

## WAVE PROPAGATION ALONG AN EDGE OF A SOLID

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Investigation results of waves propagating along edges of isotropic and anisotropic bodies are presented in this paper. Also methods of producing wave-guides, generation methods of line waves, the influence of anisotropy on distribution of vibrations and applications of these waves in the acoustic convolution process are discussed.

### 1. Introduction

Beside volume and surface waves also waves propagating along the intersection edges of planes occur in limited solid bodies. These waves are called edge or line waves. They have three components of particle motion and their amplitude decreases with the distance from the edge of both planes and inside the material. There are two fundamental line wave modes in an isotropic body, differing in the deformation manner. In an anisotropic body the deformation manner of an edge when the wave is propagating is more complex and pure modes do not occur.

Wave-guides of edge waves are characterized by high power density due to strong wave energy concentration. Acoustic power is concentrated near the edge in a range of a section of the order of  $\lambda^2$ .

Making a sharp edge is the main difficulty in producing such wave-guides. Difficulties in obtaining edges with an adequate quality limit the frequency range, in which line wave wave-guides can be applied. In the frequency range up to 200 MHz, adequately sharp edges are obtained with the utilization of the phenomenon of cleavage. Edges used in the range up to 100 MHz are obtained with the application of a diamond saw with a high rotational speed (about 20 000 r.p.m.) [1].

## 2. Methods of generating line waves

There are several methods of generating line waves. One of them consists in the transformation of a RAYLEIGH surface wave into a line wave. The transformation occurs when the surface wave incides under an adequate angle onto the edge. It was experimentally proved that in case of glass, at the incidence angle of  $75^\circ$ , about 60% of the energy of the incident wave is converted into the energy of a line wave. Therefore, this is a quite efficient generation process.

Also plate piezoelectric transducers located next to the edge, directly on the wave-guide can be applied in line wave generation (Fig. 1). Transducers with adequate vibrations generate a line wave of a definite type, depending on the location of transducers in terms of the plane of symmetry of the dihedral angle.

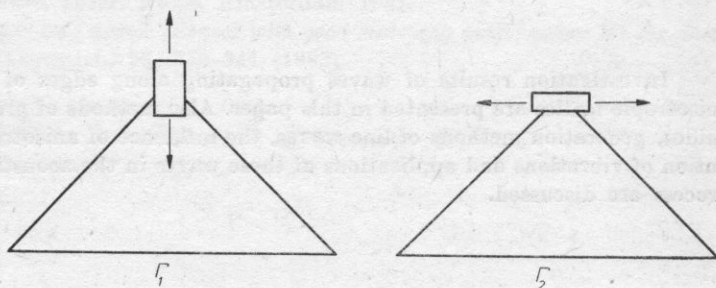


Fig. 1. Generation of line waves with plate transducers

Line waves can be also generated with the application of a interdigital, sawtooth or a single-phase electrode array transducer, located near the edge. In this case an adequate electromechanic coupling must occur in order to generate a wave. This generation method can be rationalised by applying a modified multistrip coupler (MSC), which matches the phase of the generated by the transducer Rayleigh wave with the line wave [1].

## 3. Propagation of a line wave on an isotropic body

The symmetric mode  $\Gamma_1$ , and antisymmetric mode,  $\Gamma_2$ , are the basic modes of line waves on an isotropic body. In the  $\Gamma_1$  mode deformations on the edge are located on the plane of symmetry of the dihedral angle and deformations on planes forming this angle are symmetric (Fig. 2a). In the  $\Gamma_2$  mode, deformations on the edge are perpendicular to the plane of symmetry and deformations on planes forming the angle are antisymmetric in respect to the plane of symmetry of the dihedral angle (Fig. 2b). Theoretical analysis proves that the  $\Gamma_1$  mode is a dispersive mode, while  $\Gamma_2$  is a non-dispersive mode. Experiments confirm this conclusion.

The propagation of line waves along the edges of glass samples has been investigated experimentally. Fig. 3 and 4 present oscillograms of  $\Gamma_1$  and  $\Gamma_2$  wave pulses on glass; excitation with *PZT* plate transducers, frequency 4 MHz. Pressure exerted on the edge caused signal attenuation.

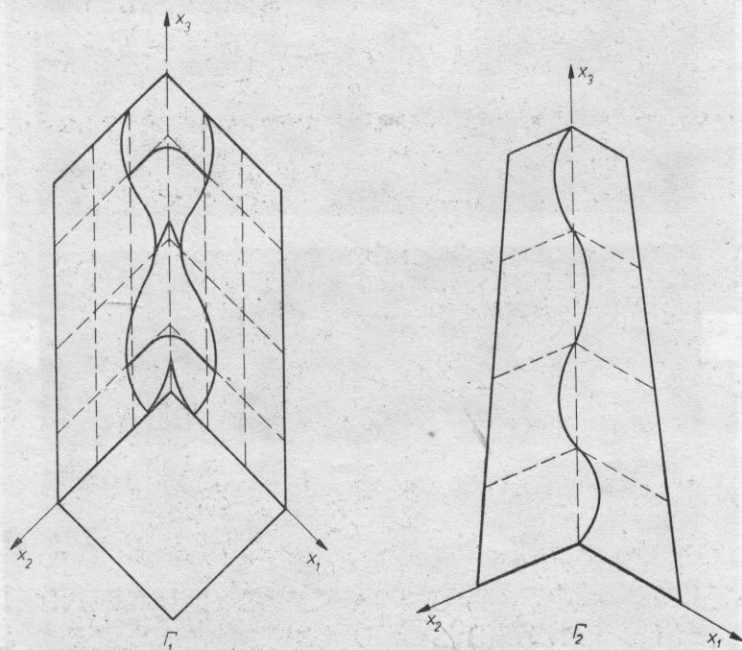


Fig. 2. Line waves on an isotropic body. a — symmetric mode  $\Gamma_1$ , b — antisymmetric mode  $\Gamma_2$ .

Both modes can propagate along curved edges. Fig. 5 presents pulses of the  $\Gamma_2$  wave, propagating along the edge of a glass half-disc with a 30 mm radius.

#### 4. Propagation of a line wave on an anisotropic body

The deformation of the edge during wave propagation is more complicated in an anisotropic body. Due to the anisotropy, line waves are only more or less close to the  $\Gamma_1$  or  $\Gamma_2$  mode in dependence on the prevailing deformation — parallel or perpendicular to the plane of symmetry of the dihedral angle.

Investigations were carried out on quartz rods placed along the *X* axis with *Y* and *Z* planes and on rods placed along the *Y* axis with *X* and *Z* planes. Conducted calculations [2] have led to the determination of the particle vibration components in the *Y* and *Z* or *X* and *Z* planes, respectively. Fig. 6 presents relative amplitudes of vibrations for quartz rods *X* in terms of the distance from the edge for two directions. The calculated mode approximates the symmetric

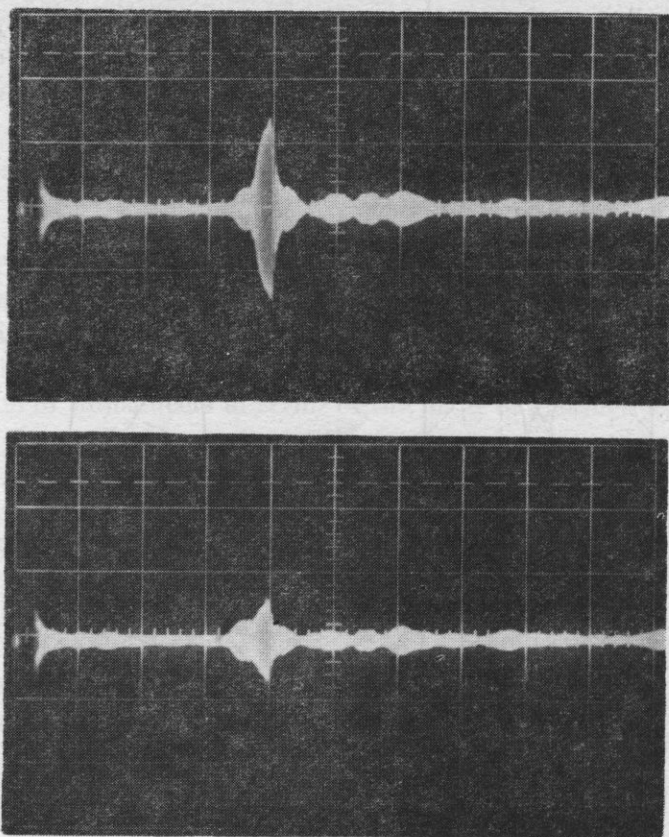


Fig. 3. Wave  $\Gamma_1$  on glass. a — free surfaces, b — pressure on the edge

mode  $\Gamma_1$ , because the resultant vector of particle motion on the edge is inclined to the bisector of the dihedral angle at an angle of  $18^\circ$ . In this case also a mode approximating the  $\Gamma_2$  mode is generated. The angle between the resultant vector of particle motion and the bisector of the dihedral angle is equal  $66^\circ$  for this mode. Relative amplitudes of the  $x, y, z$  components of a line wave in a quartz rod  $Y$  are shown in Fig. 7. The calculated angle between the bisector of the dihedral angle and the resultant vector of particle motion on the edge equals  $63^\circ$ ; the measured angle is  $69^\circ$ . Therefore, in this case the wave approximates the  $\Gamma_2$  mode. Similar calculations and measurements have been also conducted for  $ST$  quartz. Fig. 8 presents the calculated vibration distribution. Line waves in quartz samples have been generated by  $\text{SiO}_2$  plate transducers ( $Y$ -cut) which work on the fundamental frequency and on harmonics [2].

Experiments were also conducted on lithium niobate wave-guides, where the edge of intersection of the cleavage plane and the turned  $Y$  plane, inclined to the cleavage plane at an angle of  $84^\circ 54'$ , was utilized; direction of propagation  $X$ . The cleavage plane (012) is positioned along the  $X$  axis at an angle of

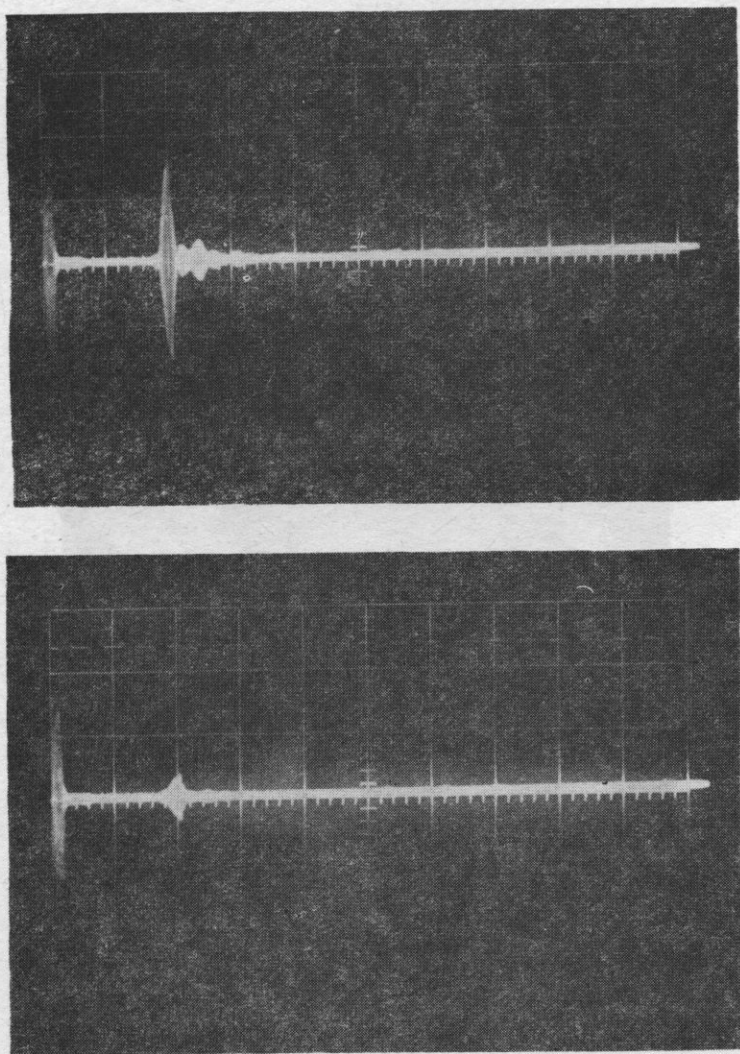


Fig. 4. Wave  $\Gamma_2$  on glass. a — free surfaces, b — pressure on the edge

$57^{\circ}14'$  to the  $Y$  axis [3]. Waves were excited by  $\text{LiNbO}_3$  plate transducers ( $Y$  163-cut); fundamental frequency 10 MHz. Fig. 9 presents oscillograms of signals for a wave approximating the  $\Gamma_2$  mode. For this orientation of  $\text{LiNbO}_3$  the resultant vector of particle motion on the edge is almost perpendicular to the bisector of the dihedral angle and the deviation equals  $4^{\circ}30'$ . Photographs show the volume wave pulse preceding the line wave pulse. This pulse comes from the transverse volume wave, which propagates in the direction  $X$ . These pulses can be separated by extending the sample, due to the velocity difference of the volume (about 4300 m/s) and line wave (about 3500 m/s). Lithium

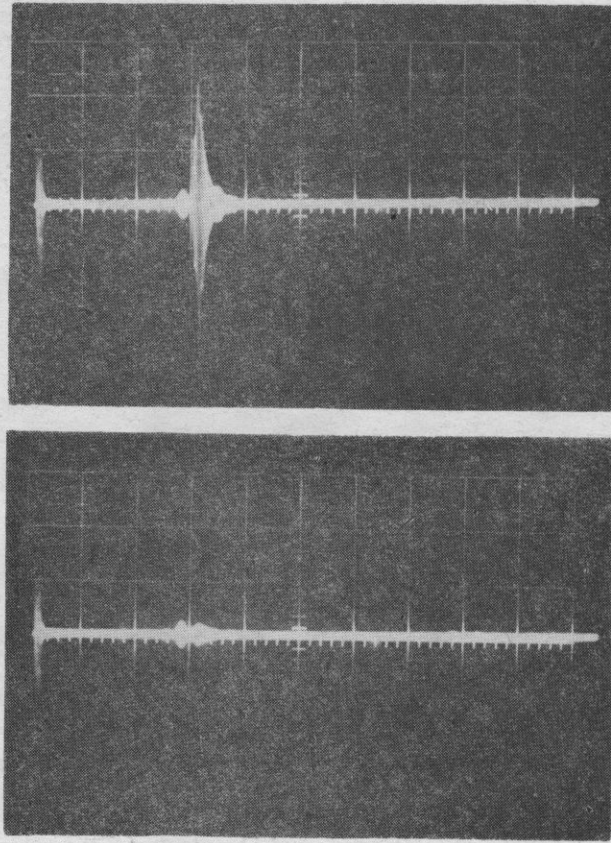


Fig. 5. Line wave on a glass semi-disc, 4 MHz. a — free surfaces, b — pressure on the edge

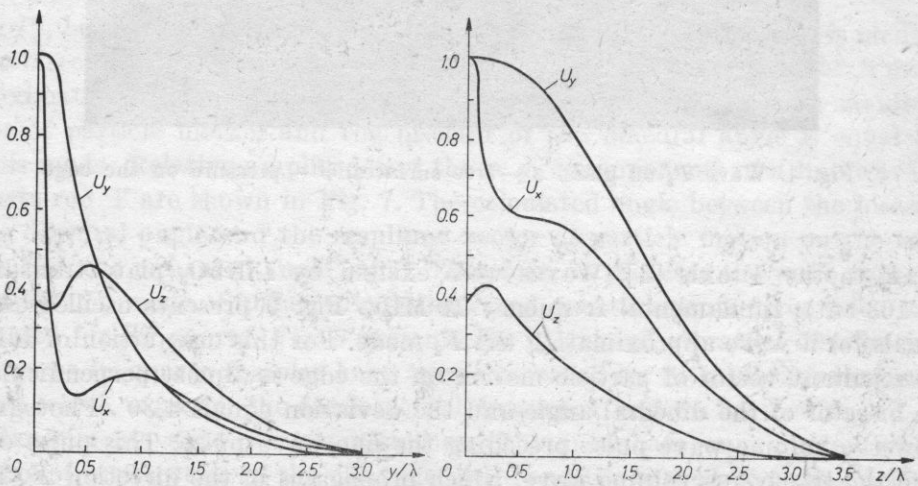


Fig. 6. Relative amplitudes of vibration components of a line wave in terms of distance  $l/\lambda$  from the edge,  $\text{SiO}_2$ , direction of propagation  $X$

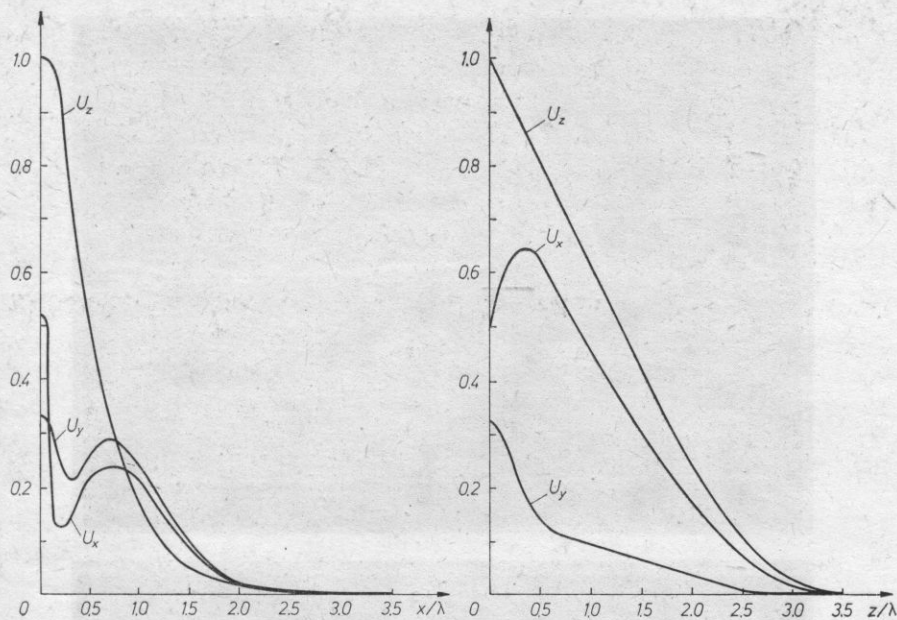


Fig. 7. Relative amplitudes of vibration components of a line wave in terms of distance  $l/\lambda$  from the edge,  $\text{SiO}_2$ , direction of propagation  $Y$

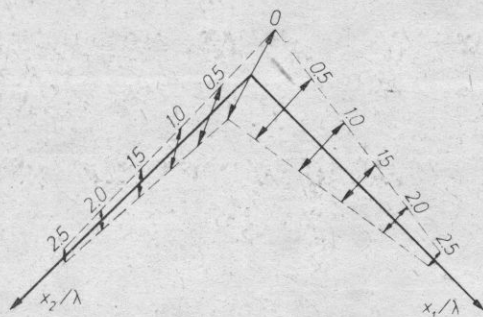


Fig. 8. Distribution of vibrations for a line wave on  $ST$  quartz sample,  $x_1$  — distance from the edge on plane  $ST$

niobate with edges on the intersection of the  $X$  and  $Y$  planes (direction of propagation  $Z$ ), were also investigated. Fig. 10 presents oscillograph records for a wave approximating the  $I_2$  mode; frequency 9.6 MHz.

### 5. Convolution with line waves

Convolution systems have found application in signal processing. They make it possible to obtain the convolution of two input signals. If two signals,  $V_1(t)$  and  $V_2(t)$ , are supplied to the system, then on the output we receive

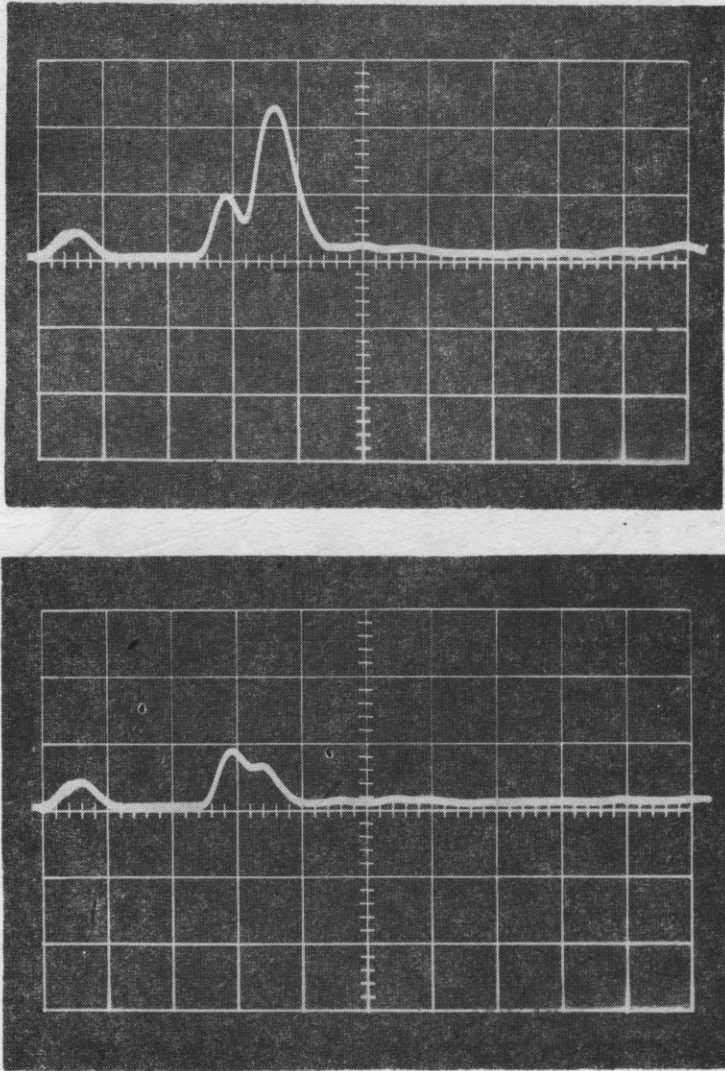


Fig. 9. Line wave on a  $\text{LiNbO}_3$  sample with an edge formed by the intersection of the cleavage plane and turned plane Y. a - free surfaces, b - pressure on the edge

a signal

$$V_3(t) = A \int_{-\infty}^{+\infty} V_1(\tau) V_2(2t - \tau) d\tau. \quad (1)$$

When signals  $V_1(t)$  and  $V_2(t)$  are identical, rectangular pulses with a  $f_1$  carrier frequency, then the pulse on the output has a triangular envelope and  $2f_1$  carrier frequency. The output signal pedestal width and the width of input pulses are equal. A time scale change occurs due to the opposite propagation



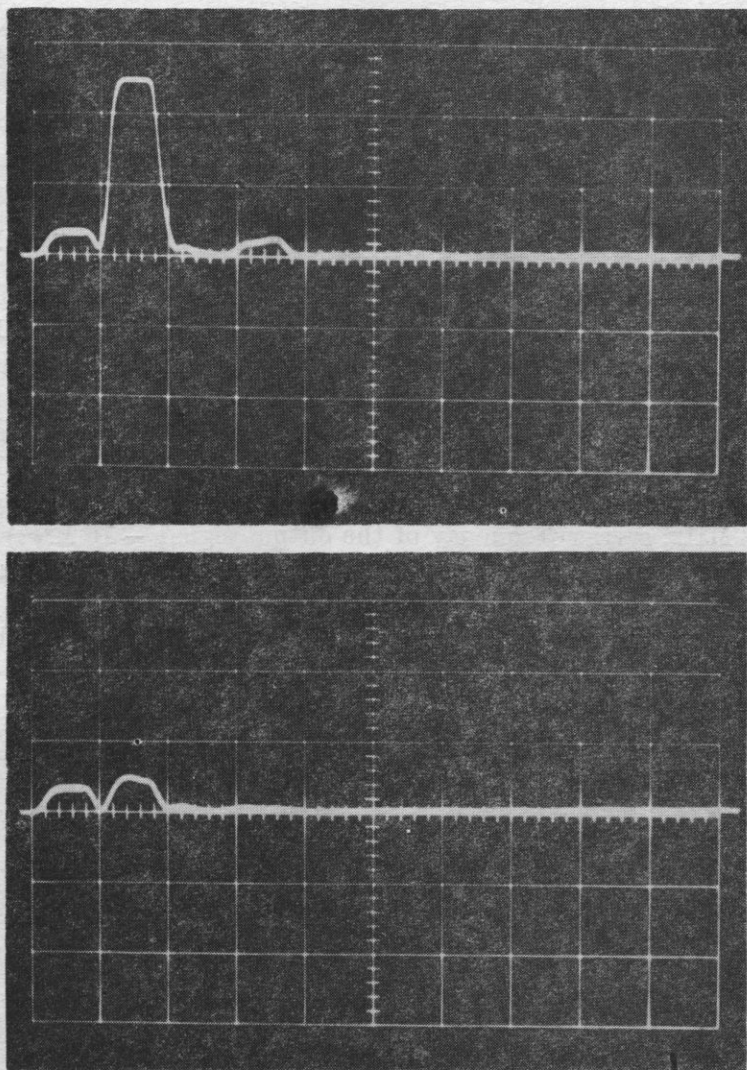


Fig. 10. Line wave on a  $\text{LiNbO}_3$  sample with an edge formed by the intersection of planes  $X$  and  $Y$ . a — free surfaces, b — pressure on the edge

direction of waves. The output voltage is proportional to the input signal width and acoustic power per unit width of the wave beam.

Line waves are strongly confined near the edge. Nonlinear effects can be observed due to high power density. The power density of  $600 \text{ W/mm}^2$  can be obtained for  $\text{LiNbO}_3$  at a frequency of approximately  $200 \text{ MHz}$ . In similar conditions, the power density of about  $20 \text{ W/mm}^2$  is obtained in  $\text{LiNbO}_3$  for a Rayleigh wave. The lack of dispersion (mode  $I_2$ ) and diffraction effect, which cause signal distortion and efficiency decrease in convolution systems with Rayleigh waves, is an additional advantage of line waves [4]. Dispersion also

limits the possibility of applying thin-layer wave-guides, which focus the beam of surface waves, in convolution systems [5].

Non-linear interactions of two line waves of the  $\Gamma_2$  mode, which propagate opposite each other, lead to the formation of a surface potential, symmetric in relation to the bisector of the dihedral angle [6]. In such a case symmetrically arranged in relation to the edge receiving electrodes can not be applied. The convolution electrode has to be situated at an approximately  $\lambda$  distance from the edge; the second surface is entirely metalized.

Convolution systems were observed in *PZT* ceramics and in lithium niobate. The lateral faces of the edge and the direction of the edge for  $\text{LiNbO}_3$  were selected as follows:  $Y$ - $128^\circ$  and natural cleavage planes, direction of propagation  $X$ ;  $Y$  and  $X$  planes, direction of propagation  $Z$ . Convolution electrodes were located on the  $Y$ - $128^\circ$  and  $Y$  plane, respectively. Fig. 11 presents oscillograms of signals of a  $\Gamma_2$  type line wave on a *PZT* sample; frequency 8.75 MHz. Convolution signals for a line wave on *PZT* ceramics (carrier frequency of input signals — 8 MHz, carrier frequency of the output signal — 16 MHz) are shown in Fig. 12. Fig. 13 presents a signal obtained from the convolution of two double signals — top oscillogram, bottom oscillogram — the shape of the input signal. A signal convolution was obtained also for line wave on  $\text{LiNbO}_3$  for both orientations [7].

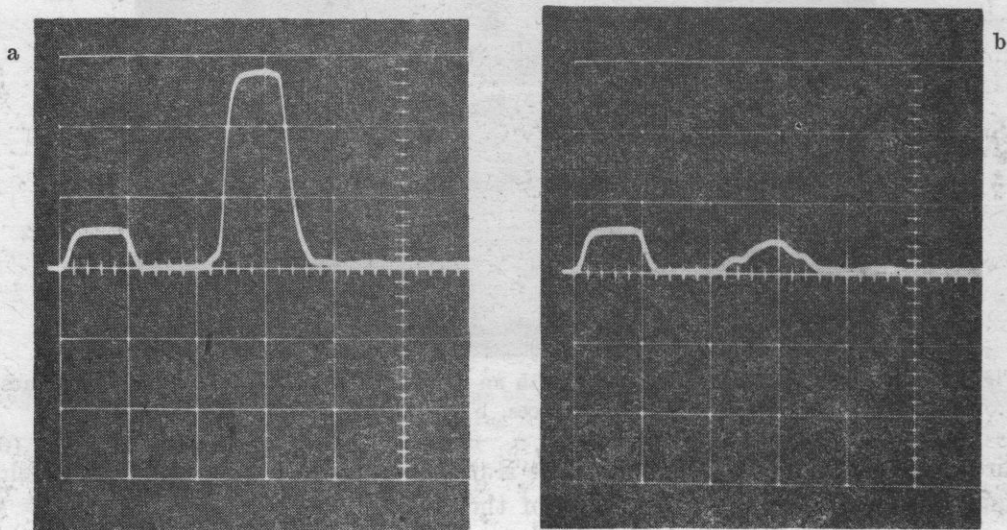


Fig. 11. Line wave on *PZT* ceramics. a — free surfaces, b — pressure on the edge

Investigations have been performed in a carrier frequency range of input signals from 8 to 12 MHz.

An increase of frequency would lead to a stronger wave concentration and an increase of the output signal at the same level of input power. The bilinear

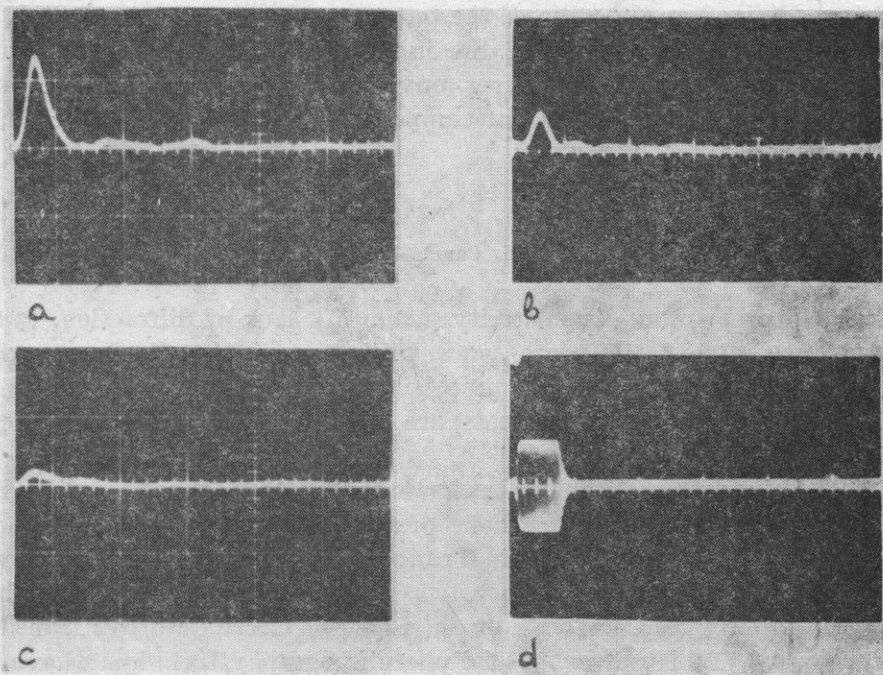


Fig. 12. Signals in the acoustic convolution, line waves on *PZT* ceramics. a — width of input pulse,  $4\mu\text{s}$ , b — width of input pulse,  $3\mu\text{s}$ , c — pressure on the edge, d — shape of input signal

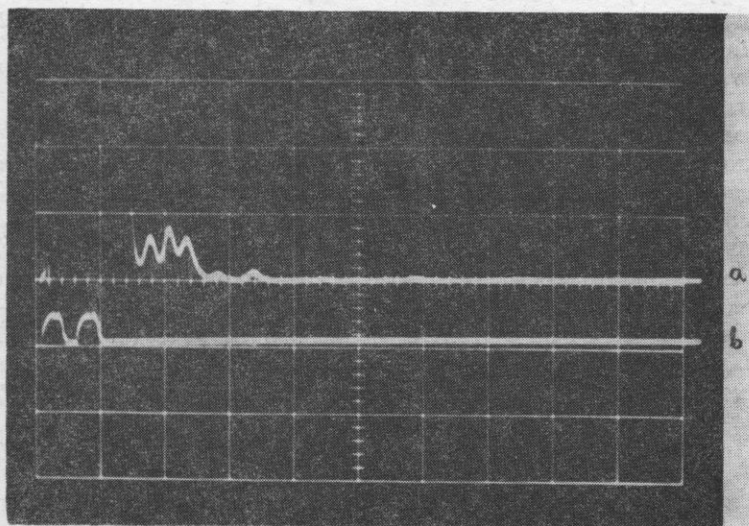


Fig. 13. Convolution of two double pulses, line waves on *PZT* ceramics. a — output signal, 16 MHz, b — input signal, 8 MHz

coefficient, defined as the ratio of the output power and the product of input acoustic powers, is a measure of the interaction efficiency in a convolution system. The application of line waves convolution systems subjected to optimization can give a bilinear coefficient comparable to the coefficient obtained in a diode system [8].

### 6. Conclusions

Such features as non-dispersivity (mode  $\Gamma_2$ ), lack of diffraction, low propagation losses and strong beam concentration prove that line waves can be utilized in delay lines and signal processing systems. Non-linear effects (convolution of signals, parametric effects) are easily obtainable due to high power density [6].

It is decisive for practical application to master the technology of producing sufficiently sharp edges and methods of effective generation of line waves. Our experiments have shown that in a range up to several tens MHz line waves can be easily and effectively generated with plate transducers. For higher frequencies interdigital, sawtooth or single-phase electrode array transducers have to be used [1]. However, special photolithography techniques have to be applied in order to introduce these structures at a distance of about  $\lambda/3$  from the edge.

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*Received on November 30, 1984; revised version on April 21, 1986*