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RECTANGULAR SOURCES AND SYSTEMS OF SOUND SOURCES WITH LARGE DIRECTIONALITY

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This paper presents a method for constructing plane, rectangular sound sources with large directionality of vibration energy radiation into the far field. The directional properties of sound sources of this type were analysed for uniform, Hanning and Blackmann distributions of the vibration velocity amplitude on their surface. It was found that a rectangular sound source with large directionality of vibration energy radiation into the far field can be realised in practice in the form of a plane, rectangular mosaic system of sound sources with a discrete Hanning distribution of their relative bulk efficiencies.

1. Introduction

In a large number of ultrasonic applications, e.g. in metrology, diagnostics or ultrasonic technology, there is the need for using sound sources with large directionality of vibration energy radiation into the far field, in order to obtain the appropriate shape of and ultrasonic wave beam or the required energy concentration in some region of a medium.

The construction and study of the properties of sound sources with large directionality of vibration energy radiation have to date been the subjects of a large number of theoretical and experimental investigations (e.g. [2], [3], [10], [12], [13]). The authors of these papers paid most attention to the selection of an appropriate vibration velocity amplitude distribution on the surface of a planar or spherical sound source with given shape, most often a circular one. In the 1970's much interest was enjoyed by the properties of a sound source with a Gaussian vibration velocity amplitude distribution on its surface [2], [3], [10] and [13]. Recently, intensive research has been carried out on the properties of systems of sound sources (e.g. [5], [7] and [8]).

This paper presents a method for constructing rectangular mosaic system of sound sources with large directionality of vibration energy radiation into the far field.

2. Radiation directionality of a planar sound source

Let us assume that in the plane $z = 0$ (Fig. 1), which is an ideal rigid baffle S_0, there is a planar sound source σ_0 which vibrates harmonically at the frequency f_0. Let it radiate vibration energy into the half-space $z > 0$, filled with a lossless and homogeneous liquid medium with density ϱ, in which the sound wave propagates at the velocity c. Let us assume that, as a result of the vibration

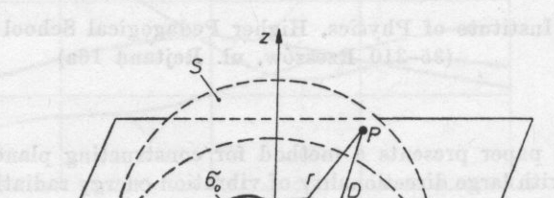

Fig. 1. The sound source σ_0 placed in the baffle S_0 and radiating vibration energy into the medium filling the half-space over the baffle

of the sound source σ_0, the distribution of the normal component of the vibration velocity in the plane of the baffle S_0 is defined as

$$v(S_0, t) = \varkappa(S_0)\exp(-j2\pi f_0 t), \tag{1}$$

where

$$\varkappa(S_0) \neq 0 \quad \text{for the surface of the source } \sigma_0, \tag{2}$$
$$\varkappa(S_0) \neq 0 \text{ for the other part of the baffle } S_0;$$

is a function determining the vibration velocity amplitude distribution on the plane S_0.

In the half-space $z > 0$, let us consider the surface of the hemisphere S with the radius

$$r \geqslant r_g = \pi f_0 r_{0m}^2/c, \tag{3}$$

surrounding the sound source σ_0 in its far field [4], [9], where r_{0m} is the longest of the distances between points of the contour of the sound source σ_0 and the origin of the coordinate system. The acoustic pressure distribution generated by the sound source σ_0 on the surface of the hemisphere S can be expressed by the formula [4], [9]

$$p(S, t) = P(S)\exp(-j2\pi f_0 t), \tag{4}$$

where

$$P(S) = -j\frac{f_0\varrho}{r}\exp(j2\pi r f_0/c)\int\limits_{S_0}\varkappa(S_0)\exp[-j2\pi f_0 r_0\cos(r_0, r)/c]dS_0 \tag{5}$$

is a function determining the acoustic pressure amplitude distribution on the surface of the hemisphere S. Since the function [4], [9]

$$P_0(S) = -j\frac{f_0\varrho}{r}\exp(j2\pi r f_0/c) \tag{6}$$

defines the acoustic pressure amplitude distribution generated on the surface of the hemisphere S by a point sound source with unit efficiency, placed in the baffle S_0 instead of the planar sound source ϱ_0, expression (5) can then be represented as

$$P(S) = P_0(S)R(S), \tag{7}$$

where

$$R(S) = \int\limits_{S_0}\varkappa(S_0)\exp[-j2\pi f_0 r_0\cos(r_0, r)/c]dS_0 \tag{8}$$

is a function defining the relative acoustic pressure amplitude distribution on the surface of the hemisphere S. It can be noted that the function $R(S)$ does not depend on the radius r. In view of this, this function determines the relative acoustic pressure amplitude distribution on the surface of the hemisphere S with any radius r which satisfies condition (3). Since at a given vibration frequency f_0 of the sound source δ_0 the function $R(S)$ depends only on the angle between the radius r and the axis $0z$, the function $R(S)$ can then be used in evaluating the sound source σ_0 in terms of directionality of acoustic pressure wave radiation in the far field. In comparing sound sources with respect to one another, it is more convenient to use the normalised function $\bar{R}(S)$, i.e. the function $\bar{R}(S)$ satisfying the condition

$$\bar{R}(S) = 1 \quad \text{for the angle } (r, r_0) = \pi/2. \tag{9}$$

It follows from dependence (8) that for the angle $(r, r_0) = \pi/2$ the function $R(S)$ takes the value

$$V_0 = \int\limits_{S_0}\varkappa(S_0)dS_0, \tag{10}$$

equal to the bulk efficiency of the sound source σ_0 placed in the baffle S_0. In view of this and from (8),

$$\bar{R}(S) = R(S)V_0, \tag{11}$$

The function $\bar{R}(S)$ is called the directional characteristic of the sound source σ_0 [4].

3. The spatial spectrum of the vibration velocity amplitude distribution function

Let us represent expression (8) in a rectangular coordinate system. Let $\nu = f_0/c$ denote the spatial frequency of a sound wave radiated by the sound source σ_0 into the medium filling the half-space $z > 0$. Since [4], [9]

$$\nu r_0 \cos(r_0, r) = \nu r_0 [\cos(r_0, x)\cos(x, r) + \cos(r_0, y)\cos(y, r)], \tag{12}$$

then, when

$$x = r_0 \cos(r_0, x), \tag{13}$$

$$y = r_0 \cos(r_0, y) \tag{14}$$

are the coordinates of points of the baffle S_0, and

$$\nu_x = \nu \cos(x, r), \tag{15}$$

$$\nu_y = \nu \cos(y, r) \tag{16}$$

are the components of the spatial frequencies of partial planar waves whose spatial superposition represents the wave radiated by the sound source σ_0 into the medium filling the half-space $z > 0$ [1], [15], the function $R(S)$ can be given in the form

$$R(\nu_x, \nu_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varkappa(x, y) \exp[-j2\pi(x\nu_x + y\nu_y)]\,dx\,dy. \tag{17}$$

It follows from dependencies (15) and (16) that in the half-space $z \geqslant 0$ the spatial frequencies ν_x and ν_y can take values from an interval defined by the inequality

$$\sqrt{\nu_x^2, \nu_y^2} \leqslant \nu^2. \tag{18}$$

It can be noted that expression (17) has a form analogous to a simple, two-dimensional Fourier transform [1], [15]. Let us consider the spatial spectrum of the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0, defined by the expression

$$K(\nu_x, \nu_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varkappa(x, y) \exp[-j2\pi(x\nu_x + y\nu_y)]\,dx\,dy, \tag{19}$$

where $-\infty < \nu_x, \nu_y < +\infty$. It follows from dependencies (17) and (19) that the distribution $R(\nu_x, \nu_y)$ of the relative acoustic pressure amplitude on the surface of the hemisphere S can be determined from the spatial spectrum $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0. Namely,

$$R(\nu_x, \nu_y) = K(\nu_x, \nu_y) H_\nu(\nu_x, \nu_y), \tag{20}$$

where

$$H_\nu(\nu_x, \nu_y) = 1 \quad \text{for } \sqrt{\nu_x^2, \nu_y^2} \leqslant \nu^2, \tag{21}$$

$$H_\nu(\nu_x, \nu_y) = 0 \quad \text{for the other } \nu_x, \nu_y,$$

with $-\infty < \nu_x, \nu_y < +\infty$. It follows from dependencies (20) and (21) that the baffle S_0 plays the role of a low-pass spatial filter [1] with the spatial transmittance $H_\nu(\nu_x, \nu_y)$, causing restriction of the spatial spectrum $K(\nu_x, \nu_y)$ of the distribution $\varkappa(x, y)$ to the region of spatial frequencies defined by expression (18).

It can be noted in turn ((10) and (19)) that the bulk efficiency of the sound source σ_0 is given by the expression

$$V_0 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varkappa(x, y)\, dx\, dy = K(0, 0), \tag{22}$$

where $K(0, 0)$ is a component of the spatial spectrum $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ for the spatial frequencies $\nu_x = \nu_x = 0$. In view of this and from (11) and (20), the directional characteristic of the sound source σ_0 can be determined in the following way:

$$\bar{R}(\nu_x, \nu_y) = K(\nu_x, \nu_y) H_\nu(\nu_x, \nu_y) / K(0, 0). \tag{23}$$

It follows hence that the directional characteristic of the sound source σ_0, for a given spatial frequency ν, can be determined from the spatial spectrum $K(\nu_x, \nu_y)$ of the function $\varkappa(x, y)$ defining the vibration velocity amplitude distribution in the plane of the baffle S_0, which contains the sound source σ_0.

4. The effect on the directional characteristic of the sound source of its shape and that of the vibration velocity amplitude distribution on its surface

The function $\varkappa(x, y)$, defining the vibration velocity amplitude distribution in the plane of the baffle S_0, can be represented as the product of the function $f(x, y)$, which was used to define the shape of the vibration velocity amplitude distribution on the surface of the sound source σ_0, and the distribution $z(x, y)$, defining the shape of this source. Namely,

$$\varkappa(x, y) = f(x, y) z(x, y), \tag{24}$$

where

$$z(x, y) = 1 \quad \text{for the surface of the source } \sigma_0, \tag{25}$$

$$z(x, y) = 0 \quad \text{for the other part of the baffle } S_0.$$

Let us determine the spatial spectrum of the distribution function (24). The use of the theorem on the Fourier transform of the product of the function and the distribution [1] gives

$$K(\nu_x, \nu_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\mu_x, \mu_y) Z(\nu_x - \mu_x, \nu_y - \mu_y)\, d\mu_x\, d\mu_y$$

$$= F(\nu_x, \nu_y) * Z(\nu_x, \nu_y). \tag{26}$$

Hence, it follows that the spatial spectrum $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0, containing the planar sound source σ_0, is a convolution of the spatial spectrum $F(\nu_x, \nu_y)$ of the function $f(x, y)$, which was used to define the shape of the vibration velocity amplitude distribution on the surface of the source σ_0, with the spatial spectrum $Z(\nu_x, \nu_y)$ of the distribution $z(x, y)$, defining the shape of this source. In view of this and from (11) and (23), the directional characteristic of the sound source σ_0 can be determined by the expression

$$\bar{R}(\nu_x, \nu_y) = [F(\nu_x, \nu_y) * Z(\nu_x, \nu_y)] H_\nu(\nu_x, \nu_y) V_0. \tag{27}$$

The authors of previous papers [2], [3], [13] and [14], on the study of the properties of sound sources with large directionality of vibration energy radiation into the far field, did not go beyond the analysis of the effect of chosen functions of the shape of the distribution $f(x, y)$ on the directional characteristic of the sound source, neglecting the effect on this characteristic, of the distribution $z(x, y)$ defining the shape of the source. Further considerations here will propose a method for constructing sound sources with large directionality of vibration energy radiation, consisting in selection of an appropriate function defining the shape of the vibration velocity amplitude distribution on the surface of a source with prescribed form.

5. Sound sources with large directionality of vibration energy radiation

The absolutely directional sound source will be understood here to be a source radiating vibration energy only towards the axis $0z$. Accordingly, the spatial spectrum of the vibration velocity amplitude distribution function in the plane of the baffle S_0, containing such a source, can be determined in the following way:

$$K(\nu_x, \nu_y) = \varkappa_0 \delta(\nu_x, \nu_y), \tag{28}$$

where $\delta(\nu_x, \nu_y)$ is a Dirac distribution.

Let us determine the vibration velocity amplitude distribution in the plane of the baffle S_0, containing the absolutely directional sound source. The use of the theorem of the inverse Fourier transform of the Dirac distribution [1] gives

$$\varkappa(x, y) = \varkappa_0, \tag{29}$$

Hence, it follows that the absolute directional sound source is a source with infinitely large size and uniform vibration velocity amplitude distribution on its surface. Using this idealized model, unrealizable practically, of the sound source, it is possible to determine the method for constructing sound source with large directionality of vibration energy radiation. Since the distribution $\delta(\nu_x, \nu_y)$ can be defined as the limit of the function series $K(\nu_x, \nu_y; A, B)$ satisfying the conditions [11]:

$$\lim_{\substack{A\to 0\\ B\to 0}} K(\nu_x,\nu_y; A, B) = 0 \quad \text{for } \nu_x, \nu_y \neq 0, \tag{30}$$

and

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K(\nu_x, \nu_y; A, B)\, d\nu_x d\nu_y = 1 \quad \text{for all } A, B > 0, \tag{31}$$

the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0, containing a sound source with large directionality of vibration energy radiation into the far field, should be chosen in such a way that the function series $K(\nu_x, \nu_y; A, B)$, derived from the Fourier transform $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ would satisfy the conditions given above. Accordingly, it can be shown that the sound source with a Gaussian vibration velocity amplitude distribution analysed in papers [2], [3], [10] and [13] is a sound source with large directionality of vibration energy radiation in the sense defined above, since the function series $K(\nu_x, \nu_y; A, B)$, derived from the spatial spectrum $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0, containing this source, which has the following form in a rectangular coordinate system [13]:

$$K(\nu_x, \nu_y; A, B) = K(\nu_x; A) K(\nu_y; B), \tag{32}$$

where

$$K(\nu_x; A) = \frac{1}{A}\exp(-\pi\nu_x^2/A^2), \tag{33}$$

whereas

$$K(\nu_y;\ B) = \frac{1}{B}\exp(-\pi\nu_y^2/B^2), \tag{34}$$

satisfies [10] conditions (30) and (31). Since a sound source with a Gaussian distribution, as the absolutely directional sound source, is a source with infinitely large size, therefore, strictly speaking, it is realizable in practice realizable only in approximation [2]. Further considerations will show that it is not possible to construct practically realizable sound sources with large directionafity of vibration energy radiation into the far field.

6. Rectangular sound sources with large directionality of vibration energy radiation

Let us now construct a rectangular sound source with the sides a and b showing large directionality of vibration energy radiation into the far field. The following series will be used for that purpose [11]:

$$K(\nu_x,\ \nu_y;\ A,\ B) = K(\nu_x;\ A)K(\nu_y;\ B), \tag{35}$$

where

$$K(\nu_x;\ A) = \frac{1}{A}\operatorname{sinc}(\nu_x/A), \tag{36}$$

whereas

$$K(\nu_y;\ B) = \frac{1}{B}\operatorname{sinc}(\nu_y/B), \tag{37}$$

satisfying conditions (30) and (31), with $A = 1/a$, $B = 1/b$, while

$$\operatorname{sinc}(z) = \frac{\sin(\pi z)}{\pi z}. \tag{38}$$

Let us consider uniform, Hanning and Blackmann distributions. The Fourier transforms of these distributions are functions of the form of (35) [6].

a) Uniform distribution. Let us assume that the vibration velocity amplitude distribution in the plane of the baffle S_0 is defined in the following way (Fig. 2):

$$\varkappa(x,\ y) = \varkappa_0\varkappa(x)\varkappa(y), \tag{39}$$

where

$$\varkappa(x) = 1 \quad \text{for } |x| \leqslant a/2, \qquad \varkappa(x) = 0 \quad \text{for } |x| > a/2, \tag{40}$$

whereas

$$\varkappa(y) = 1 \quad \text{for } |y| \leqslant b/2, \tag{41}$$

$$\varkappa(y) = 0 \quad \text{for } |y| > b/2,$$

Let us determine the spatial spectrum of the distribution function (39). The use of the theorem on the ourier transform of the product of distribution with separated variables and the theorem on the transform of the distribution *sgn*(z) [11] gives

$$K(\nu_x, \nu_y) = \varkappa_0 K(\nu_x) K(\nu_y), \tag{42}$$

where

$$K(\nu_x) = a \operatorname{sinc}(a\nu_x), \tag{43}$$

whereas

$$K(\nu_y) = b \operatorname{sinc}(b\nu_y). \tag{44}$$

The transform $K(\nu_x)$ (Fig. 2) of the distribution function (40) is a function having the main maximum for $\nu_x = 0$ and side extremes decreasing as $|\nu_x|$ increases, at a rate of 20 dB/decade. The highest of the side extremes are larger by 13 dB than the main one.

b) Hanning distribution. Let us assume that the vibration velocity amplitude distribution in the plane of the baffle S_0 is defined in the following way (Fig. 2):

$$\varkappa(x, y) = \varkappa_0 \varkappa(x) \varkappa(y), \tag{45}$$

where

$$\varkappa(x) = 0.5 + 0.5\cos(2\pi x/a) \quad \text{for } |x| \leqslant a/2, \tag{46}$$

$$\varkappa(x) = 0 \quad \text{for } |x| > a/2,$$

whereas

$$\varkappa(y) = 0.5 + 0.5\cos(2\pi y/b) \quad \text{for } |y| \leqslant b/2,$$

$$\varkappa(y) = 0 \quad \text{for } |y| > b/2. \tag{47}$$

Let us determine the spatial spectrum of the distribution function (45). This gives

$$K(\nu_x, \nu_y) = \varkappa_0 K(\nu_x) K(\nu_y), \tag{48}$$

where

$$K(\nu_x) = a[0.5\operatorname{sinc}(a\nu_x) + 0.25\sin(a\nu_x - 1) + 0.25\operatorname{sinc}(a\nu_x + 1)], \tag{49}$$

whereas

$$K(\nu_y) = b[0.5\operatorname{sinc}(b\nu_y) + 0.25\operatorname{sinc}(b\nu_y - 1) + 0.25\operatorname{sinc}(b\nu_y + 1)]. \tag{50}$$

The transform $K(\nu_x)$ (Fig. 2) of the distribution function (46) is a function having the main maximum for $\nu_x = 0$ and side extremes decreasing as $|\nu_x|$ increases, at a rate of 60 dB/decade. The highest of the side extremes are larger by 32 dB than the main maximum. Compared with the uniform distribution, the transform $K(\nu_x)$ of the distribution function $\varkappa(x)$ for the Hanning distribution has a wider main maximum, but lower and more slowly decreasing side extremes.

c) Blackmann distribution. Let us assume that the vibration velocity amplitude distribution in the plane of the baffle S_0 is defined in the following way (Fig. 2):

$$\varkappa(x, y) = \varkappa_0 \varkappa(x) \varkappa(y), \tag{51}$$

where

$$\varkappa(x) = 0.42 + 0.5\cos(2\pi x/a) + 0.08\cos(4\pi x/a) \quad \text{for } |x| \leqslant a/2,$$
$$\varkappa(x) = 0 \quad \text{for } |x| > a/2, \tag{52}$$

whereas

$$\varkappa(y) = 0.42 + 0.5\cos(2\pi y/b) + 0.08\cos(4\pi y/b) \quad \text{for } |y| \leqslant b/2,$$
$$\varkappa(y) = 0 \quad \text{for } |y| > b/2. \tag{53}$$

Let us determine the spatial spectrum of the distribution function (51) This gives

$$K(\nu_x, \nu_y) = \varkappa_0 K(\nu_x) K(\nu_y), \tag{54}$$

where

$$K(\nu_x) = a[0.42\,\text{sinc}(a\nu_x) + 0.25\,\text{sinc}(a\nu_x - 1) + 0.25\,\text{sinc}(a\nu_x + 1) + 0.04\,\text{sinc}(a\nu_x - 2) + 0.04\,\text{sinc}(a\nu_x + 2)], \tag{55}$$

whereas

$$K(\nu_y) = b[0.42\,\text{sinc}(b\nu_y) + 0.25\,\text{sinc}(b\nu_y - 1) + 0.25\,\text{sinc}(b\nu_y + 1) + 0.04\,\text{sinc}(b\nu_y - 2) + 0.04\,\text{sinc}(b\nu_y + 2)]. \tag{56}$$

The transform $K(\nu_x)$ (Fig. 2) of the distribution function (52) is a function having the main maximum for $\nu_x = 0$ and side extremes decreasing as $|\nu_x|$ increases, at a rate of 34 dB/decade. The highest of the side extremes are larger by 57 dB than the main maximum. Compared with the Hanning distribution, the transform $K(\nu_x)$ of the distribution function $\varkappa(x)$ for the Blackmann distribution has a wider main maximum, but lower and more slowly decreasing side extremes.

It follows from these considerations that each of the distributions: uniform, Hanning's and Blackmann's, can be used to construct a rectangular sound source with large directionality of vibration energy radiation into the far field. To achieve this, for a given frequency f_0 of vibration of the surface of the sound source, appropriately large size of the source, compared with the length of the sound wave in the medium where this source will radiate vibration energy,

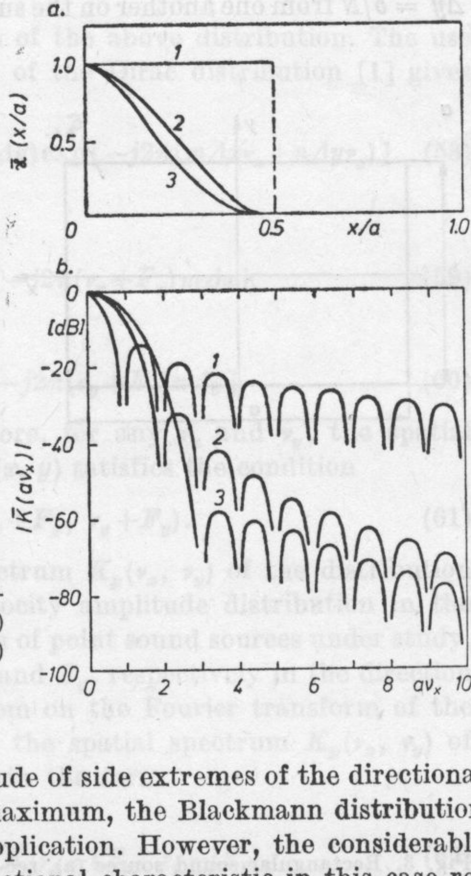

Fig. 2. Vibration velocity amplitude distribution function (a) and their Fourier transforms (b) for uniform (1), Hanning (2) and Blackmann (3) distributions

should be selected. In view of the magnitude of side extremes of the directional characteristic, compared with its main maximum, the Blackmann distribution seems to be most useful for practical application. However, the considerable width of the main maximum of the directional characteristic in this case requires the use of sound sources with appropriately large size, compared with the wavelength, much larger than are necessary in the case of the Hanning distribution. In view of this, a rectangular sound source with the Hanning distribution of the vibration velocity amplitude on its surface seems to be more useful for application as a source with large directionality of vibration energy radiation into the far field than one with uniform or Blackmann distributions.

7. Rectangular systems of sound sources with large directionality of vibration energy radiation

Let us consider a rectangular sound source σ_0 with the sides a and b (Fig. 3) showing large directionality of vibration energy radiation in the far field. Let us assume that the vibration velocity amplitude distribution in the plane of the baffle S_0, containing this source, is defined by the function $\varkappa(x, y)$.

Let us replace the sound source σ_0 by a system consisting of a number of point sound sources σ_{mn}, spaced respectively at intervals $\Delta x = a/M$ and $\Delta y = b/N$ from one another on the surface of the rectangle occupied by the sound

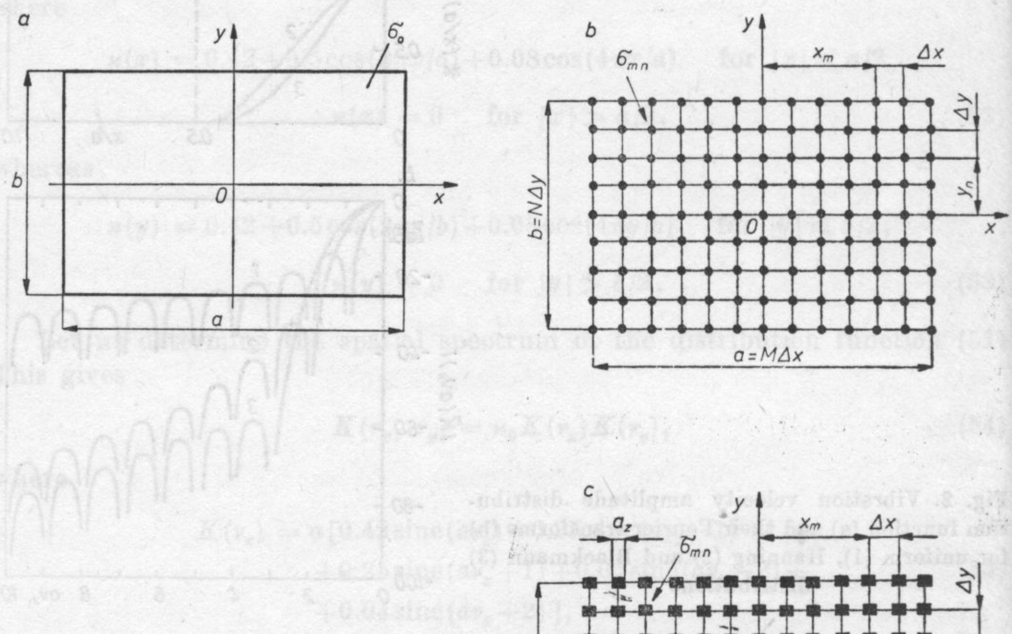

Fig. 3. Rectangular sound source (a), rectangular system of point sound sources (b), rectangular system of planar sound sources

source σ_0 (Fig. 3). Let us assume that the efficiencies of individual sources in the system are equal to the values $\varkappa(m\Delta x, n\Delta y)$ of the distribution function $\varkappa(x, y)$. Accordingly, the vibration velocity amplitude distribution in the plane of the baffle S_0, containing the system of point sound sources under study, is defined by the expression (Fig. 4)

$$\varkappa_p(x, y) = \varkappa(x, y)\frac{1}{\Delta x\,\Delta y}\,\mathrm{III}(x/\Delta x, y/\Delta y)$$

$$= \varkappa(x, y)\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\delta(x-m\Delta x, y-n\Delta y) \tag{57}$$

$$= \sum_{m=-M/2}^{M/2}\sum_{n=-N/2}^{N/2}\varkappa(m\Delta x, n\Delta y)\,\delta(x-m\Delta x, y-n\Delta y).$$

Let us determine the spatial spectrum of the above distribution. The use of the theorem on the Fourier transform of the Dirac distribution [1] gives

$$K_p(\nu_x, \nu_y) = \sum_{m=-M/2}^{M/2}\sum_{n=-N/2}^{N/2}\varkappa(m\Delta x, n\Delta y)\exp[-j2\pi(m\Delta x\nu_x+n\Delta y\nu_y)]. \tag{58}$$

Since

$$\exp(-j2\pi\nu_x m\Delta x) = \exp[-j2\pi(\nu_x+F_x)m\Delta x] \tag{59}$$

and

$$\exp(-j2\pi\nu_y n\Delta y) = \exp[-j2\pi(\nu_y+F_y)n\Delta y], \tag{60}$$

where $F_x = 1/\Delta x$ and $F_y = 1/\Delta y$, therefore, for any ν_x and ν_y, the spatial spectrum $K_p(\nu_x, \nu_y)$ of the distribution $\varkappa_p(x, y)$ satisfies the condition

$$K_p(\nu_x, \nu_y) = K_p(\nu_x+F_x, \nu_y+F_y). \tag{61}$$

Hence, it follows that the spatial spectrum $K_p(\nu_x, \nu_y)$ of the distribution $\varkappa_p(x, y)$, which defines the vibration velocity amplitude distribution in the plane of the baffle S_0 containing the system of point sound sources under study, is a periodic function with the periods F_x and F_y, respectively in the direction of the axes $0\nu_x$ and $0\nu_y$. From the theorem on the Fourier transform of the distribution $1/\Delta x\Delta y\ \mathrm{III}(x/\Delta x, y/\Delta y)$ [1], the spatial spectrum $K_p(\nu_x, \nu_y)$ of distribution (57) can also be represented in the form

$$K_p(\nu_x, \nu_y) = F_xF_y\sum_{k=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}K(\nu_x-kF_x, \nu_y-lF_y). \tag{62}$$

Hence, it follows that the spectrum $K_p(\nu_x, \nu_y)$ of the distribution $\varkappa_p(x, y)$, defining the vibration velocity amplitude distribution in the plane of the baffle S_0 containing the system of point sound sources under study, is the sum of the spectra $K(\nu_x, \nu_y)$ of the distribution function $\varkappa(x, y)$ of the vibration velocity amplitude in the plane of the baffle S_0, containing the rectangular sound source σ_0 (Fig. 4), multiplied by F_xF_y and displaced by F_x and F_y with respect to each other, respectively in the direction of the axes $0\nu_x$ and $0\nu_y$. In view of this and from (20), the relative acoustic pressure amplitude distribution on the surface of the hemisphere S, surrounding the system of point sources under study in its far field, is defined by the expression

$$R_p(\nu_x, \nu_y) = K_p(\nu_x, \nu_y) H_\nu(\nu_x, \nu_y). \tag{63}$$

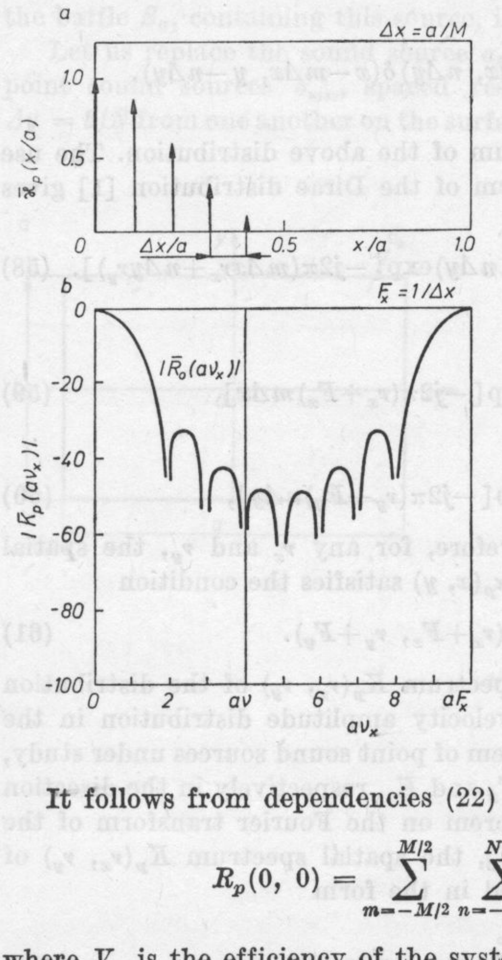

Fig. 4. Vibration velocity amplitude distribution (a) and its Fourier transform (b) for a discrete Hanning distribution

It follows from dependencies (22) and (58) that

$$R_p(0, 0) = \sum_{m=-M/2}^{M/2} \sum_{n=-N/2}^{N/2} \varkappa(m\Delta x, n\Delta y) = V_p, \tag{64}$$

where V_p is the efficiency of the system of point sound sources under study. In view of this and from (23), the directional characteristic of this system of sound sources is given by the expression

$$\bar{R}_p(\nu_x, \nu_y) = R_p(\nu_x, \nu_y)/V_p. \tag{65}$$

It follows from dependencies (62), (63) and (65) that when $F_x, F_y \gg 2\nu$, i.e. when $\Delta x, \Delta y \ll \lambda/2$, where $\lambda = 1/\nu$, then

$$\bar{R}_p(\nu_x, \nu_y) \simeq \bar{R}(\nu_x, \nu_y). \tag{66}$$

Hence, it follows that when the intervals among the individual sources of a system of point sound sources are sufficiently small compared with half the length of the sound wave radiated, the directional characteristic of this system has a shape close to that of the directional characteristic of the rectangular sound source σ_0 (Fig. 4).

Let us now replace the rectangular sound source σ_0 by a practically realizable system of planar sound sources σ_{mn}, spaced at the respective intervals $\Delta x = a/M$ and $\Delta y = b/N$, on the surface of the rectangle occupied by the sound source σ_0 (Fig. 3). Let us assume that the vibration velocity amplitude distribution in the plane of the baffle S_0, containing the central source σ_{00} of the system, set apart from the other sources, is defined by the function $\varkappa_{00}\varkappa_z(x, y)$. Let us assume that the coefficients $\varkappa_{mn}$ for the individual sources σ_{mn} in the system are equal to the values $\varkappa(m\Delta x, n\Delta y)$ of the distribution function $\varkappa(x, y)$. Accordingly, the vibration velocity amplitude distribution in the plane of the baffle S_0, containing the system of planar sound sources now under study, can be defined in the following way:

$$\varkappa_u(x, y) = \varkappa_z(x, y) * \varkappa_p(x, y), \tag{67}$$

with

$$\varkappa_p(x, y) = \sum_{m=-M/2}^{M/2} \sum_{n=-N/2}^{N/2} \varkappa(m\Delta x, n\Delta y)\,\delta(x - m\Delta x, y - n\Delta y). \tag{68}$$

Let us now determine the spatial spectrum of distribution (67). The use of the theorem on the Fourier transform of the convolution of the function and the distribution [1] gives

$$K_u(\nu_x, \nu_y) = K_z(\nu_x, \nu_y) K_p(\nu_x, \nu_y), \tag{69}$$

where

$$K_p(\nu_x, \nu_y) = \sum_{m=-M/2}^{M/2} \sum_{n=-N/2}^{N/2} \varkappa(m\Delta x, n\Delta y) \exp[-j2\pi(m\Delta x\nu_x + n\Delta y\nu_y)]. \tag{70}$$

In view of this and from (20), the relative acoustic pressure distribution on the surface of the hemisphere S, surrounding the system of planar sound sources under study in its far field, is defined by the expression

$$R_u(\nu_x, \nu_y) = [K_z(\nu_x, \nu_y) K_p(\nu_x, \nu_y)] H_\nu(\nu_x, \nu_y) = R_z(\nu_x, \nu_y) R_p(\nu_x, \nu_y). \tag{71}$$

In view of this and from (22),

$$R_u(0, 0) = R_z(0, 0) R_p(0, 0) = V_z V_p = V_u, \tag{72}$$

where V_u is the bulk efficiency of the system of sources under study. From dependencies (64) and (72), it can be written that

$$V_u = \sum_{m=-M/2}^{M/2} \sum_{n=-N/2}^{N/2} V_{mn}, \tag{73}$$

where

$$V_{mn} = V_z \varkappa(m\Delta x,\ n\Delta y) \tag{74}$$

is the bulk efficiency of the source σ_{mn} of the system. Hence, it follows that the series of the values $\varkappa(m\Delta x,\ n\Delta y)$ of the distribution function $\varkappa(x,\ y)$ is defined by a discrete distribution of the relative bulk efficiencies of the sources σ_{mn} in the system, referred to the efficiency V_z of its central source σ_{00}. From (23), (75) and (72), the directional characteristic of the rectangular system of planar sound sources is defined by the expression

$$\bar{R}_u(\nu_x,\ \nu_y) = R_u(\nu_x,\ \nu_y)/V_u = \bar{R}_z(\nu_x,\ \nu_y)\bar{R}_p(\nu_x,\ \nu_y). \tag{75}$$

It follows from the above dependence that the directional characteristic of the system of planar sound sources is the product of the directional characteristic of the central source σ_{00} of this system and the directional characteristic of a system of point sources derived by replacing each of the sources σ_{mn} by a point source. In a case when the dimensions of each of the sources σ_{mn} in the system are small compared with the length $\lambda = 1/\nu$ of the wave radiated, it can be assumed that

$$\bar{R}_z(\nu_x,\ \nu_y) \simeq 1. \tag{76}$$

In this case, expression (75) can be represented as

$$\bar{R}_u(\nu_x,\ \nu_y) \simeq R_p(\nu_x,\ \nu_y). \tag{77}$$

In view of this, in a case when the dimensions of each of the sources σ_{mn} in the system of planar sound sources under study are sufficiently small compared with the length of the wave radiated, the directional characteristic of this system has a shape close to that of the directional characteristic of a rectangular system of point sources derived by replacing each source δ_{mn} in the system of planar sound sources by a point source.

In summary, it follows from the considerations made that the directionality of vibration energy radiation in the far field, close to that showed by a rectangular sound source σ_0 with the vibration velocity amplitude distribution on its surface defined by the function $\varkappa(x, y)$, will occur for a mosaic system of sound sources, composed of planar sound sources with sufficiently small dimensions compared with the length of the wave radiated, spaced on the surface of the rectangle occupied by the source δ_0, at intervals which are also sufficiently short compared with half the wavelength. In addition, the relative bulk efficiencies of the sources in this system must be equal to the values of the distribution function $\varkappa(x, y)$ at the intervals at which its individual sources are spaced. Fig. 5 shows the directional characteristic of a rectangular mosaic system of sound sources, determined for a discrete Hanning distribution of the relative bulk efficiencies of the sources in this system. It was assumed that $a = b = 4\lambda$,

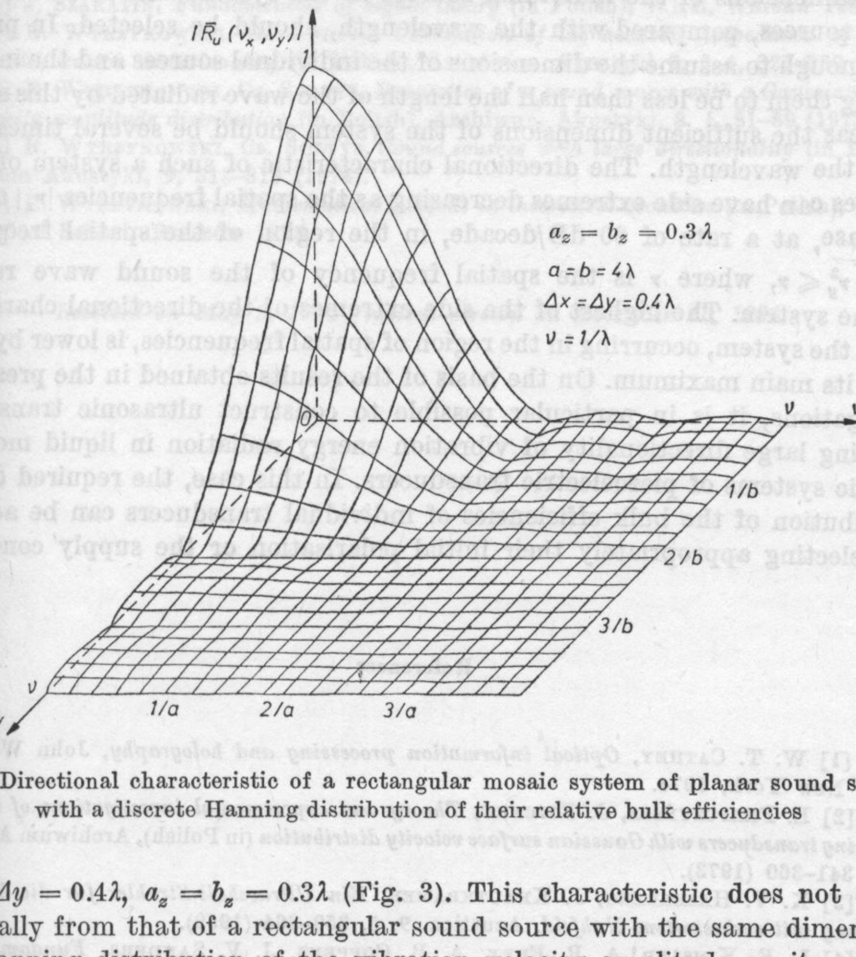

Fig. 5. Directional characteristic of a rectangular mosaic system of planar sound sources with a discrete Hanning distribution of their relative bulk efficiencies

$\Delta x = \Delta y = 0.4\lambda$, $a_z = b_z = 0.3\lambda$ (Fig. 3). This characteristic does not differ practically from that of a rectangular sound source with the same dimensions and Hanning distribution of the vibration velocity amplitude on its surface (Fig. 2).

8. Conclusions

It follows from these considerations that a rectangular sound source with large directionality of vibration energy radiation into the far field can be realized in practice in the form of a rectangular mosaic system of planar sound sources, vibrating in phase, with a discrete Hanning distribution of their bulk efficiencies. To achieve this, for a given frequency of vibration of the surfaces of the sound sources, it is necessary to select sufficiently large dimensions of the system of sources, compared with the length of the sound wave in the medium in which this system will radiate vibration energy. In addition, as small as possible dimensions of the individual sources in the system and intervals among those sources, compared with the wavelength, should be selected. In practice, it is enough to assume the dimensions of the individual sources and the intervals among them to be less than half the length of the wave radiated by this system, whereas the sufficient dimensions of the system should be several times larger than the wavelength. The directional characteristic of such a system of sound sources can have side extremes decreasing as the spatial frequencies $|\nu_x|$ and $|\nu_y|$ increase, at a rate of 60 dB/decade, in the region of the spatial frequencies $\sqrt{\nu_x^2+\nu_y^2} \leqslant \nu$, where ν is the spatial frequency of the sound wave radiated by the system. The highest of the side extremes of the directional characteristic of the system, occurring in the region of spatial frequencies, is lower by 32 dB than its main maximum. On the basis of the results obtained in the present investigations, it is in particular possible to construct ultrasonic transducers, showing large directionality of vibration energy radiation in liquid media, as mosaic systems of piezoelectric transducers. In this case, the required discrete distribution of the bulk efficiencies of individual transducers can be achieved by selecting appropriately their initial polarisation or the supply conditions.

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