ARCHIVES OF ACOUSTICS
12, 3–4, 215–224 (1987)

KINETICS OF THE TRANSPORT OF AEROSOL PARTICLES UNDER THE INFLUENCE OF DRIFT FORCES IN THE STANDING WAVE FIELD

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In the paper the time constants of the aerosol particles transport process under the influence of drift forces appearing in the standing wave field have been estimated. The transport phenomenon which causes particle concentration in the neighbourhood of the minimum of drift force potential significantly supports the coagulation microprocesses by reducing the distances between particles. It has been demonstrated that the growth of particle concentration around the points of stable equilibrium is exponential. The time constant of this growth was estimated and the formulae which precisely determine the time needed to obtain the assumed concentration increase have been derived. Parameters of spatial particle distribution in the equilibrium state and time necessary for reaching such distribution have been estimated under an assumption of lack of interactions between particles.

Basic notation

x spatial coordinate in the direction of wave propagation
t time,
f frequency,
ω angular frequency,
λ wave-length,
k wave-number,
ϱ_g gas density,
ϱ_p density of the aerosol particle,
r radius of the particle,
m particle mass,
η gas viscosity,
k_B Boltzmann constant,
T temperature.

1. Introduction

In the field a standing wave defined by the deflection function of particles of the medium

$$\xi = \xi_0 \sin kx \sin \omega t, \tag{1.1}$$

the particles of aerosol, in the first approximation, exhibit a harmonic motion with amplitude and phase depending on the particle magnitude, frequency and other quantities characteristic for the particle, the medium and the acoustic wave. If the second order effects are taken into consideration, such as radiation pressure, asymmetry of medium oscillations resulting from the standing wave amplitude dependence on the position or periodical changes of viscosity then it can be concluded that the real motion of a particle is a superposition of the oscillating and translatory motion [1]. Averaging of the second-order-approximation solutions of the equations of motion indicates that the motion of instantaneous equilibrium position of a particle is induced by the resistance force and a certain force F_D which depends on the position according to the formula [1]

$$F_D(x) = F_0 \sin 2kx. \tag{1.2}$$

irrespectively of the considered mechanism of the phenomenon.

Such forces are called the drift forces. Depending on the sign of the constant F_0 which denotes the maximum value of the force, these forces cause the constant movement of aerosol particles towards the nodes (given by an equation $kx = n\pi$ or loops $kx = \left(n+\frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$) of the standing wave. In the case $F_0 > 0$ the minimum of the force potential given with the formula

$$U_D(x) = F_0 (2k)^{-1} \cos 2kx + \text{const}, \tag{1.3}$$

are in the loops of the wave. On the contrary, if $F_0 < 0$, the drift forces make the aerosol particles move towards the nodes.

In approximation which is valid for small Reynolds numbers the equation of motion of the average position of the particle is [1]

$$m_p \frac{d^2 x}{dt^2} + 6\pi\eta r \frac{dx}{dt} - F_0 \sin 2kx = 0, \tag{1.4}$$

where the second summand represents the resistance force in the Stokes approximation. By dividing this equation by the particle mass we obtain

$$\frac{d^2 x}{dt^2} + \frac{1}{\tau}\frac{dx}{dt} - A_D \sin 2kx = 0, \tag{1.5}$$

where $\tau = m_p/(6\pi\eta r) = 2r^2 \varrho_p/(9\eta)$ is the relaxation time and $A_D = F_0/m_p$ denotes the maximum value of acceleration induced by the drift force. This value which, by analogy to other force fields can be considered as the intensity of the drift force field, depends strongly on the particle size and the nature of drift. In our previous paper [2] we have presented the qualitative analysis of the intensity of several drift types. We have demonstrated that the drift connected with the radiation pressure called the radiation drift (abbrev.: R drift) dominates in the case of relatively large particles with radii about 10^{-4} m. The order of magnitude of the value of field intensity for this drift is 10m/s^2 and decreases rapidly with the decrease of radius of particles. Because this value is near to the value of gravity acceleration, the motion of large particles under the influence of R drift forces with simultaneous settling is frequently investigated [3, 4]. The L drift concerned with periodical temperature changes and appropriate viscosity changes in the standing wave dominates in the range of radii between 10^{-5} and 10^{-6} m, where it attains maximum of intensity. For the frequency 10 kHz and wave energy density 100 J/m^3 this maximum is greater than 10^3 m/s^2. Similar values characterize the A drift, concerned with the phenomenon of asymmetry of the particle motion in the standing wave field, when the influence of the wave on the particle changes according to the decrease of vibration amplitude of particles of the medium together with the decrease of distance to the wave node. Maximum of intensity of this drift appears for the particles of radii slightly smaller than 10^{-6} m.

Equation (1.5) is analogical to the equation of motion of a pendulum in a viscous medium. Despite the simplicity of the phenomenon it represents, this equation does not have an elementary solution. The small displacements approximation which is applied in the case of a pendulum can not be applied, because here the motion of particles at the points far from equilibrium position is of special interest. Analogy with the pendulum allows to expect the quasiperiodical solutions with amplitude damping and the monotonic (aperiodical) solutions, if the damping is large enough, the driving force is small, and the particle inertia is negligible. Analysis of the solution type, carried out by means of graphical and numerical methods in the paper [2], allowed for finding the relation between the constants of Eq. (1.5) (i.e. relaxation time τ, drift forces field intensity A_D and wavenumber k), which constitutes the monotonicity criterion of solutions. This relation is

$$\tau^{-1}(2kA_D)^{-1/2} > 2, \tag{1.6}$$

what, after some transformations and after substituting the equation parameters expressed by basic quantities characterising the particle and the wave gives the conditions

$$A_D < 81\eta^2 c/(64\pi r^4 \varrho_p^2 f). \tag{1.7}$$

For fixed values of viscosity, wave velocity and frequency and density of a particle, the above condition relates the drift force field intensity to the particle radius. As it has been demonstrated in the paper [2], this condition is fulfilled for all drift types, if the particles radii are restricted to the values smaller than 10^{-5} m.

The motion of aerosol particles consists then in monotonically approaching the stable equilibrium point. From the physical viewpoint this is the evidence of the negligible inertia of a particle. Hence, in the analysis of the motion in the range of applicability of the condition (1.7), we shall consider the differential equation

$$\frac{1}{\tau}\frac{dx}{dt} = A_D \sin 2kx, \tag{1.8}$$

obtained from eq. (1.5) by neglecting the inertial component. This equation, called the King–St. Claire equation for the names of the scientists who for the first time applied it in the theory of drift (cf. [4], [5]), can be integrated by separation of variables. Simple calculations lead to the solution

$$\mathrm{tg}\,kx = \mathrm{tg}\,kx_0 \exp(2\tau A_D kt), \tag{1.9}$$

where $x = x_0$ is put for $t = 0$. The solution (1.9) does not depend on the initial velocity of particles, being integral of a first order equation. Usefulness of this approximation in the range of drift force intensities and particle radii defined by the condition (1.7) has been confirmed by numerical calculations [1].

2. Estimation of the time constants of the motion

The solution (1.9) anticipates that for positive values of constant A_D which represents the drift forces field intensity the aerosol particles will move towards the points in which $kx = (n+\frac{1}{2})\pi$, i.e. to the standing wave loops. On the other hand, the wave nodes are the points of unstable equilibrium and particles which initial positions satisfy the equation $kx_0 = n\pi$ will remain in the nodes infinitely long, putting aside the random forces which can throw them off such points. For the negative values of the constant A_D the roles of loops and modes exchange; anyway, this does not change the kinematics of the transport process, because the change of sign of the exponent in Eq. (1.9) is equivalent to the linear change of variables $kx \to kx - \frac{\pi}{2}$. In practice the sign of the constant A_D is positive only in the case of radiation drift, and only for relatively heavy particles, for which $\varrho_p/\varrho_g > 5/2$ [4]. For lighter particles the sign of A_D for the R drift, as well as for the L and A drifts, is negative in the whole range of particle density. Hence as a rule aerosols are concentrated around the nodes.

A particle, the motion of which is described by Eq. (1.9), tends to the equilibrium position asymptotically and reaches if after a theoretically infinite period. The concentration of particles increases then around such a point and decreases in the region between a loop and a node. Hence, calculation of the period needed for attaining the equilibrium position is aimless. Instead of this, we shall investigate the changes of particle concentration as a function of time and position.

Let us assume that at instant $t = 0$ all the particles are in rest and that their distribution along the x axis perpendicular to the nodes and loops plane is uniform. Let us denote the number of particles present in a unit interval of this axis at the point x and at the time t as $N(x, t)$. We have then

$$N(x, t)|_{t=0} = N_0 = \text{const}. \tag{2.1}$$

The type of the approximation of the equation of motion as well as the form of the solution indicate that particles tending to a loop or a mode do not overtake each other (it could have been so in the case when, in contradiction to our assumption, particles were placed farther from the points of balance had and larger initial velocity). If all particles start their motion at the time $t = 0$, then those from a point x_0 will be at a point x, at the time t and those from a point $x_0 + dx_0$ at the same time will be at a point $x + dx$ (cf. Fig. 1). It can be seen at once that if the velocity of

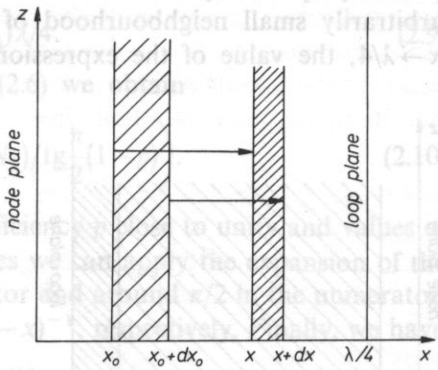

Fig. 1. Concentration of aerosol as the effect of an asymptotic travel of aerosol particles towards the loop planes

particles drops while approaching the equilibrium point, then their concentration grows. Relative concentration (with respect to the initial one) is then equal to

$$\frac{N(x, t)}{N_0} = \frac{dx_0}{dx}. \tag{2.2}$$

The value of the derivative in the above equation can be found by taking differential of both sides of (1.9):

$$\sec^2 kx\,dx = \sec^2 kx_0 \exp(2\tau A_D kt)\,dx_0. \tag{2.3}$$

Eliminating x_0 by means of eq. (1.9) we obtain

$$N(x, t) = N_0(\cos^2 kx \exp(\alpha t) + \sin^2 kx \exp(-\alpha t))^{-1}, \tag{2.4}$$

where $\alpha = 2\tau A_D k$.

As it can be easily seen at the standing wave nodes ($kx = n\pi$) the concentration decreases exponentially as $N_0 \exp(-\alpha t)$ and at the loops it grows as $N_0 \exp(\alpha t)$. Time constant of this process, equal to α^{-1}, depends on relaxation time, wavenumber and drift force intensity, thus on the drift type. From the formula (2.4) one can calculate the time after which the particle concentration in a given point x achieves a specified value. However, because the solution of Eq. (2.4) leads to a complex solution which is not very useful, we shall pass to other methods of estimating the rate of the process of concentrating particles by the drift forces.

If we fix a time period during which the concentration of particles in a specific neighbourhood of a stable equilibrium point should take place, then this time will always occur to be insufficient for a certain number of particles, initially distributed near the unstable equilibrium points. The form of the approximate solution (1.9) indicates that particles travel to the wave loop during an arbitrarily long period. It is clear when we solve Eq. (1.9) with respect to time:

$$t = \alpha^{-1} \ln(\mathrm{tg}\, kx/\mathrm{tg}\, kx_0). \tag{2.5}$$

If $x_0 \to 0$ then $t \to \infty$. On the other hand, particles move towards an equilibrium point asymptotically. Hence, one can not speak about their complete clustering in an arbitrarily small neighbourhood of a wave loop after a finite time. Indeed, for $x \to \lambda/4$, the value of the expression (2.5) also tends to infinity.

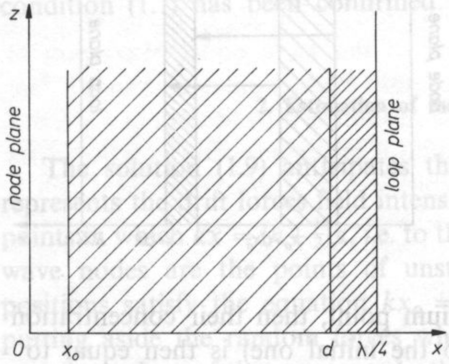

Fig. 2. Cleared zone (interval $[x_0, \lambda/4)$ and intensive coagulation zone (interval $[d, \lambda/4]$)

Let us denote by d the coordinate of a point which a particle should surpass to enter a "zone of intense coagulation", where the inter-particle processes reach the intensity high enough for an "almost immediate aggregation" (such concepts, which are useful in considerations where the nature of microcoagulation processes is neglected, were introduced in the paper [6]). Other particles which were initially displaced in the range $[x_0, d]$ will reach the point d earlier (cf. Fig. 2). This means that after the period

$$t_k = \alpha^{-1} \ln(\mathrm{tg}\, kd/\mathrm{tg}\, kx_0), \tag{2.6}$$

the zone defined by dx will be reached by the fraction $(\lambda/4 - x_0)/(\lambda/4)$ of all particles, provided that their distribution along the x axis was uniform at $t = 0$. Then, the quantity

$$p = (\lambda/4 - x_0)/(\lambda/4) = 1 - 4x_0/\lambda, \tag{2.7}$$

denotes the relative number of particles which would reach the zone defined by d in time t_k, with respect to the number of all particles. This value can be called the efficiency of the mechanism of transport of particles by the drift forces to the zone of intense coagulation. If it is assumed that particles in this zone will undergo coagulation for certain then p can be called simply the efficiency of coagulation. The latter formula makes it possible to express the value x_0 from the initial condition of Eq. (1.9) by this useful parameter

$$x_0 = (1-p)\lambda/4. \tag{2.8}$$

After the period found from Eq. (2.6) almost all the particles of aerosol will be in the zone defined by d (except for those which at $t = 0$ were in the interval $[0, x_0]$). It can be assumed then that the average concentration of particles in this zone grew as many times as the distance from point d to the loop is smaller than a quarter of wave-length. Hence, we have $N_0/N_k = (\lambda/4-d)/(\lambda/4) = 1-4d/\lambda$, from which it follows

$$d = (1-N_0/N_k)\lambda/4. \tag{2.9}$$

Substituting the two latter formulae to Eq. (2.6) we obtain

$$t_k = \alpha^{-1}\ln\left(\operatorname{tg}\frac{\pi}{2}(1-N_0/N_k)/\operatorname{tg}\frac{\pi}{2}(1-p)\right). \tag{2.10}$$

What interests us the most are the values of efficiency p close to unity and values of the quotient N_0/N_k close to zero. In these cases we can apply the expansion of the function tangent around zero in the denominator and around $\pi/2$ in the numerator, using approximations $\operatorname{tg}x \approx x$ and $\operatorname{tg}x = (\pi/2-x)^{-1}$, respectively. Finally, we have

$$t_k = \alpha^{-1}\ln\frac{4N_k/N_0}{\pi^2(1-p)}. \tag{2.11}$$

It should be reminded that the value

$$\alpha^{-1} = (2\tau A_D k)^{-1} = 9\eta c/(8\pi r^2 \varrho_p f A_D), \tag{2.12}$$

is a time constant, characteristic for the process of exponential concentration variations (cf. Eq. (2.4)). Expression (2.11) introduces a significant amount of precision into the notion of coagulation time for given efficiency of the process and specified concentration increase. In agreement with our expectations this time grows if both these parameters increase. Note that this growth is logarithmic, so in the estimations only the order of magnitude of the parameters $1-p$ and N_k/N_0 is important. For example if we put $1-p = 10^{-2}$, then the value of coagulation time given by the expression (2.11) will be $8.30\cdot\alpha^{-1}$ for $N_k/N_0 = 10^2$ and $17.52\alpha^{-1}$ for $N_k/N_0 = 10^6$. So, the growth of assumed concentration by four orders of magnitude gives only the doubling of time needed to obtain it. In addition, for average wave-lengths and typical particle dimensions, the concentration growth larger than 10^4 is unrealistic.

For the quantitative estimation of the speed of process of gathering the particles in the regions of around wave loops or nodes let us assume now in the formula (2.11) some numerical values of parameters characterizing the particle, the acoustic field and the transport process. Similarly as in the estimation of drift forces field intensities for various types of drifts in the paper [2], we shall put: $\varrho_p = 10^3$ kg/m³, $\varrho_g = 1.2$ kg/m³, $\eta = 1.85\cdot10^{-5}$ Ns/m² and $c = 340$ m/s, what corresponds with an acoustic wave in the air. If we also fix the frequency and energy of the wave (f = 10 kHz and $\bar{E} = 100$ J/m³ in our example), then, according to formula (2.11), the equation $t_k = \text{const}$ will provide a relation between the drift force field intensity A_D and the aerosol particle radius. In the Fig. 3, where the graphs of A_D for various drift types taken from paper [2] are also shown, equation $t_k = \text{const}$ is represented by a straight line. For the concentration process the following values were assumed: $1-p = 10^{-2}$, what corresponds to the efficiency equal to 99%, and $N_k/N_0 = 10^3$. The dashed line delimits the range of applicability of the King–St. Claire approximation and represents the condition (1.7). As it can be seen, for the above listed values of wave parameters, the L type drift effectively gathers the particles around the equilibrium positions in the time about 0,1 s. The R type drift induces the concentration of particles with radii larger than 10^{-5}m in the time shorter than 1 s, while the A type drift can provide for concentration of particles with radii about 10^{-6}m in the time of equal order of magnitude.

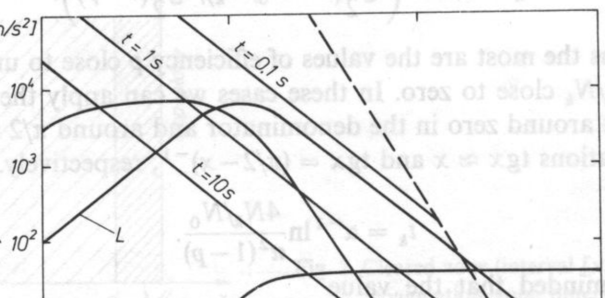

Fig. 3. Plots of intensity of the drifts type R, L and A vs particle radius and lines of constant values of coagulation times. The straight dashed line defines the region of application of the King-St. Claire approximation

3. Spatial distribution of particles around the equilibrium points

While discussing the phenomenon of transport of the aerosol particles under the influence of drift forces we have not taken into account the processes responsible for the coagulation of particles. We have assumed however that the appropriate growth of concentration, corresponding to the reduction of mutual distances between particles, will cause quick coagulation in concentration zones. In the two-particle problem a similar approach was applied by JESSEL and TIMOSHENKO [6], by introducing the concept of the „coagulation parameter" as a dimension of the zone in which particles coagulate „automatically and almost immediately". The constant of all exponential function of time which describes the decrease of concentration of aerosol particles was expressed by them with the use of the dimensions of such a zone, assumed to be an ellipsoid of revolution, elongated in the direction of wave propagation.

Theoretical approach which consists in neglecting the interactions between particles makes it possible to estimate statistically the dimensions of a zone in which the particles will gather. The distribution of equilibrium of the particles is described by the Boltzmann distribution

$$\mathcal{N}(x) = \text{const}\cdot\exp\left(-\frac{U(x)}{k_B T}\right), \tag{3.1}$$

while potential is given by Eq. (1.3). In the neighbourhood of a minimum, e.g. near the point $x = \lambda/4$, the function $U_D(x)$ has approximately the following form

$$U_D(x) \approx \text{const} + m_p A_D k\left(x - \frac{\lambda}{4}\right)^2 \tag{3.2}$$

where the terms of order $(x - \lambda/4)^{-4}$ are rejected. Then, in this approximation, the spatial distribution is of gaussian type

$$\mathcal{N}(x) = \text{const}\cdot\exp\left(-\frac{m_p A_D k\left(x-\frac{\lambda}{4}\right)^2}{k_B T}\right). \tag{3.3}$$

Dimension of a zone of particle concentration is then of the order of magnitude

$$\sigma = \left(\frac{k_B T}{2 m_p A_D k}\right)^{\frac{1}{2}} \tag{3.4}$$

The above value can be considered as the dimension of the zone of concentration of particles, if the particle interactions are neglected and the particle dimensions are not taken into account. Such estimation can be useful if the coagulation itself is caused by additional factors which are independent of the standing wave. A technical arrangement proposed by SCOTT [7] can be an example here. In this arrangement the particles are concentrated in the loop or node planes with the use of an intensive standing wave, and the coagulation is due mainly to the ortokinetical effect caused by a perpendicular, sawtooth progressive wave.

The time required to obtain an equilibrium distribution of particles near the stable equilibrium points can be estimated more precisely with the use of Eq. (2.6) which describes the time of travel of a particle from an initial point to a point of coordinate d, i.e. to the border of a given concentration zone. Putting $x_0 = d = \sigma$ we shall estimate the time necessary for establishing an equilibrium distribution of nearly all the particles, excluding those which initially were placed at the distance from the ustable equilibrium point smaller than σ. Here we shall use the same expansion of the tangent function as previously:

$$t_k = \alpha^{-1}\ln(k\sigma)^{-2} = (2\tau A_D k)^{-1}\ln\frac{2m_p A_D}{k k_B T}. \quad (3.5)$$

For the numerical values assumed in the quantitative discussion at the end of the previous paragraph and for $T = 300$ K we get $t_k \approx 14\alpha^{-1}$. This is the value of the same order of magnitude as the coagulation times obtained above.

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Received October 2, 1986.