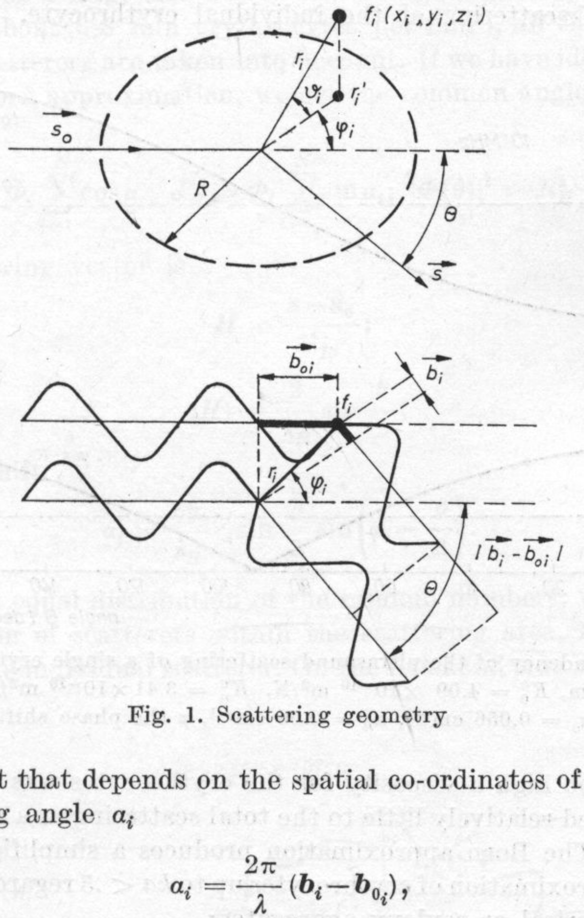

Fig. 1. Scattering geometry

goes a phase shift that depends on the spatial co-ordinates of the particle and on the scattering angle a_i

$$a_i = \frac{2\pi}{\lambda}(\boldsymbol{b}_i - \boldsymbol{b}_{0_i}), \tag{2}$$

where λ is the wavelength, $\boldsymbol{b}_{0_i}$ the projection of r_i on the unit vector $\mathbf{s}_0$ and $\boldsymbol{b}_i$ the projection on $\boldsymbol{s}$.

We must add that the scattered wave goes through an additional phase shift on each particle.

The angle-dependent scattering factor of a single erythrocyte $\varphi(\theta)$; with

$$p_s(r, \theta) = A\frac{\exp(jkr)}{r}\,\Phi(\theta), \tag{3}$$

where A is equal to the amplitude of the incident wave, $k = 2\pi/\lambda$ the number of waves and p_s the scattered ultrasound pressure; was calculated for a nonrigid sphere with a constant volume: $V_0 = 4\pi a_0^3/3$. The angle-dependent scattering factor consists of 3 components: an inhomogenity of compressibility, density and, to a lesser extent, of viscosity (Fig. 2).

Inhomogenities of compressibility and viscosity act as monopole sources. The sum of the three components yields — with regard to the phase shifts — a preferred backscattering of the individual erythrocyte.

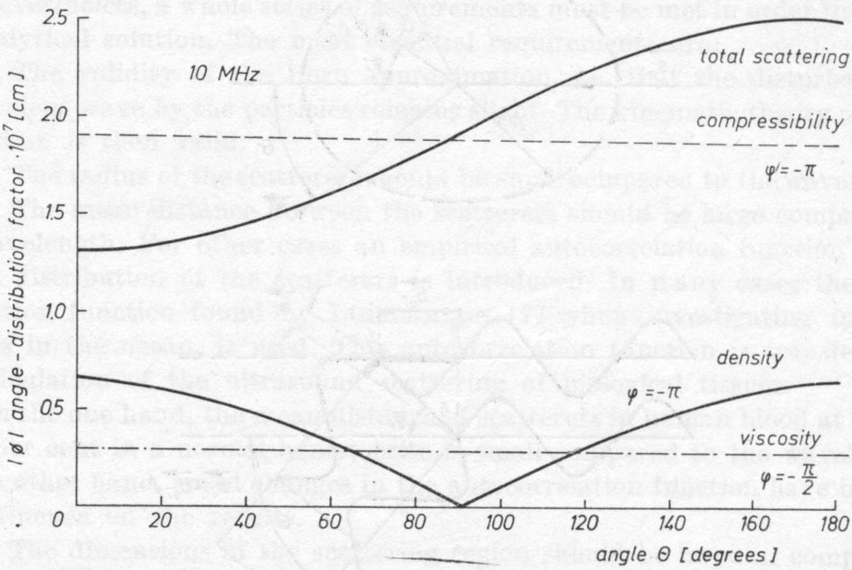

Fig. 2. Angle-dependency of the ultrasound scattering of a single erythrocyte at 10 MHz, where $a_0 = 2.75$ µm, $K_0^c = 4.09 \times 10^{-10}$ m²/N, $K_n^c = 3.41 \times 10^{-10}$ m²/N, $\varrho_0 = 1.03$ g/cm³, $\varrho_n = 1.09$ g/cm³, $\alpha_0 = 0.056$ cm^{-1}, $\alpha_n = 0.56$ cm^{-1}, φ the phase shift of the erythrocyte

Although too high a viscosity for the erythrocytes was assumed, the viscosity contributed relatively little to the total scattering as a result of the phase shift of $-\pi/2$. The Born approximation produces a simplified solution which gives a good approximation of erythrocytes up to $ka < .5$ regardless of their shape — whether spherical or rouloux aggregates:

$$\Phi_i = \frac{1}{4\pi} k_R^2 V_0 \left(\frac{K_n^c - K_R^c}{K_R^c} + \frac{\varrho_n - \varrho_R}{\varrho_n} \cos\theta \right), \tag{4}$$

where the mean number of waves in the coherent solution is

$$k_R = \varrho_R \cdot k_R \cdot \omega^2 \tag{5}$$

and the mean compressibility (K_R^c) and the mean density (ϱ_R):

$$K_R^c = K_0^c + h' \cdot (K_n^c - K_0^c); \quad \frac{1}{\varrho_R} = \frac{1}{\varrho_0} + h' \left(\frac{1}{\varrho_n} - \frac{1}{\varrho_0} \right), \tag{6}$$

with the number of hematocrites being $h' = V_0 N'$ and $N' = \bar{N}/V$ the mean number of particles $\bar{N}$ per volume V (where V_0 is the volume of the individual scatterer, K_n^c and ϱ_n the compressibility and density of the scatterer, K_0^c and ϱ_0 those the surrounding medium, and ω the radian frequency).

In the case of a large number of scatterers, for example a hematocrite of 45 per cent (about 6-8 mln erythrocytes per mm³), all the phase shifts of the individual scatterers are taken into account. If we have identical scatterers and adapt the Born approximation, we get the common angle-dependent scattering factor:

$$R_H = \Phi_i \sum_{i=1}^{N} \cos a_i; \; J_H = \Phi_i \sum_{i=1}^{N} \sin a_i; \; |\Phi(\theta)|^2 = R_H^2 + J_H^2, \tag{7}$$

where the scattering vector is

$$\boldsymbol{H} = \frac{\boldsymbol{s} - \boldsymbol{s}_0}{\lambda_R}; \qquad |H| = \frac{2}{\lambda_R} \sin \frac{\theta}{2}, \tag{8}$$

and the phase shift

$$a_i = \frac{2\pi}{\lambda_R} r_i' \sin \frac{\theta}{2} \sin\left(\varphi_i - \frac{\theta}{2}\right). \tag{9}$$

If we use an equal distribution of the random numbers, we obtain an isotropic distribution of scatterers within the scattering area. Fig. 3 shows the interference of the individual scatterer. On the left-hand side we see the projec-

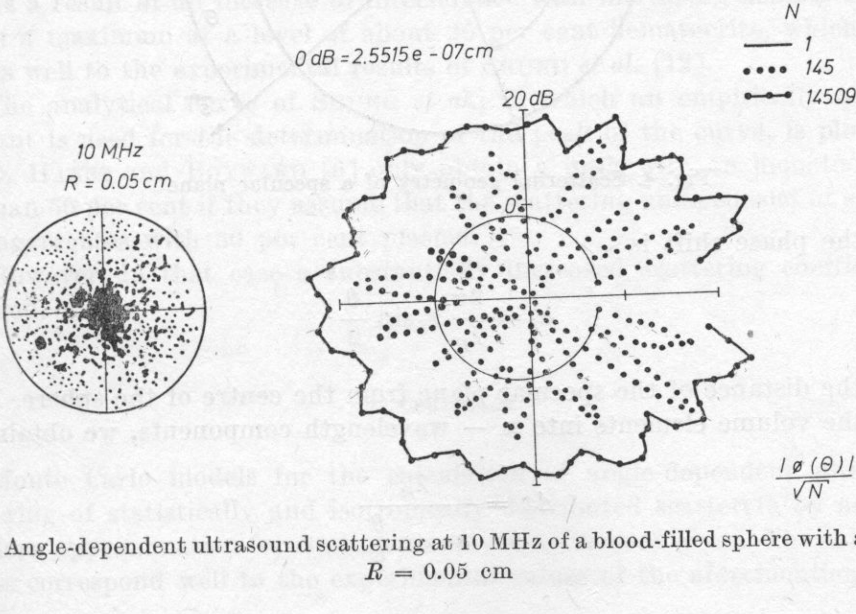

Fig. 3. Angle-dependent ultrasound scattering at 10 MHz of a blood-filled sphere with a radius; $R = 0.05$ cm

tion of the spatial distribution of the scatterers in the sphere. This is calculated using random numbers. In this example of a relatively small sphere with a radius $R = 1$ cm, we find that, in addition to a geometric effect, all contributions from the individual scatterers add up by analogy with the coherent solution (9).

However, calculations for a large number of particles would have required a relatively long period of time. For this reason we developed a second, simplified Monte Carlo model for higher hematocrite values.

3. Monte Carlo model 2

We collected all volume elements ΔV_i of the same phase shift a_i (Fig. 4). The respective volume elements were calculated to

$$\Delta V_i = 2\pi R^2 \cdot \Delta x - \frac{2}{3}\pi \Delta x^3 - 2\pi x_i^2 \Delta x, \tag{10}$$

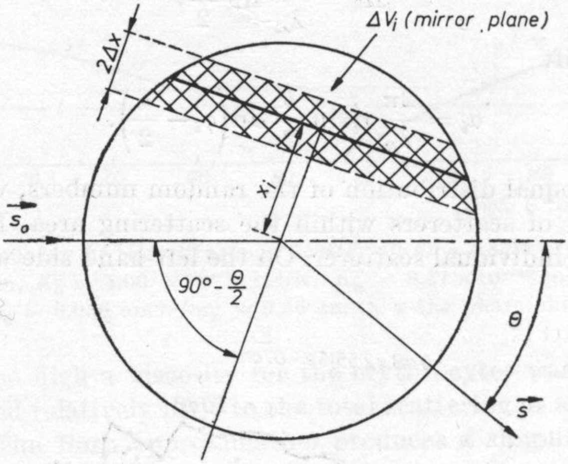

Fig. 4. Scattering geometry of a specular plane

where the phase shift is

$$a_i = \frac{2\pi}{\lambda_R} x_i \sin\frac{\theta}{2} \tag{11}$$

and x_i the distance of the specular plane from the centre of the sphere. If we divide the volume elements into n — wavelength components, we obtain

$$\Delta x = \frac{\lambda_R}{4n \sin\frac{\theta}{2}}. \tag{12}$$

The number of scattering particles that are contained in the volume element, must be calculated using a random number W_i. We assume that the number of scatterers in the volume element is Poisson distributed.

Thus:

$$W_i = \exp(-N'\Delta V_i)\,\frac{(N'\Delta V_i)^N}{N!}. \tag{13}$$

The number of scatterers N in the volume element ΔV_i with the phase shift α_i is calculated using a random number W_i.

Since the sum of Poisson distributed processes is also Poisson distributed, all specular planes of equal phase shift at the spacing

$$\Delta x_m = \frac{\lambda_R}{2\sin\frac{\theta}{2}} \tag{14}$$

can be collected.

Fig. 5 gives the calculated scattering coefficient

$$\frac{\sigma_s}{V} = \frac{|\Phi(\theta)|^2}{V} \tag{15}$$

for the back-scattering as the function of the hematocrite of the Monte Carlo calculations (model 2), in comparison to the quantities given by RAYLEIGH [11], SHUNG *et al.* [12], HANSS and BOYNARD [6], MORSE and INGARD [9].

As a result of an increase in interference with increasing hematocrite, we obtain a maximum at a level of about 20 per cent hematocrite, which corresponds well to the experimental results of SHUNG *et al.* [12].

The analytical curve of SHUNG *et al.*, in which an empirically adjusted constant is used for the determination of the peak of the curve, is plotted in Fig. 5. HANSS and BOYNARD [6] only obtain a peak with an hematocrite of less than 50 per cent if they assume that the scattering units consist of erythrocyte aggregates with 50 per cent plasma.

However, in that case a substantially increased scattering coefficient is obtained.

4. Conclusion

Monte Carlo models for the calculation of angle-dependent ultrasound scattering of statistically and isotropically distributed scatterers by adopting the Born approximation for continuous waves, have been given. The calculated results correspond well to the experimental values of the aforementioned literature. It is possible to study independently the influences of geometrical interferences, signal fluctuations, the correlation length of ultrasound impulses and the influence of the density of aggregates with this method.

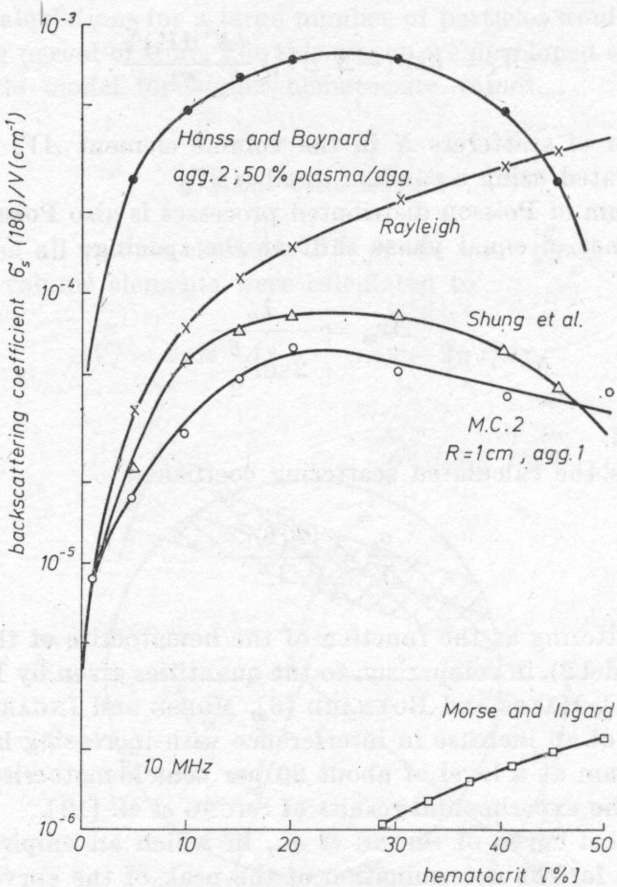

Fig. 5. The ultrasound back-scattering dependent upon the hematocrite mean values over 10 Monte Carlo calculations ○, in contrast to SHUNG *et al.* [12] △, RAYLEIGH [11] ×, and HANSS and BOYNARD [6] ●

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