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ACOUSTIC PROPERTIES OF TWO PARALLEL ELASTIC PLATES

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Two parallel elastic plates are submerget in gas. A plane sinusoidal wave propagates perpendicularily to the plates. Such a system models a paned window. In order to present the physical phenomenon without excessively developing the mathematical description, the plates are modelled by rigid plane screens suspended elastically. The displacement of the screen is proportional to the force, and the suspension rigidity is proportional to the third power of the screen thickness (as in a bended plate).

On the basis of motion equations and continuity equations, the displacement behind both plates is determined. The attenuation coefficient depends on the frequency, plate thickness and their spacing. Plates with different thicknesses give a better characteristic than those with identical thicknesses. The attenuation coefficient does not depend on the sequence of plates and it has two maximum in the acoustical range. Frequencies corresponding to them differ from the frequencies of free vibrations of the plates and the cavity.

1. Model

We are analysing two simple-supported elastic plates submerged in gas (Fig. 1). These plates have equal moduli of elasticity but different thicknesses. A plane sinusoidal wave incides perpendicularily onto the plates and passes through them undergoing attenuation. The described system is a model of a window closed by two panels. This paper is aimed at the determination of the resulting displacement field with special pressure put to the analysis of the influence of plate thickness, their distance and sequence on this field.

Let us begin with the analysis of a single circular plate. Displacement, w (in the direction of the x-axis, Fig. 1) satisfies equation [1]

$$\frac{\partial^4 w}{\partial r^4} + C_1 \frac{\partial^2 w}{\partial r^2} + q = C_2 \frac{\partial^2 w}{\partial t^2}, \tag{1}$$

where q is the external load and r is the distance from the centre.

Constant coefficients of the equation were denoted by C_1, C_2. The load q is induced by a known incident wave and by the reflected and refracted waves of unknown intensities dependent (on plane $x = 0$) only on r, $A_r(r)$, $A_t(r)$. Due to motion, every plate surface element is a source of a spherical wave which loads all plate surface elements. Therefore, the load q in point r can be expressed as integrals, over the whole plate, of the function $(A_r(r) - A_t(r))$. Substituting them in equation (1) we obtain an integral-differential equation for quantities: $w(r, t)$, $A_r(r)$, $A_t(r)$. Other, unmentioned here, equations are displacement and velocity continuity equations. Such a system of equations can not be solved analytically. Obviously, the situation for a rectangular plate or a system of plates is even more complex. For this reason we are limiting our investigations to an analysis of a simplified model presented in Fig. 2.

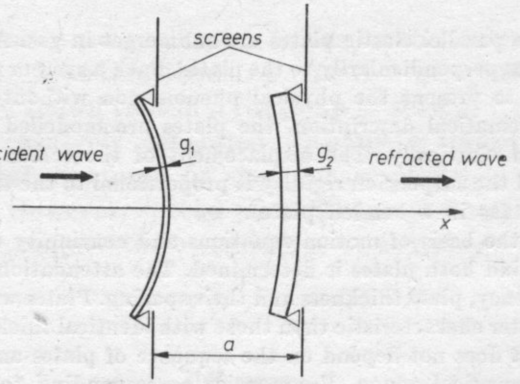

Fig. 1. Analyzed system

Two ridig diaphragms of g_1 and g_2 thicknesses are suspended elastically at a distance, a, and can move in the direction of the x-axis. In order to model a system of plates in the best possible way we assume that suspension rigidities, Q_1 and Q_2, are proportional to the third power of the diaphragm thickness, $Q_1 \approx g_1^3$, $Q_2 \approx g_1^3$ (because the rigidity of a bended plate is proportional to the third power of its thickness). Masses of a unit diaphragm surface, m_1 and m_2, are proportional to their thicknesses: $m_1 \approx g_1$, $m_2 \approx g_2$. It can be expected that the attenuation properties of the system presented in Fig. 2 have a characteristic approximating the characteristic of attenuation properties of the system presented in Fig. 1.

2. Acoustical properties of a single diaphragm

We are analysis a diaphragm of a g-thickness placed at $x = \bar{x}$. A plane sinusoidal wave, moving to the right, incides onto the diaphragm (Fig. 3):

$$A_i e^{i\omega(t - x/c)}$$

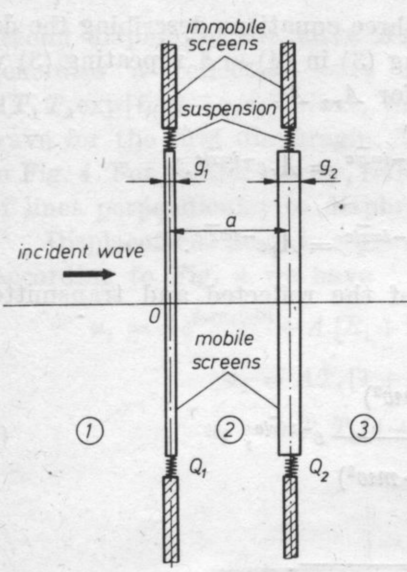

Fig. 2. Substitute model

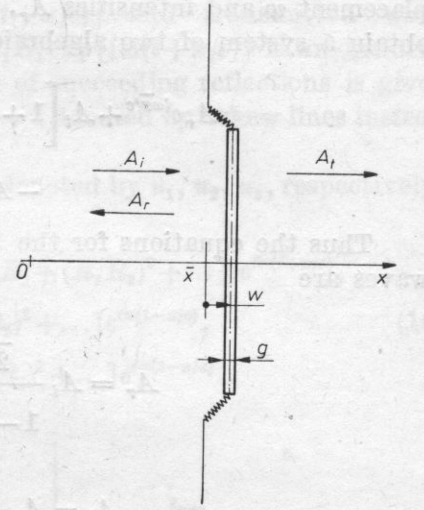

Fig. 3. System with a single diaphragm

and generates a reflected wave (moving to the left)

$$A_r e^{i\omega(t+x/c)},$$

and a transmitted wave (moving to the right) $A_t e^{i\omega(t-x/c)}$ A_i and ω are assumed known. Intensities A_r, A_t should be determined from the motion equations and continuity equations. The calculation methodology is discussed in papers [2] and [3]. Denoting the displacement on the left side of the diaphragm by u, and the right side by v, we obtain

$$u = A_i e^{i\omega(t-x/c)} + A_r e^{i\omega(t+x/c)}, \quad v = A_t e^{i\omega(t-x/c)}. \tag{2}$$

The displacement of the diaphragm is denoted by $w = w(t)$. At $x = \bar{x}$ the displacements u and v, are identical and equal w. Thus we have

$$(A_i e^{-i\omega\bar{x}/c} + A_r e^{i\omega\bar{x}/c}) e^{i\omega t} = A_t e^{-i\omega\bar{x}/c} e^{i\omega t} = w. \tag{3}$$

As the air elasticity modulus is $E = \varrho c^2$, then waves (2) interact with the mobile diaphragm with a force (acting in the direction $+x$).

$$i\omega \varrho c[(A_i - A_t) e^{-i\omega\bar{x}/c} - A_r e^{i\omega\bar{x}/c}] e^{i\omega t}.$$

Also the force $-Qw$ acts on a unit surface and it is the reaction of the elastic suspension. Including finally the force of inertia $m\ddot{w}$ we achieve the motion equation diaphragm for the

$$i\omega \varrho c[(A_i - A_t) e^{-i\omega\bar{x}/c} - A_r e^{i\omega\bar{x}/c}] e^{i\omega t} = m\ddot{w} + Qw. \tag{4}$$

Equations (3) and (4) give a system of three equations describing the displacement w and intensities A_r, A_t. Inserting (3) in (4) and repeating (3) we obtain a system of two algebraic equation for A_r, A_t

$$A_r e^{i\omega\bar{x}/c} + A_t\left[1+\frac{Q-mw^2}{i\omega c\varrho}\right]e^{-i\omega\bar{x}/c} = A_i e^{-i\omega\bar{x}/c},$$

$$-A_r e^{i\omega\bar{x}/c} + A_t e^{-i\omega\bar{x}/c} = A_i e^{-i\omega\bar{x}/c}.$$

Thus the equations for the intensities of the reflected and transmitted waves are

$$A_r = A_i \frac{\dfrac{i}{2\omega\varrho c}(Q-mw^2)}{1-\dfrac{i}{2\omega\varrho c}(Q-mw^2)} e^{-2i\bar{x}/c}, \tag{5}$$

$$A_t = A_i \frac{1}{1-\dfrac{i}{2\omega\varrho c}(Q-mw^2)}.$$

Further analysis does not require the formula for w, therefore we are not giving it here. Let's notice that for a resonance frequency $\omega^2 = Q/m$ we have.

$$A_r = 0, \quad A_t = A_i.$$

According to (5) for waves sent at $\bar{x} = 0$ (see Fig. 2) we obtain

$$A_r = R_1 A_i, \quad A_t = T_1 A_i, \tag{6}$$

where

$$R_1 = \frac{-M_1^2 + iM_1}{1+M_1^2}, \quad T = \frac{1+iM_1}{1+M_1^2}, \quad M_1 = \frac{Q_1 - m_1 w^2}{2\omega\varrho c}. \tag{7}$$

These formulas are also true for waves propagating in the direction $-x$. For a second diaphragm situated at $x = a$ we have

$$A_r = R_2 A_1, \quad A_t = T_2 A_i, \tag{8}$$

where

$$R_2 = \frac{M_2^2 + iM_2}{1+M_2} e^{-2i\omega a/c}, \quad T_2 = \frac{1+iM_2}{1+M_2^2}, \quad M_2 = \frac{Q_2 - m_2 w^2}{2\omega\varrho c}. \tag{9}$$

3. Two diaphragms

We will utilize previous results in this paragraph. The wave $A\exp[i\omega(t-x/c)]$ inciding onto the first diaphragm produces a reflected wave $AR_1\exp[i\omega(t+x/c)]$ and a transmitted wave $AT_1\exp[i\omega(t-x/c)]$. In regard to the second diaphragm the wave $AT_1\exp[i\omega(t-x/c)]$ is an incident wave, which generates a reflected wave $AT_1\exp[i\omega(t+x/c)]$ and transmitted wave $AT_1T_2\exp[i\omega(t-x/c)]$. Next, the wave $AT_1R_1\exp[i\omega(t+x/c)]$ is an incident wave for the first diaphragm. The diagram of succeeding reflections is given in Fig. 4. For greater clarity, reflected waves are denoted by askew lines instead of lines perpendicular to diaphragms.

Displacement fields in regions 1, 2, 3 are denoted by u_1, u_2, u_3, respectively. According to Fig. 4 we have

$$u_1 = Ae^{i\omega(t+x/c)}+A\{R_1+T_1^2R_2[1+R_1R_2+(R_1R_2)^2+\ldots]\}e^{i\omega(t+x/c)},$$

$$u_2 = AT_1[1+R_1R_2+(R_1R_2)^2+\ldots]e^{i\omega(t-x/c)}, \tag{10}$$

$$u_3 = AT_1T_2[1+R_1R_2+(R_1R_2)^2+\ldots]e^{i\omega(t-x/c)}.$$

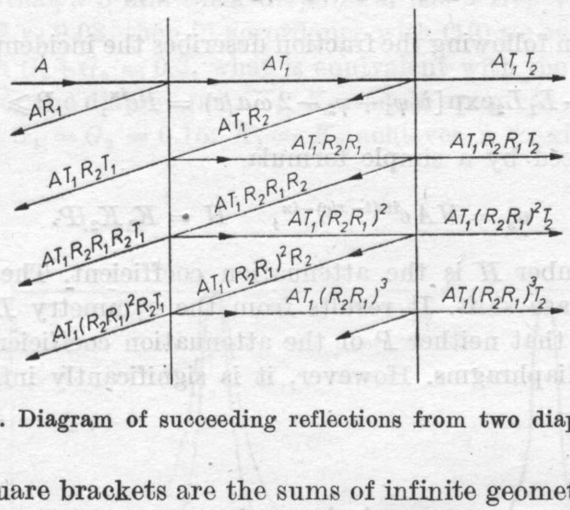

Fig. 4. Diagram of succeeding reflections from two diaphragms

Terms in square brackets are the sums of infinite geometrical progressions. On the basis of formulas for the sums of such progressions we achieve

$$u_1 = Ae^{i\omega(t-x/c)}+A\left(R_1+\frac{T_1^2R_2}{1-R_1R_2}\right)e^{i\omega(t+x/c)},$$

$$u_2 = A\frac{T_1}{1-R_1R_2}e^{i\omega(t-x/c)}+A\frac{T_1R_2}{1-R_1R_2}e^{i\omega(t+x/c)}, \tag{11}$$

$$u_3 = A\frac{T_1T_2}{1-R_1R_2}e^{i\omega(t-x/c)}.$$

From the point of view of noise control the formula for u_3 is the most important. Having numeric calculations in mind we will express the coefficients T_1, T_2, R_1, R_2 in the exponential form

$$T_1 = K_1e^{i\varphi_1},\quad T_2 = K_2e^{i\varphi_2},\quad R_1 = L_1e^{i\psi_1},\quad R_2 = L_2e^{i\psi_2'}, \tag{12}$$

where K_1, K_2, L_1, L_2 are non-negative constants. In complience with formulas (7) and (9) we have

$$K_1 = \frac{1}{\sqrt{1+M_1^2}}, \quad K_2 = \frac{1}{\sqrt{1+M_2^2}}, \quad L_1 = K_1|M_1|, \quad L_2 = K_2|M_2|, \tag{13}$$

$$\varphi_1 = \operatorname{arctg} M_1, \quad \varphi_2 = \operatorname{arctg} M_2, \quad \psi_1 = \operatorname{arctg}(1/M_1)+\pi,$$

$$\psi_2 = \operatorname{arctg}(1/M_2)+\pi,$$

where M_1, M_2 are described by fromulas (7) and (9).

Applying (12), (13) we obtain from formula (11)

$$u_3 = \frac{K_1K_2}{1-L_1L_2\exp[i(\psi_1+\psi_2-2\omega a/c)]} Ae^{i\omega(t-x/c)}. \tag{14}$$

The expression following the fraction describes the incident wave. Denoting

$$1-L_1L_2\exp[i(\psi_1+\psi_2-2\omega a/c) = Pe^{i\varkappa}, \quad P \geqslant 0 \tag{15}$$

(14) can be replaced by a simple formula

$$u_3 = HAe^{iw(t-x/c)-i\varkappa}, \quad H = K_1K_2/P. \tag{16}$$

The real number H is the attenuation coefficient. The term $\exp(-i\varkappa)$ represents the phase shift. It results from the symmetry $L_1L_2 = L_2L_1$ and $\varphi_1+\varphi_2 = \psi_2+\psi_1$, that neither P or the attenuation coefficient H depends on the sequence of diaphragms. However, it is significantly influenced by their thickness.

4. Attenuation

Diagrams of the functions K_1, K_2 and H in terms of frequence ω, will be given here. We will introduce dimensionless thicknesses

$$G_1 = g_1/a, \quad G_2 = g_2/a \tag{17}$$

and the following frequencies

$$\omega_0 = \frac{\varrho c}{\varrho_s a}, \quad \omega_1 = \sqrt{\frac{Q_1}{m_1}}, \quad \omega_2 = \sqrt{\frac{Q_2}{m_2}}, \quad \omega_a = \pi\frac{c}{a}. \tag{18}$$

Frequencies, ω_1 and ω_2, are the frequencies of free vibrations of diaphragms 1 and 2. The frequency ω_a is the free frequency of the space between the diaphragms. It is proportional to the number of distances, a, which the wave travels during a unit of time.

In accordance with the assumption that $Q_1 \approx g_1^3 m_1 \approx g_1$ and in accordance with (18) we have

$$\omega_1 = CG_1, \quad \omega_2 = CG_2, \tag{19}$$

where C is a certain factor of proportionality.

Further calculations are conducted for typical values equivalent for a window in a building, namely:

diaphragm spacing	$a = 0.1$ m
wave velocity in air	$c = 330$ m/s
vave velocity in glass	$c_1 = c_2 = 4000$ m/s
air density	$\varrho = 1$ kg/m³
glass density	$\varrho_s = 3300$ kg/m³

We assume that a 3-mm-thick diaphragm has a free vibration frequency of 30 s^{-1}. Since $G = 0.03$, then in accordance with (19) we accept $C = 1000\ s^{-1}$.

We establish $G_1 + G_2 = 0.3$, what is equivalent with the assumption about a constant mass of the diaphragms. K_1, K_2 and H are given as frequency functions. In Fig. 5, $G_1 = G_2 = 0.15$. $K_1 = K_2$ achieves a maximum of 1 for the

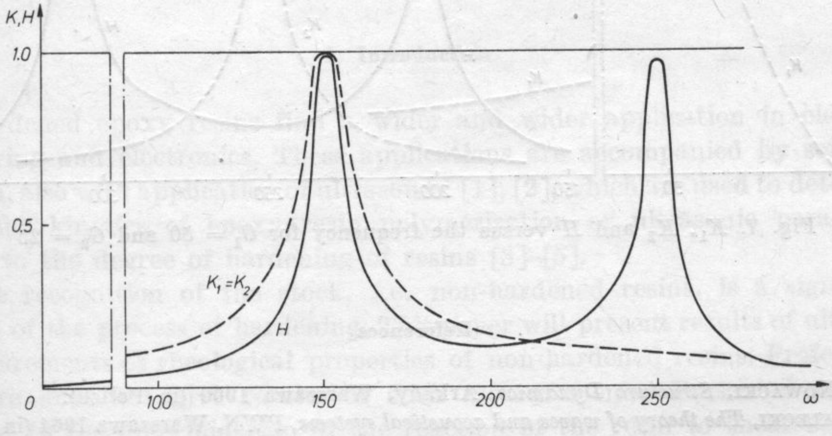

Fig. 5. $K_1 = K_2$ and H versus the frequency for $G_1 = 0.15$

frequency $\omega = 150\ s^{-1}$. Also H achieves a maximum equal to 1 for this frequency. For $\omega = 250\ s^{-1}$ function H achieves next maximum, lower than 1. In Fig. 6, $G_1 = 0.1$, $G_2 = 0.2$. The maximal values of the attenuation coefficient are lower than 1. Maxima are shifted in regards to the maxima of coefficients K_1, K_2. Fig. 7 presents K_1, K_2 and H for $G_1 = 0.5$, $G_2 = 25$. Also in this case H is everywhere lower than 1.

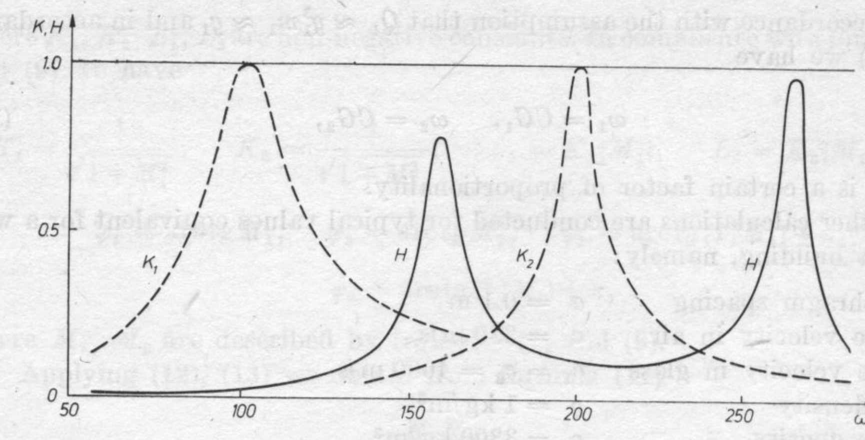

Fig. 6. K_1, K_2 and H versus the frequency for $G_1 = 0.1$ and $G_2 = 0.2$

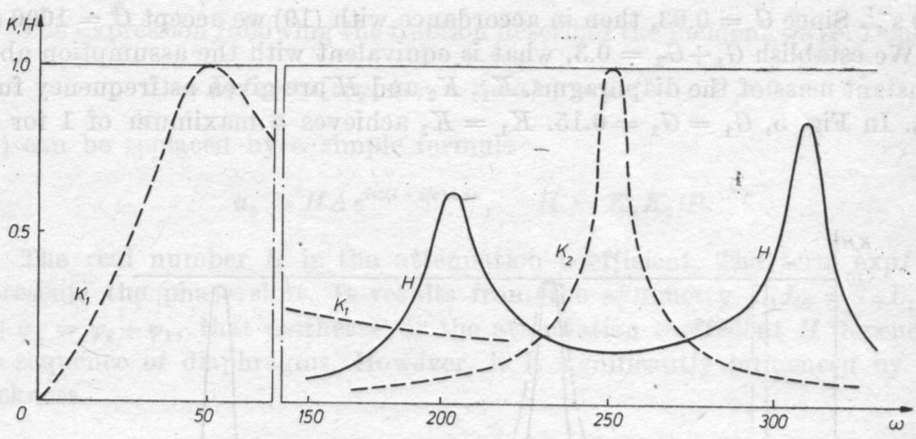

Fig. 7. K_1, K_2 and H versus the frequency for $G_1 = 50$ and $G_2 = 25$

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