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EVALUATION OF THE PHASE ERROR IN SOUND INTENSITY MEASUREMENT

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This paper presents an analysis of phase dependencies occurring in investigations of sound intensity, in particular those of the effect of the phase error of measurement equipment on the value and direction of the intensity vector measured at some point. Two methods of intensity measurement, based on measurement of the particle velocity from the pressure gradient, were taken into consideration: the direct method (formulae (5), (10), (12), (13)) and the one based on the cross-spectrum of acoustic pressure signals (formulae (6), (23)).

The phase error $\Delta\varphi$ causes changes in the directional characteristic of the system (formula (28), Fig. 7), changes in the values of intensity, measured with changed order of the measurement channels (formula (30), Fig. 6), the ratio of the values of the real part of the cross-spectrum of the pressure processes from the two microphones to that of its imaginary part (Figs. 9–10), and also in the existence of the imaginary part of their cross-spectrum when the two microphones are affected by the same acoustic field (formula (42), Fig. 8).

This paper presents theoretical considerations and specific examples of phase error evaluation in equipment used in investigations (Fig. 1) on the basis of the changes in question, which cause it.

The phase error of equipment causes considerable distortion of results, involving changes in the measured values, direction and also the sign of the intensity vector in some cases (see Fig. 11), therefore it is important to interpret it correctly.

Notation

$E(f)$ — expected value of the function of f
$e_{\Delta x}$ — unity vector in the direction Δx
F_x — Fourier transform of the function of x
F_x^* — conjugate value of F_x
$G_{ik}(j\omega)$ — measured auto ($i = k$) or cross spectral density function

$g_{ik}(j\omega)$ – real spectral density function undistorted by measurement error
$H_i(j\omega)$ – transmittance of the channel i
$|H_i|$ – amplification factor of the channel i
$\boldsymbol{I}(r)$ – intensity vector at the point r
$I(t)$ – intensity as a function of time
$I(\omega)$ – intensity spectrum
$I(\Delta\omega)$ – intensity component measured in the band $\Delta\omega$
k – wave number
T – analysis time
α – angle between the straight line connecting the fronts of the microphones and the wave incidence direction
Δx – spacing of the microphones in the probe
ε – effective error of the measured quantity
τ_i – pulse response of the channel i
Φ – phase shift between the pressure signals from the two microphones
$\Delta\varphi$ – phase error of equipment
$\Phi_{\Delta x}$ – phase shift of pressure behaviour caused by spacing the microphones at the distance Δx
$\psi(r)$ – phase difference between the pressure and velocity signals at the point r

1. Introduction

The value of the sound intensity vector at some point r of the field can be given by the formula

$$\boldsymbol{I}(r) = E[p(r,t)v_n(r,t)], \tag{1}$$

where p is the acoustic pressure of the wave and v_n is the particle velocity of the medium in the direction n.

Assuming in general that the phase shift in sinusoidal waves with frequency ω, between the wave pressure and the particle velocity, as expressed by the formulae

$$p = |p|\cos(\omega t-\psi),\quad v = |v|\cos\omega t \tag{2}$$

is ψ, the sound intensity can be given by the dependence

$$I(r) = \frac{|p|\,|v|}{2T}\left[\frac{1}{2}\int\limits_0^T \cos\psi dt + \frac{1}{2}\int\limits_0^T \cos(2\omega t-\psi)\,dt\right]. \tag{3}$$

The first term of the formula denotes energy propagation and is independent of time. The mean value of the second integral tends to zero when $T\to\infty$.

The value of the phase shift ψ depends on the structure of the field at a given point and determines the impedance of the medium at this point. It is difficult, even for simple sources, to estimate theoretically the value of the angle at any point of the field. In the far field, where all waves can approximately be considered plane, the angle $\psi = \pi/2$.

The practical evaluation of the acoustic intensity is based on the measurement of the particle velocity as the pressure gradient by the two-microphone method:

$$\boldsymbol{I}(r) = -\frac{1}{\varrho} E\left[p(r,t)\int_0^t \textbf{grad}\; p(r,t)\,dt\right]. \tag{4}$$

The direct method requires the measurement of the sum and difference between the pressure signals p_1 and p_2 from the two microphones and that the operations should be carried out according to the dependence [6], [7]

$$\boldsymbol{I}(r,t) = \boldsymbol{e}_{\Delta x}\lim_{T\to\infty}\left\{1/2\varrho\Delta x T\int_0^T\left[(p_1+p_2)\int_0^T (p_2-p_1)\,dt\right]dt\right\}, \tag{5}$$

where T is the measurement time, ϱ is the density of the medium, Δx-the spacing of the microphones, the direction of the vector $\boldsymbol{I}$ is defined by Δx.

In turn, the method of intensity calculation by means of the function of the cross-spectrum is based on the formula [2], [3] [5]

$$\boldsymbol{I}(\omega) = (\boldsymbol{e}_{\Delta x})\,(1/\varrho\Delta x\omega)\,\mathrm{Im}G_{12}(j\omega). \tag{6}$$

The error involved in the velocity evaluation based on the pressure gradient measurement according to the following dependence, which is basic for the two methods,

$$v(r,t) = -(1/\varrho\Delta x)\int_0^t [p_2(t)-p_1(t)]\,dt, \tag{7}$$

is affected by the phase error of equipment $\Delta\varphi(\omega)$, which is the phase difference between two channels, summing up with the physical phase difference between the pressure signals, resulting from the value of the spacing of the microphones, Δx, in the microphone probe, $\Phi_{\Delta x}$,

$$\Phi(\omega) = \Phi_{\Delta x}(\omega) \pm \Delta\varphi(\omega). \tag{8}$$

The behaviour of the acoustic pressure processes registered by the two microphones is thus described by the dependencies

$$p_1 = A\cos\omega t,$$

$$p_2 = B\cos[\omega t + \Phi_{\Delta x}(\omega) \pm \Delta\varphi(\omega)]. \tag{9}$$

2. Effect of the phase error of equipment on the results of the intensity measurement

In spite of all technical operations, there is usually a slight phase difference between channels. Below is presented the effect of the phase error on the results obtained by the direct method of intensity measurement and that of cross-spectrum. Fig. 1 shows a schematic diagram of the equipment used in the investigations to confirm the considerations.

2. 1. *Effect of the phase error in the direct method*

When between measurement channels there is the phase difference $\Delta\varphi$ and also that in amplification, then, instead of the acoustic pressures $p_1(t)$ and $p_2(t+\Delta x/c)$, the measurement system registers the pressures (Fig. 2) $P_1 =$

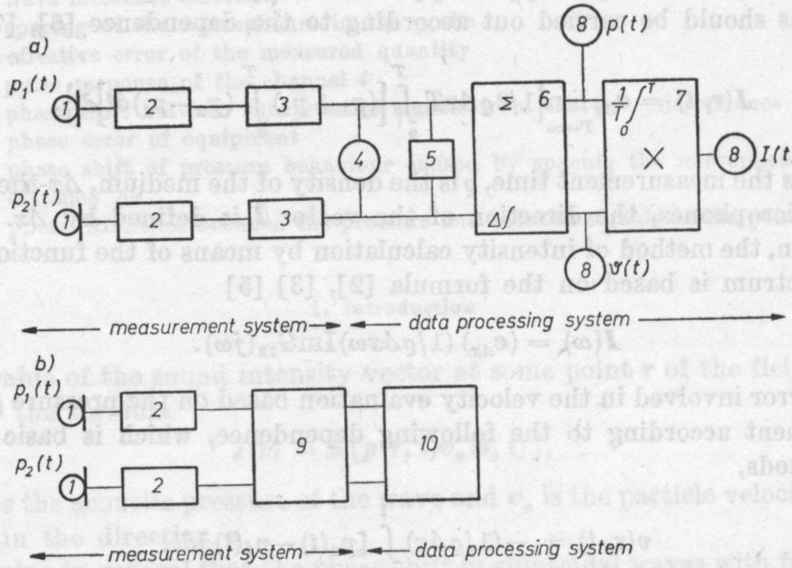

Fig. 1. A general diagram of the equipment used in the intensity investigations. The measurements system is the same for the direct and cross-spectral methods, with different measurement data processing system for each (a) for the direct method, (b) for the cross-spectral method. 1 — MV201 1/2′ microphones, 2 — 00017 RFT amplifiers, 3 — system of 00017 RFT 1/3 octave filters, 4 — SS 4100 Iwatsu two-stream oscilloscope, 5 — BK 2971 phasemeter, 6 — author's own signal summation and differentiation-integration system, 7 — DISA 55 D 75, 52B25 multiplication and integration system, 8 — V 541 digital voltmeter, 9 — magnetic recorder (with different types used (see the text)), 10 — energy spectrum analyser, Universal Digital Analyser Plurimat S, or 3720 and 3721 Hewlett-Packard correlator and integration system

$p_1(t)/\tau_1$ and $P_2 = p_1(t+\Delta x/c)/\tau_2$. Thus, the system measures some intensity value $I(r, t)$, which is different from the real intensity $I(r)$ (formula (5)):

$$\hat{\boldsymbol{I}}(r, t) = e_{\Delta x}(1/2T\Delta x\varrho)\int_0^T \left\{\left[p_1(t)/\tau_1 + p_2\left(t+\frac{\Delta x}{c}\right)/\tau_2\right]\int_0^t \left[p_2\left(t+\frac{\Delta x}{c}\right)/\tau_2 - p_1(t)/\tau_1\right]dt\right\}dt, \tag{10}$$

where τ_i is the pulse response of the channel i.

When using this method the two channels should show as high phase and amplitude agreement as possible, since it is impossible to compensate for the differences in a simple way at the stage of intensity calculations. It is easy to eliminate differences in amplification, but some inevitable left-over phase difference causes error to arise in evaluation, which can be represented for the current sinusoidal wave in the following way.

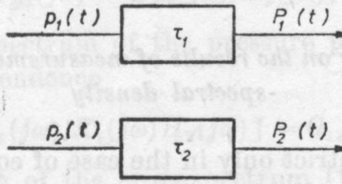

Fig. 2. A schematic diagram of the time transformation of the channels of the measurement system. τ_i denotes the pulse response of the channel i

The pressures registered by the two microphones can be given by the harmonic series

$$p_1(t) = \sum_i A_i \cos[\omega_i t + (k_i \Delta x \pm \Delta\varphi(\omega_i)/2],$$
$$p_2(t) = \sum_i B_i \cos[\omega_i t - (k_i \Delta x \pm \Delta\varphi(\omega_i)/2]. \quad (11)$$

The phase error causes a shift in the pressure phase, and thus at the same time, it affects the value of intensity measured by method represented by formula (5). This is expressed by the following formula, obtained as a result of the substitution of (11) in (5),

$$I(r, t) = (1/2\varrho\Delta x) \sum_i [(A_i B_i/\omega_i) \sin(k_i \Delta x \pm \Delta\varphi(\omega_i)]. \quad (12)$$

A more accurate result of the intensity measurement can be obtained by determining the arithmetic mean from two measurements carried out with changed order of microphones with respect to the wave incidence direction. In one of the measurements, the phase difference between the channels sums up with the phase difference caused by the spacing of the microphones, and it detracts in the other, and thus the mean value calculated for any bands $\Delta\omega$ from two measurements can be expressed by the following dependence:

$$I_{av}(r, \Delta\omega) = (|I(r, \Delta\omega)^{(a)}| + |I(r, \Delta\omega)^{(b)}|)/2$$
$$= I(r, \Delta\omega)[\sin(k\Delta x - \Delta\varphi) + \sin(k\Delta x + \Delta\varphi)]/2k\Delta x$$
$$= I(r, \Delta\omega)\cos\Delta\varphi \sin k\Delta x/k\Delta x. \quad (13)$$

The values of k and $\Delta\varphi$ are calculated for the centre frequency of the band $\Delta\omega$.

In the case when the amplification factors of channels, $H_1(\omega)$ and $H_2(\omega)$, are different from unity, they also have to be considered in the intensity measurement:

$$I(r, t) = e_{\Delta x}(1/2H_1 H_2 \varrho \Delta x) \sum_i (A_i B_i \sin k_i \Delta x / k_i \Delta x). \tag{14}$$

2.2 Effect of the phase error on the results of measurements by the method of cross-spectral density

Formula (6) would be strict only in the case of equipment with ideal transmission, undistorted by amplitude and phase error. In practice, instead of the signals p_1 and p_2, at the output of the measurement equipment there are the signals P_1 and P_2, resulting from the passage through channels with definite pulse responses τ_1 and τ_2, or, in the frequency domain, with the respective

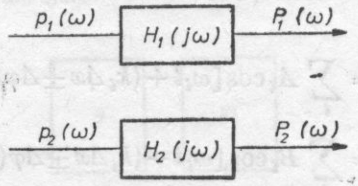

Fig. 3. A schematic diagram of the transfer function of the channels of the measurement system. $H(j\omega)$ denotes the transmittance of the measurement system

transmittances $H_1(j\omega)$ and $H_2(j\omega)$ (Fig. 3). The transmittances $H_1(j\omega)$ and $H_2(j\omega)$ are the complex functions of frequency

$$\begin{aligned} H_1(j\omega) &= |H_1| \exp[-j\varphi_1(\omega)], \\ H_2(j\omega) &= |H_2| \exp[-j\varphi_2(\omega)], \end{aligned} \tag{15}$$

and thus the real cross-spectrum of the signals, $g_{12}(j\omega)$, of the pressures present at points 1 and 2 will in a general case be different from the cross-spectrum of the signals registered by the equipment, $G_{12}(j\omega)$. The real cross-spectrum of the signals p_1 and p_2 is given by the formula

$$g_{12}(j\omega) = E\{F_{p1}(j\omega) F_{p2}^*(j\omega)\}, \tag{16}$$

where E is the mean value.

Since the measured signals P_1 and P_2 are related to the real pressure values by the dependencies between their transforms:

$$F_{p1}(j\omega)H_1(j\omega) = F_{P1}(j\omega),$$
$$F_{p2}(j\omega)H_2(j\omega) = F_{P2}(j\omega), \quad (17)$$

and hence,

$$F_{p1}(j\omega) = F_{P1}(j\omega)/H_1(j\omega),$$
$$F_{p2}(j\omega) = F_{P2}(j\omega)/H_2(j\omega), \quad (18)$$

herefore, the real cross-spectrum of the pressure processes is related to the measured one by the dependence

$$g_{12}(j\omega) = E[F_{P1}(j\omega)F_{P2}(j\omega)/H_1(j\omega)H_2(j\omega)] = G_{12}(j\omega)/H_1(j\omega)H_2(j\omega), \quad (19)$$

where, from the definition of the cross-spectrum [1],

$$G_{12}(j\omega) = E[F_{P1}(j\omega)F^*_{P2}(j\omega)]. \quad (20)$$

The intensity value calculated from measurements is given by the formula

$$I(j\omega) = e_{\Delta x}\mathrm{Im}\{G_{12}(j\omega)\exp(-j(\varphi_2-\varphi_1))\}/2\pi f\Delta x\varrho|H_1|\,|H_2|$$
$$= e_{\Delta x}\mathrm{Im}\{G_{12}(j\omega)[\cos\Delta\varphi(\omega)\pm j\sin\Delta\varphi(\omega)]\}/2\pi f\varrho\Delta x|H_1|\,|H_2|, \quad (21)$$

where $\Delta\varphi(\omega) = \varphi_2-\varphi_1$ is the phase difference between channels for the frequency ω.

Calculation of the value of the imaginary part of formula (21) gives the dependence

$$\mathrm{Im}\{G_{12}(j\omega)[\cos\Delta\varphi(\omega)\pm j\sin\Delta\varphi(\omega)]\} = \mathrm{Im}G_{12}(\omega)\cos\Delta\varphi(\omega)$$
$$+R_eG_{12}(\omega)\sin\Delta\varphi(\omega). \quad (22)$$

Substitution of formula (22) into (6) gives the following formula for the intensity value, accounting for the phase error of the equipment,

$$I(j\omega) = [\mathrm{Im}G_{12}(\omega)\cos\Delta\varphi(\omega)\pm\mathrm{Re}G_{12}(\omega)\sin\Delta\varphi(\omega)]/\omega\varrho\Delta x|H_1|\,|H_2|. \quad (23)$$

On the assumption of low phase error, formula (23) becomes the same as (6).

Two measurements, involving changed order of microphones, permits total elimination of the phase error [4], [9] in calculating intensity as the geometric mean of the two imaginary parts of the cross-spectrum.

3. Phase calibration of the measurement system

Irrespective of the further means of signal processing, an intensity measurement system consists of a probe (Fig. 4) – a system of two microphones spaced at Δx, preamplifiers, amplifiers and possibly a magnetic recorder. Despite careful selection of the elements, it is inevitable for some phase differences to occur between channels; particularly significant sources of phase shift are multi-head tape recorders with different recording and reproducing heads.

In most of the investigations, the author used a two-channel tape recorder which was modified in the laboratory to serve measurement purposes. This

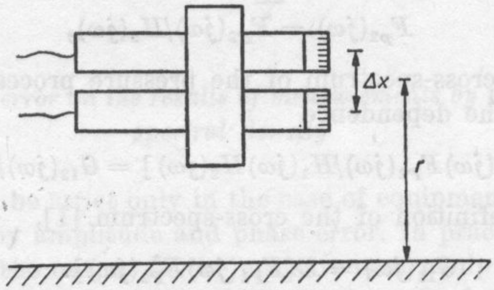

Fig. 4. The microphone probe used in the investigations. 1/2′ microphones spaced at Δx at the distance r from the source

tape recorder with one recording and reproducing head does not cause any phase shift within the resolution capacity of the equipment, 0.5°, in contrast to high-class multi-channel tape recorders with separate recording and reproducing heads, e.g. in the 8–channel Schlumberger tape recorder (where the heads are two four-channel units) only one pair of channels was found to involve low phase shift. Very small inaccuracies in the setting of heads (different for recording and reproducing ones) cause high phase differences. An error of a few thousandths of a millimetre can cause phase error of 90° and 180°, depending on the tape velocity (the error decreases as the velocity increases) and on the frequency (the phase error is lower at higher frequencies), e.g. for 5 kHz at the tape velocity of 190 mm/s the 0.01 mm displacement of the heads causes the phase error $\Delta\varphi = 45°$. In a high-class tape recorder like Schlumberger, under these conditions, error close to 90° was observed, and for a tape velocity of 95 mm/s it was almost 180°. In a Nagra IV *SJ* tape recorder, which was used in some measurements, the phase difference-small at low frequencies: 100 Hz −1°, 200 Hz −2° − increases to 40° at 4000 Hz.

The system can be calibrated electrically or acoustically in the plane wave field, e.g. in a tube of standing waves or in the far field.

Measurements of the phase shift between channels for electrical signals supplied from the generator to the inputs of the amplifiers permit; when the first element of system, i.e. the microphone, is neglected; the selection of appropriate elements of the system (e.g. tape recorder tracks) and, in some cases, compensation for the phase differences found.

The microphones themselves are only a slight source of phase error, whereas when they are part of the probe system, they cause some mutual field perturbations [6], [11].

The phase differences between channels can be measured by a phasemeter (or by a two-stream oscilloscope), when the probe is in the plane wave field with varying frequency. The result depends to some extent on the order and the slope (rising or falling) of the release signal, which indicates the purposiveness of averaging of result series. Microphones, together with channels, are changed in position in the course of measurements (positions *a* and *b*), which changes their position with respect to the wave front. When the phase measurement process is released by a signal from channel 1 and the measured value of the phase difference between the channels is $\Phi_{2/1}$, the equipment error can be determined rom the dependence

$$\begin{aligned} \Delta\varphi(\omega) &= \Phi_{2/1}^{(a)}(\omega)-\Phi_{\Delta x}(\omega), \\ \Delta\varphi(\omega) &= \Phi_{2/1}^{(b)}(\omega)+\Phi_{\Delta x}(\omega). \end{aligned} \tag{24}$$

When the processes are released by a signal from channel 2, the measured phase difference is $\Phi_{1/2}$ and the equipment error $\Delta\varphi$ results from the equations

$$\begin{aligned} \Delta\varphi(\omega) &= -\Phi_{1/2}^{(a)}(\omega)-\Phi_{\Delta x}(\omega), \\ \Delta\varphi(\omega) &= -\Phi_{12}^{(b)}(\omega)+\Phi_{\Delta x}(\omega). \end{aligned} \tag{25}$$

It is purposeful to carry out a large number of various phase error measuref ments, since phase fluctuations can be observed in the measurement system (a problem signalled in the literature).

4. Evaluation of the phase difference between channels on the basis of the directional responses of the microphone system

The directional responses of the microphone system were made in the far field of a loudspeaker fed from a generator. The microphone system was fixed in the axis of a Drehtisch 02012 *RFT* turntable with remote control programmed for measurements every 15°. The fronts of the microphones described a circle with a radius of 2 cm, which in the far field ensured in this region signals with close values for the two microphones. A few measurements of the responses were carried out, giving good repeatability of the basic structure of the response (the measurements were densified at the characteristic points).

The microphone system shows large directionality at medium and high frequencies (an example for the probe investigated is shown in Fig. 5), permitting the maximum values, and even more distinctly the minimum values, as a function of the angle of the rotation of the probe with respect to the wave incidence direction, to be found. The maximum values are dozen-odd times as large as the minimum ones, and the difference in their levels is dozen-odd dB.

It is interesting to observe changes in the phase angle between the behaviours p and v for the rotation of the axes of the microphone with respect to the source (Fig. 6) in a free field. When the axes of the microphones are set

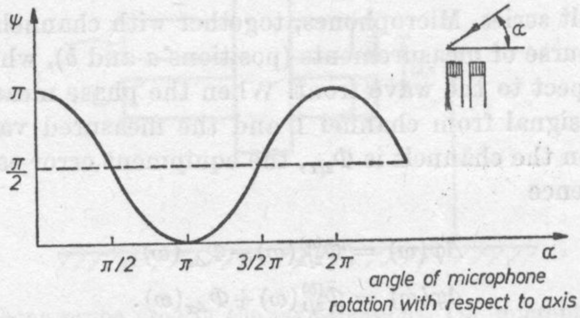

Fig. 5. A change in the angle ψ between the pressure behaviour and the particle velocity as a result of the rotation of the microphones with respect to the axis, relative to the incident wave, observed on the display of a twostream oscilloscope, for the system shown in Fig. 1a

parallel to the direction of the wave ($\alpha = 90°$) it is possible to observe the phase shift $\psi = 90$. and, as a consequence, zero value of intensity. In turn the angle $\psi = 0°$ and the related intensity maximum can be observed close to the angle $\alpha = 180°$, whereas the value $\psi = 180°$ and the maximum intensity value with the opposite sign occur close to the angle $\alpha = 360°$. A change in the wave incidence angle α with respect to the axes of the microphones is reflected in a change in the value and sign of the intensity vector, according to the cosine function

$$I = e_{\Delta x} |I| \cos \alpha. \tag{26}$$

The behaviour of directional responses obtained for the microphone system used in papers [7–9] (see Fig. 1a) requires interpretation based on theoretical considerations. An ideal directional response would be symmetrical, minimum intensity values would occur for the incidence angles of 90° and 270°. The real responses are not symmetrical and the minima are shifted. This effect, when neglecting some measurement error, results from the existence of the phase error $\Delta\varphi$ in equipment and permits this error to be calculated.

It is seen from formula (12) that the intensity takes a zero value when the argument of the sine function is zero, i.e.

$$k \Delta x \cos \alpha = \Delta\varphi(\omega),$$

or (27)

$$k \Delta x \cos \alpha = \Delta\varphi(\omega) + \pi$$

thus for the following values of the incidence angles α

$$\alpha = \begin{cases} \cos^{-1}(\Delta\varphi(\omega)/k\Delta x), \\ \cos^{-1}[(\pi+\Delta\varphi(\omega)/k\Delta x]. \end{cases} \tag{28}$$

When the angle $\alpha_{\min}$ corresponding to $I_{\min}$ is found from the directional response, it is possible to calculate the phase error for a given frequency from the formula

$$\Delta\varphi(\omega) = k\Delta x \cos\alpha_{\min}. \tag{29}$$

The following values were calculated for the 0.5 kHz directional response shown in Fig. 4:

$$\alpha_{\min} = 115°, \text{ i.e. } \Delta\varphi = 2°; \; \alpha_{\min} = 285°, \text{ i.e. } \Delta\varphi = 2°.$$

The phase error of equipment is also reflected in the nonsymmetry of the sensitivity of the system, i.e. the different maximum intensity values for $\alpha \cong 0°$ and $\alpha \cong 180°$. A decrease in the sensitivity in one direction is accompanied by its increase in another. The ratio of the maximum amplitudes registered in those cases can be represented by the formula

$$\sin\left(k\Delta x+\varphi(\omega)\right)/\sin\left(k\Delta x-\varphi(\omega)\right) = K, \tag{30}$$

which permits the phase error of equipment to be calculated. The value of the phase error of the example shown in Fig. 5, which was calculated by this method is only $\Delta\varphi = 0.5°$ for $K = 1.2$. When $k\Delta x$ takes the value of $\Delta\Phi$, the directional response changes radically, a double increase in the value measured in one direction is accompanied by a decrease in the intensity value to zero in another, which results from the following dependencies:

$$I_1 = |I|[\sin(k\Delta x+\Delta\varphi)]/k\Delta x = |I|(\sin 2k\Delta x)/k\Delta x,$$

$$I_2 = |I|[\sin(k\Delta x-\Delta\varphi)]/k\Delta x = 0. \tag{31}$$

The intensity change towards the intensity maximum is

$$\varepsilon = I_1/I_{\varphi=0} = 2\cos k\Delta x \simeq 2, \tag{32}$$

corresponding to an increase of $\Delta L = 3$ dB in the measured intensities for low $k\Delta x$. It can be seen in the example of the directional response of the probe discussed, shown in Fig. 6, for the frequency $f = 0.2$ Hz, how strongly the effect of the phase error on the form of the directional response depends on frequency. For one minimum with the angle $\alpha_{\min} = 150°$, the error calculated from formula (29) is only 1.5° which almost causes one of the branches of the response to vanish (a similar effect is cited in ([10]).

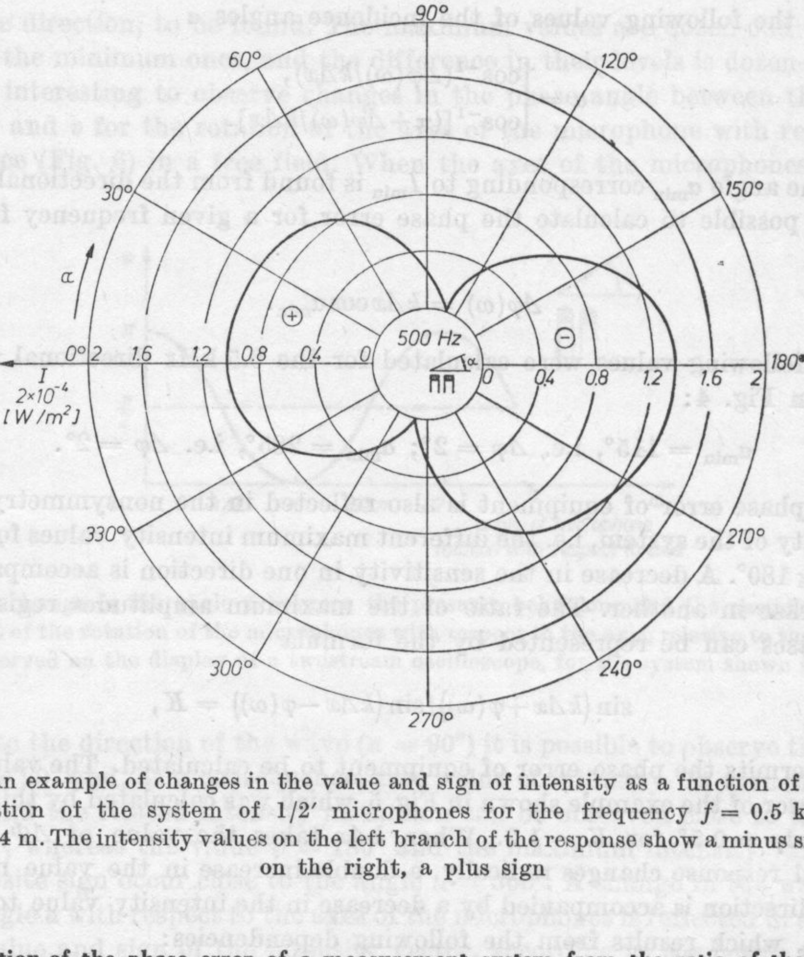

Fig. 6. An example of changes in the value and sign of intensity as a function of the angle α of rotation of the system of 1/2′ microphones for the frequency $f = 0.5$ kHz, with $\Delta x = 0.014$ m. The intensity values on the left branch of the response show a minus sign; those on the right, a plus sign

5. Evaluation of the phase error of a measurement system from the ratio of the real and imaginary parts of the cross-spectral density of the pressure behaviours from the two microphones of the probe

The test of the lack or presence of phase shifts between channels over the whole frequency range is the behaviour of the values of the function $\mathrm{Im}G_{12}(j\omega)$ when the input signals are equal for the two channels in terms of amplitude and phase, which results from the following calculations.

When the microphone system is in the sinusoidal wave field, the microphones register processes which can be expressed by formula (9). The auto spectral density functions of the processes are represented by the following definitions [1]:

$$g_{11}(\omega) = 2 \lim_{T\to\infty} E[F_{p1}(j\omega)F^*_{p1}(j\omega)], \tag{33}$$

$$g_{22}(\omega) = 2 \lim_{T\to\infty} E[F_{p2}(j\omega)F^*_{p2}(j\omega)],$$

while the cross-spectral density of the two behaviours is given as

$$g_{12}(j\omega) = 2 \lim_{T\to\infty} E[F_{p1}(j\omega)F_{p2}^*(j\omega)], \tag{34}$$

where F_{p1} and F_{p2} are Fourier transforms of the pressure behaviours p_1 and p_2. In turn, the values of the transforms F_{p1} and F_{p2} are, from the definitions, given by the formulae

$$F_{p1}(j\omega) = (A/2)\int_0^\infty \{[\exp(j\omega_0 t)+\exp(-j\omega_0 t)]\exp(-i\omega t)\}dt = jA\omega/(\omega_0^2-\omega^2), \tag{35}$$

$$F_{p2}(j\omega) = (B/2)\int_0^\infty \{[\exp(j\omega_0 t+\varphi)+\exp[-j(\omega_0 t+\varphi)]]\exp j\omega t\}dt$$
$$= [B/(\omega_0^2-\omega^2)](\omega_0\sin\varphi - j\omega\cos\varphi). \tag{36}$$

Substitution of the expressions of the pressure transforms (35) and (36) in formulae (33) and (34) gives the following dependencies:

$$g_{11}(\omega) = A^2/(\omega_0^2-\omega^2)^2,$$

$$g_{22}(\omega) = [B^2/(\omega_0^2-\omega^2)^2](\omega_0^2\sin^2\varphi+\omega^2\cos^2\varphi) \tag{37}$$

and

$$g_{12}(j\omega) = [AB\omega/(\omega_0^2-\omega^2)](\omega\cos\varphi - j\omega_0\sin\varphi). \tag{38}$$

The modulus of the cross energy density is thus expressed by the formula

$$|g_{12}(\omega)| = [AB\omega/(\omega_0^2-\omega^2)](\omega^2\cos^2\varphi+\omega_0^2\sin^2\varphi)^{1/2}; \tag{39}$$

its real part is

$$\mathrm{Re}\,g_{12}(\omega) = [AB\omega^2/(\omega_0^2-\omega^2)]\cos\varphi \tag{40}$$

and the imaginary part

$$\mathrm{Im}\,g_{12}(\omega) = [jAB\omega\omega_0/(\omega_0^2-\omega^2)]\sin\varphi. \tag{41}$$

There follows the following relationship of the phase difference between signals from the two microphones

$$\tan\varphi(\omega) = \tan[\varphi_{\Delta x}(\omega)\pm\Delta\varphi(\omega)] = \mathrm{Im}G_{12}(\omega)/\mathrm{Re}G_{12}(\omega). \tag{42}$$

With zero phase shift between the process p_1 and p_2, the imaginary part of the cross-spectrum is zero, whereas the real part reaches a maximum value. Fig. 8 shows successively the spectrum of the imaginary part, of the real part and the modulus of the cross-spectrum between the pressure behaviours registered by the measurement system (Fig. 1a) when the microphones in the tube are affected by a signal which is the same in terms of amplitude and phase.

In the case shown in Fig. 8, the value of the phase error $\Delta\varphi$, calculated from the ratio of the real and imaginary parts of the cross-spectrum, is 2° for $f = 0{,}5$ kHz.

An example of the difficulties occurring in the interpretation of results when the phase error of equipment is large with respect to $k\Delta x$, drawn from the author's own investigations, is given below. In some case of excitation of a plate by a tone, with signals being recorded by a Nagra tape recorder, a series of

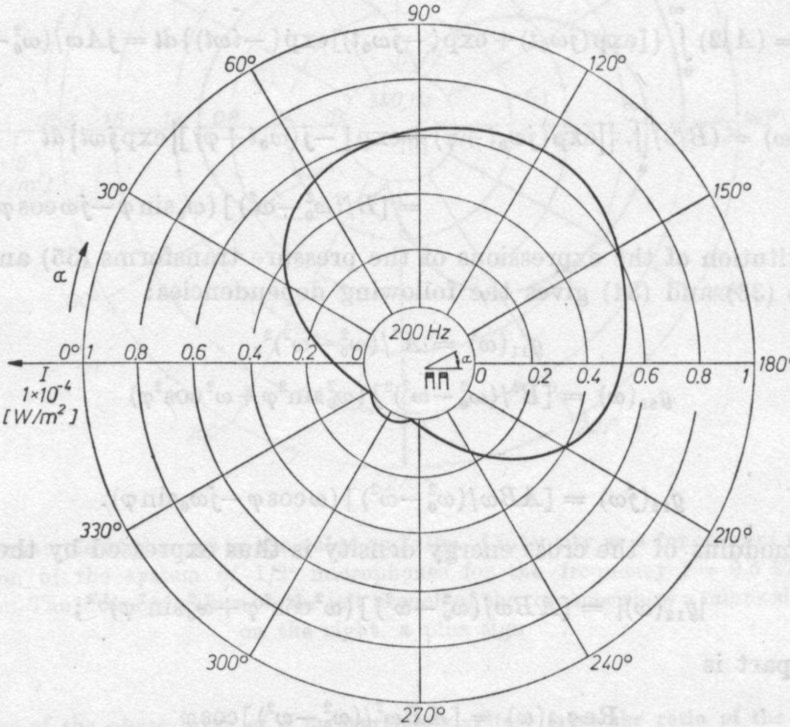

Fig. 7. An example of changes in the directional response of the system of 1/2' microphones for the frequency $f = 0.2$ kHz as a result of the existence of phase error with the value $\Delta\Phi = 1.5°$ in the system shown in Fig. 1a. The microphones spaced at $\Delta x = 0.014$ m

results was obtained, where the sign of $\mathrm{Im}G_{12}(\omega)$ varied depending on the frequency emitted by the plate, irrespective of the measurement point on the plate (Fig. 8). The measurement for one of the positions of the probe gave probable results and that for the opposite localisation yielded results which it was difficult to interpret. The mean value of the phase error of equipment, $\Delta\varphi(0.5) = 4°$ (with phasemeter measurements indicating 5°), was calculated from the ratio $\mathrm{Re}G_{12}(\omega)/\mathrm{Im}G_{12}(\omega)$ for all the measurement points. In this case (the frequency of the excitation signal $f = 0.5$ kHz, with the resonance frequency of the plate $f = 21$ Hz, the microphone spacing $\Delta x = 0.014$ m, the distance between the probe and the source $r = 0.4$ m) the value of $\Phi_{\Delta x}$ was 7.5°, and thus the results of the intensity measurements can be represented, in keeping with formula (41), for known values of $\Delta\varphi$ and $\Phi_{\Delta x}$, in the following way:

$$\mathrm{Im}\,G_{12}^{(a)}(0.5) \sim \text{const. } 0.21, \qquad \mathrm{Im}\,G_{12}^{(b)}(0.5) \sim \text{const. } 0.03.$$

It can be seen from Fig. 9 that in this case it is difficult to compare the calculated and measured values, although, considering the signs of the components, the mean value of intensity in the band is close to zero. (Because of the difficulties in interpretation this method was abandoned in practical applications).

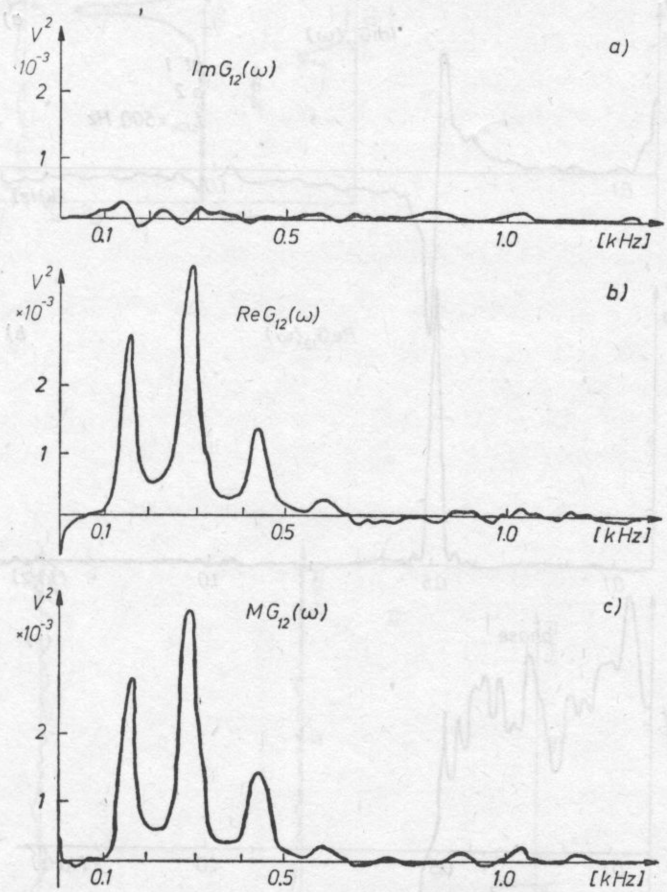

Fig. 8. An example of the behaviour of the cross-spectral density $G_{12}(j\omega)$ of signals from the microphone system placed in the same acoustic field in a tube of standing waves. a) the imaginary part of the spectrum, b) the real part, c) the modulus. The results permit the phase error of equipment to be calculated from formula (42) for any frequency. The signal was white noise, the microphones were spaced at $\Delta x = 0.014$ m. The calculations were carried out on a Hewlett-Packard system (Fig. 1b), with a Nagra tape recorder. The conditions of the calculations: the width of the frequency band analysed B = 15 Hz, the number of countings $N = 128 \times 1024$, summation averaging

The sine function in formula (41) changes its sign for the emitted frequency, there is a phase jump by π. The resultant phase difference, in the case of using a Nagra tape recorder, is too small for correct results to be obtained. The change of the sign of $\mathrm{Im}G_{12}(\omega)$ from $+$ to $-$, or conversely (Fig. 9), depends, it was found, on the arbitrarily assumed order of signal processing.

It was found that the ratio of the real parts for two positions of the probe $\mathrm{Re}G_{12}^{(a)}(\omega)/\mathrm{Re}G_{12}^{(b)}(\omega) = 1$, which results from calculations (formula (40)) and measurements (Figs. 9–11). However, it was established that when the tone exciting

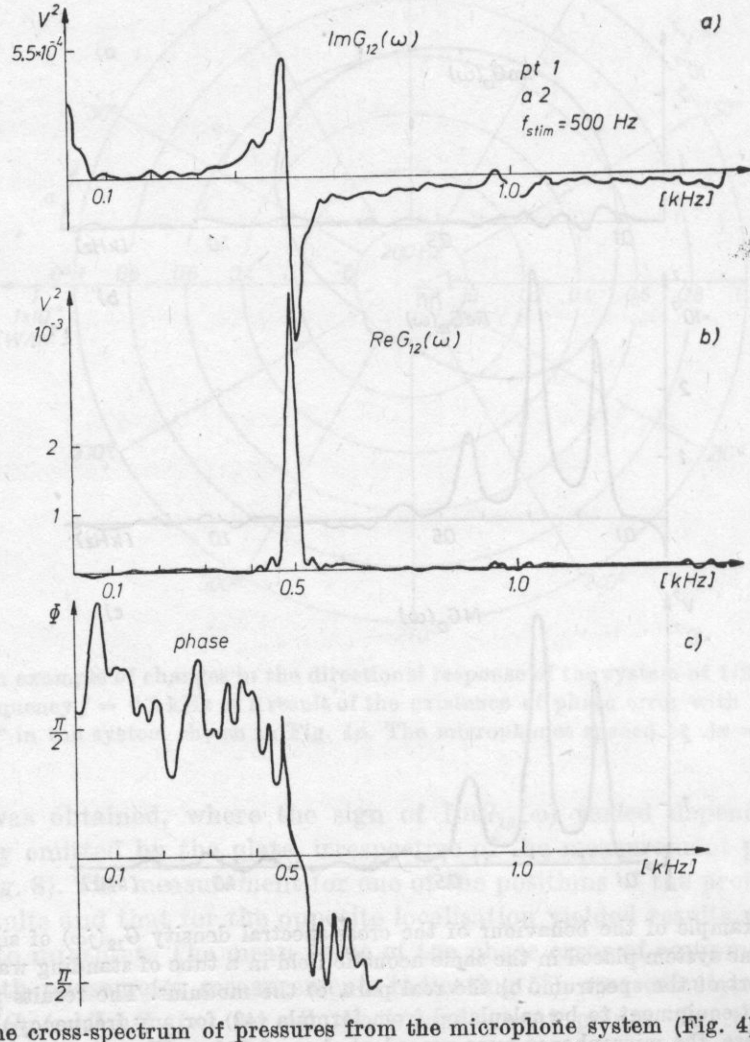

Fig. 9. The cross-spectrum of pressures from the microphone system (Fig. 4), placed over a vibrating plate excited by a tone with the frequency $f = 0.5$ kHz. a) the imaginary part of the spectrum, b) the real part, c) phase. The conditions of the measurements: the microphones spaced at $\Delta x = 0.014$ m, the distance between the probe and the plate $r = 0.04$ m. The conditions of the calculations are the same as in Fig. 8

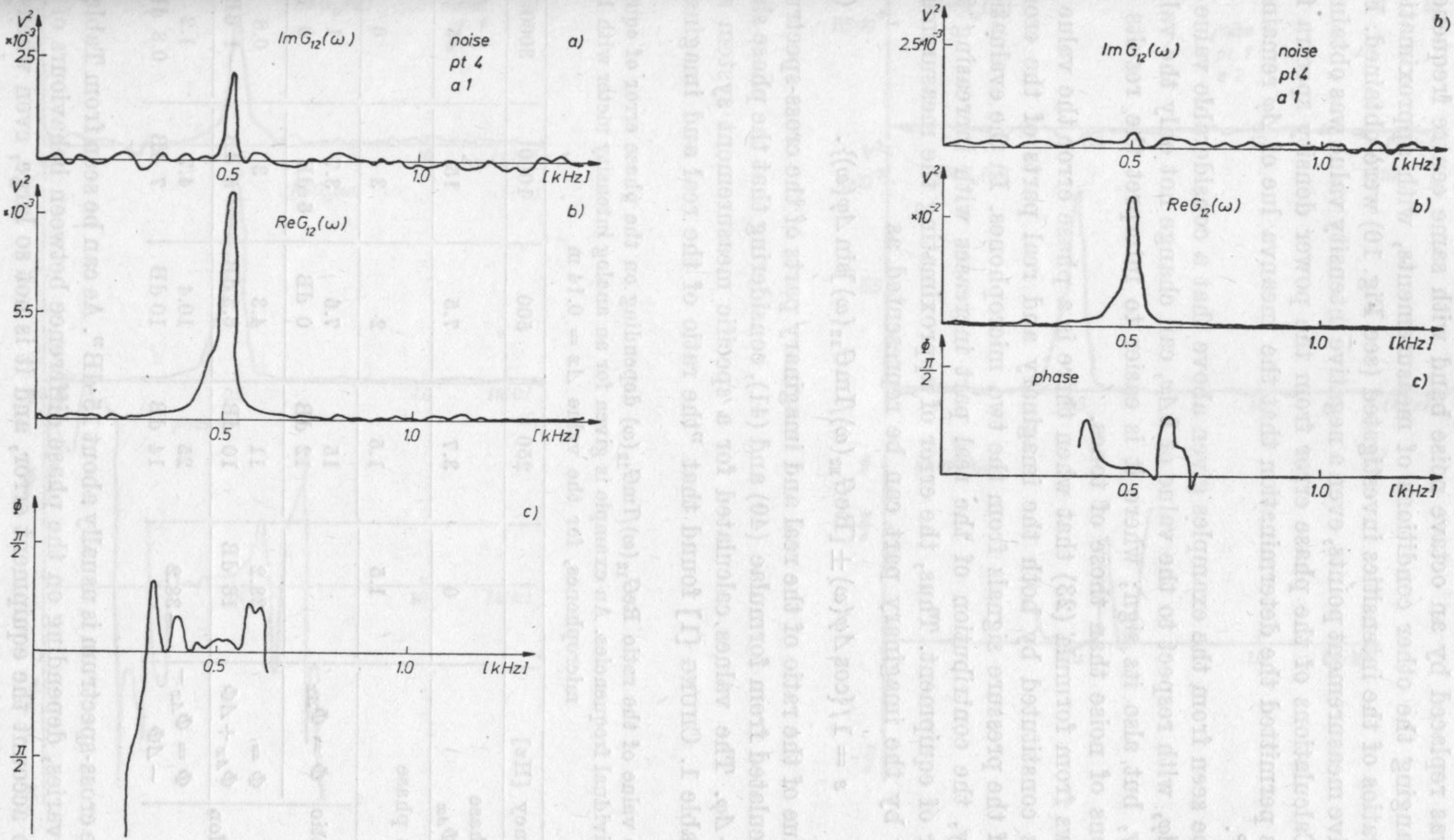

Fig. 10. The cross-spectrum of pressures from the microphone system, placed over a plate excited to vibration by a noise band with the centre frequency $f = 0.5$ kHz. The results were obtained for two positions of the probe (a) and after reversal (b). The other conditions of the measurements and calculations were the same as those in the description of Fig. 9

the plate was replaced by an octave noise band with same centre frequency, without changing the other conditions of measurements, with approximation the above ratios of the intensities investigated (see Fig. 10) were obtained. For one of the five measurement points, even a negative intensity value was obtained (Fig. 11). Calculations of the phase error from the power density spectra for noise bands permitted the determination that the meanva lue of $\Delta\varphi$ remained equal to 4°.

It can be seen from the examples given above that a considerable value of the error $\Delta\varphi$, with respect to the value of $k\Delta x$, can change not only the value of intensity, but also its sign; where it is easier to interpret the results of investigations of noise than those of tones.

It follows from formula (23) that when there is a phase error the value of intensity is constituted by both the imaginary and real parts of the cross-spectrum of the pressure signals from the two microphones. In the evaluation of intensity, the contribution of the real part increases with increasing the phase error of equipment. Thus, the error of approximating the measurement result only by the imaginary part can be represented as

$$\varepsilon = 1/\{\cos\Delta\varphi(\omega) \pm [\mathrm{Re}G_{12}(\omega)/\mathrm{Im}G_{12}(\omega)]\sin\Delta\varphi(\omega)\}. \qquad (43$$

The value of the ratio of the real and imaginary parts of the cross-spectrum can be calculated from formulae (40) and (41), considering that the phase shift $\Phi = k\Delta x + \Delta\varphi$. The values calculated for a specific measurement system are given in Table 1. CHUNG [1] found that "the ratio of the real and imaginary

Table 1. The value of the ratio $\mathrm{Re}G_{12}(\omega)/\mathrm{Im}G_{12}(\omega)$ depending on the phase error of equipment for individual frequencies. An example is given for an analog intensity meter with 1/2' microphones, for the value $\Delta x = 0.14$ m

Frequency [Hz]			250	500	1000	3000
Physical phase difference $\Phi_{\Delta x}$		0	3.7	7.5	15	45
Equipment phase error $\Delta\Phi$		1.5	1.5	2	3	6
Value of ratio	$\Phi = \Phi_{\Delta x}$		15 12 dB	7.6 0 dB	3.7 6 dB	1 0
$\frac{\mathrm{Re}G_{12}(\omega)}{\mathrm{Im}G_{12}(\omega)}$ for	$\Phi = \Phi_{\Delta x} + \Delta\Phi$	38.2 16 dB	11 10 dB	4.3 6.3 dB	3 5 dB	0.8 −1 dB
	$\Phi = \Phi_{\Delta x} - \Delta\Phi$	−38.2	25 14 dB	10.4 10 dB	4.7 7 dB	1.2 0.8 dB

parts of the cross-spectrum is usually about 15 dB". As can be seen from Table 1, this value varies, depending on the phase difference between behaviours of Φ, taking into account the equipment error, and it is not so large, even when no phase error occurs (see Table 1, case *a*).

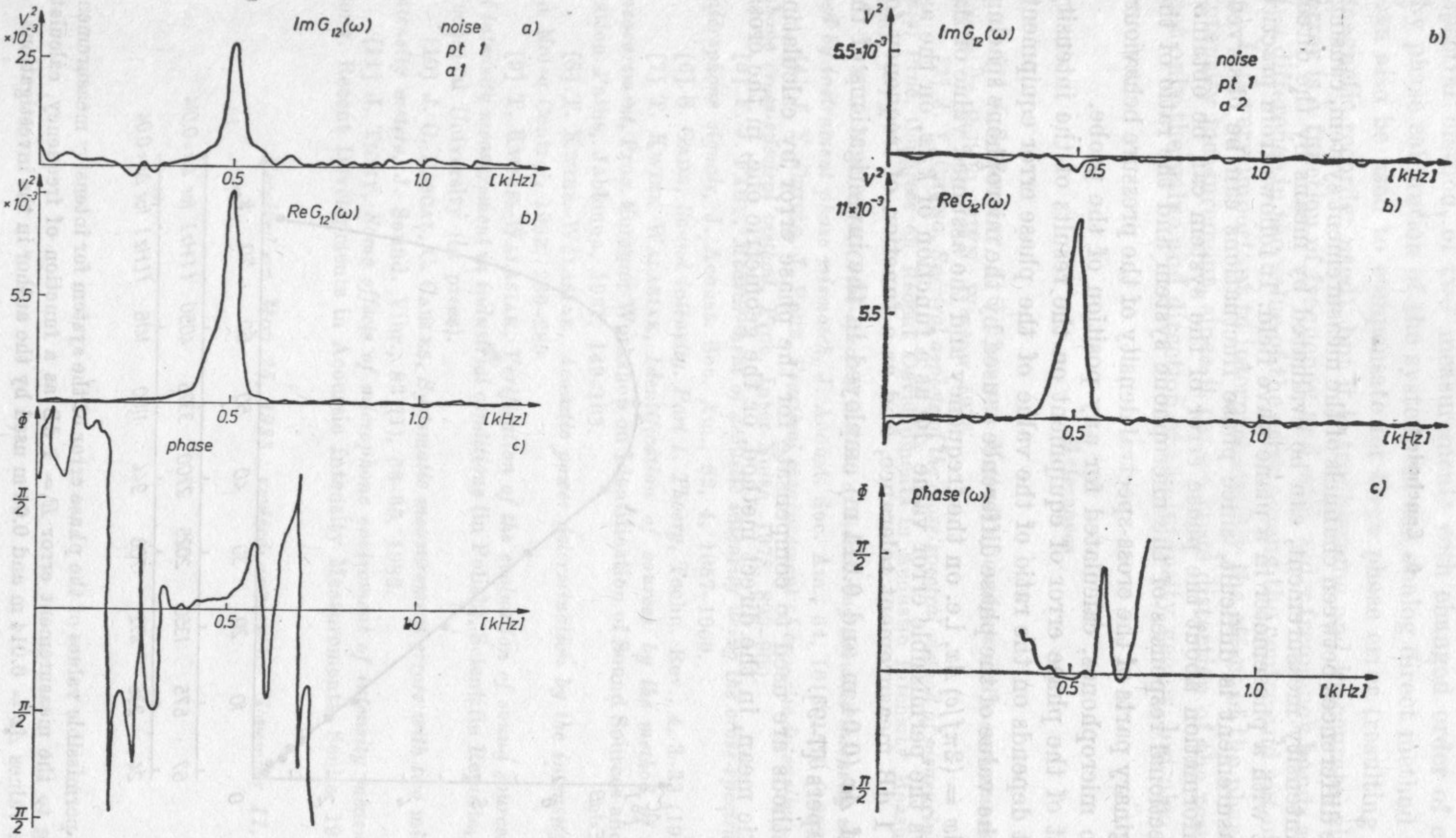

Fig. 11. The results as in Fig. 10, obtained for one of the measurement points, characterized by a change in the sign of the intensity vector after the reversal of the probe (b), as a result of the existence of large-compared with $k\Delta x$ – phase error of the equipment system

6. Conclusion

The phase differences between channels of the measurement system, causing error in the intensity measurement, can be evaluated by means by the direct measurement with a phasemeter in a plane wave field. It follows from practice that the measurement is difficult, since phase fluctuations can be observed. Additional information about the phase error of the system can be obtained from the directional responses of the microphone system and the ratio of the real and imaginary parts of the cross-spectral density of the pressure behaviours from the two microphones, calculated for any position of the probe.

The effect of the phase error of equipment on the results of the intensity measurement depends on the ratio of the value of the phase error equipment, $\Delta\varphi(\omega)$, and the value of the phase difference caused by the microphone spacing, $\Phi_{\Delta x}(\omega) = k\Delta x = (2\pi f/c)\Delta x$, i.e. on the frequency and the assumed value of Δx. Fig. 11 shows the permissible error value $\Delta\Phi$ as a function of $k\Delta x$, on the assumption of 1 dB measurement tolerance, and as a function of frequency for the values of Δx (0.04 m and 0.014 m) employed in the investigations of the author of papers [7–9].

Some methods are used to compensate for the phase error by calculating the arithmetic mean, in the direct method, or the geometric one, in the cross-

Fig. 12. The permissible values of the phase error of the system for intensity measurements, corr esponding to the measurement error $E = 1$ dB as a function of frequency, calculated for the values $\Delta x = 0.014$ m and 0.04 m used by the author in the investigations

spectral method, of two measurements with changed order of circuits [4], or by phase calibration of the system. In the analog direct method phase shifters can also be used to compensate for large phase error (resulting e.g. from the magnetic recorder used), but it is troublesome and inaccurate, since the phase error is a function of frequency and with wider frequency bands analysed there is a difference in the value of the error among the centre and cut-off frequencies.

It is very important to interpret strictly the effect of the phase error on the intensity measurement, since it causes large distortion of the results, changing not only the value and direction of the intensity vector, but also its sign.

References

[1] J. S. Bendat, A. G. Piersol, *Methods of analysis and measurement of random functions* (in Polish), PWN, Warsaw 1976.

[2] J. Y. Chung, *Spectral aspects of the cross-spectral method of measuring acoustic intensity*, Proc. Congress on Recent Developments in Acoustic Intensity Measurement, Senlis, 1981, 1–11.

[3] J. Y. Chung, *Cross-spectral method of measuring acoustic intensity without error caused by instrument phase mismatch*, J. Acoust. Soc. Am., 64, 1613–1616 (1978).

[4] J. Y. Chung, J. Pope, *Practical measurement of acoustic intensity – the two microphone cross-spectral method*, Proc. Inter-Noise, 1978, 893–900.

[5] F. J. Fahy, *Measurement of acoustic intensity using the cross-spectral density of two microphone signals*, J. Acoust. Soc. Am., 62, 4, 1057–1059.

[6] S. Gade, *Sound intensity. Part I. Thoery*, Techn. Rev., 4, 3–33 (1982).

[7] T. Kwiek-Walasiak, *Identification of sources by the method of sound intensity measurement*, Proc. Summer Workshop on Identification of Sound Sources and Sound Propagation Paths, Jabłonna, 1981, 149–163.

[8] T. Kwiek-Walasiak, *Acoustic power determination by the intensity method*, Proc. on Noise Control, 1982, 286–289.

[9] T. Kwiek-Walasiak, *Verification of the evaluation of sound sources by the method of intensity measurement in industrial conditions* (in Polish), Scientific Reports, Świętokrzyska Technical University (in press).

[10] J. C. Pascal, C. Carles, *Systematic measurement errors with two microphone sound intensity meters*, J. Sound. Vibr., 83 (1), 56–65, 1982.

[11] J. Tichy, *Some effects of microphone enviroment of intensity measurements*, Proc. Cong. Recent Developments in Acoustic Intensity Measurements, Senlis, 1981, 25–31.

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