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RECTANGULAR PHASE SOUND SOURCES FOCUSING RADIATED ENERGY OF VIBRATIONS

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This paper is concerned with the acoustic field in the Fresnel zone of a focusing rectangular phase sound source with the following amplitude distributions of the vibration velocity: uniform, HAMMING'S, HANNING'S and BLACKMAN'S. Amplitude distributions of the acoustic potential were determined along the main axis of such a sound source and in planes parallel to the main axis. It was found that in the case of HAMMING'S, HANNING'S and BLACKMAN'S amplitude distributions of the vibration velocity, the amplitude distribution of the acoustic potential along the amin axis has only one maximum, which is situated near the focal point. This maximum has the highest value for HAMMING'S distribution. When the dimensions of the source are increased, the value of the maximum increases and it is shifted towards the focal point, while its width decreases. Amplitude distributions of the acoustic potential in planes parallel to the axis of the source have relatively narrow maxima, which occur along this axis. They are narrower than for a Gaussian amplitude distribution of the vibration velocity, which was analysed in previous papers. Besides the main maximum also side maxima occur in the focal plane. They are strongly damped for HAMMING'S, HANNING'S and BLACKMAN'S distributions.

Therefore, the acoustic field of a focusing rectangular phase sound source with HAMMING'S, HANNING'S and BLACKMAN'S distributions is relatively uniform in the Fresnel zone.

1. Introduction

In order to achieve the highest possible transverse resolving power of ultrasonic diagnostic systems, sources are applied, which essure the radiation of a possibly narrow beam of ultrasonic waves with a homogeneous internal structure, i.e. without local maxima and minima. HASELBERG and KRAUTKRÄMER have proved [3] that a plane sound source with a Gaussian distribution of the amplitude of vibration velocity radiates a wave beam with the required internal structure. Also a spherical, focusing sound source with a Gaussian distribution of the amplitude of vibration velocity can produce a relatively narrow beam of ultrasonic waves with an uniform internal structure, as it was shown by FILIPCZYŃSKI and ETIENNE [2]. Paper [9] deals with the far acoustic field of a rectangular sound source with the following distributions of the amplitude of vibration velocity: uniform, HAMMING'S, HANNING'S and BLACKMAN'S. It was found that the directional pattern of such a source with HANNING'S distribution has a relatively narrow main maximum and sufficiently damped side maxima. It was also shown that a sound source, which radiates the energy of vibrations to the far field with a sufficiently large directionality can be realized by a rectangular mosaic system of plane sound sources with discrete HANNING'S distributions of their relative bulk efficiencies. This paper investigates the near field of a rectangular phase source with uniform, HAMMING'S, HANNING'S and BLACKMAN'S distributions of the amplitude of vibration velocity. Results obtained prove further research on the possibility of producing a focusing sound source in the form of a rectangular mosaic phase system of planar sound sources worth continuing.

2. The acoustic field of a plane sound source in Fresnel's zone

Let us assume (Fig. 1) that a plane sound source σ_0, which vibrates with a simple periodic motion with frequency f_0, is situated on plane $z = 0$, which is a perfectly rigid baffle S_0. Let it radiate energy of vibrations into half-space $z > 0$, filled with a lossless and homogeneous fluid medium, in which the acoustic wave propagates with velocity c. It was also accepted that the sound source σ_0 in the plane of the baffle S_0 produces an amplitude distribution of the normal component of the vibration velocity described by the following function

$$\begin{aligned} &\varkappa(x, y) \neq 0 \text{ for the surface } \sigma_0; \\ &\varkappa(x, y) = 0 \text{ for the rest of the baffle } S_0. \end{aligned} \tag{1}$$

Let us consider an arbitrary plane S_z in the half-space $z > 0$, parallel to the plane of the baffle S_0 (Fig. 1). z will denote the distance between these two planes. The distribution of the complex amplitude of the acoustic potential in an arbitrary plane S_z can be determined from [4, 10]

$$\Phi(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varkappa(x, y)\frac{\exp(j2\pi\nu d)}{2\pi d}dxdy, \tag{2}$$

where $\nu = f_0/c$ is the spatial frequency. It results from Fig. 1 that the distance between point P_z on plane S_z and point P_0 on baffle S_0 is determined from expression

$$d = z\sqrt{1+\frac{(x-\xi)^2+(y-\eta)^2}{z^2}}. \tag{3}$$

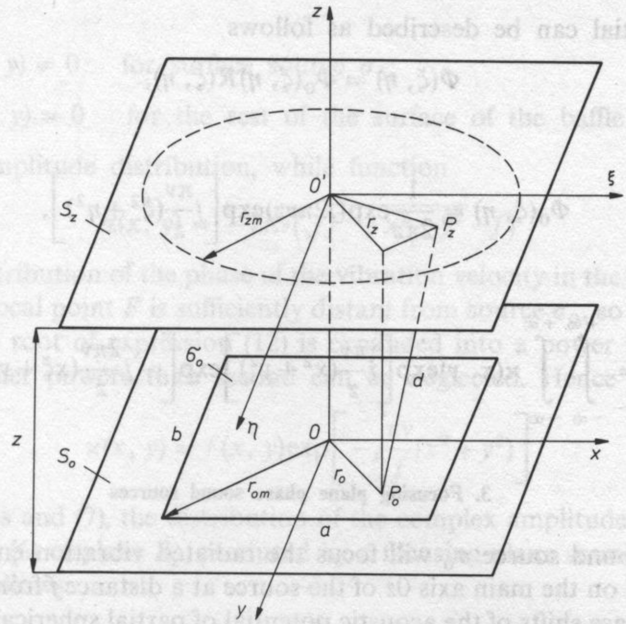

Fig. 1. Plane sound source σ_0 located in baffle S_0 radiating energy of vibrations into a medium which fills the half-space above the baffle

By r_{om} we denote (Fig. 1) the greatest distance of points of the source contour σ_0 from the origin of coordinates, and by r_{zm} — the radius of the neighbourgood of point 0' in which axis 0z passes through plane S_z. Let $r_{om} \gg \lambda/v$, where λ is the wave length. It was assumed that the distance z is great enough in relation to r_{om} and r_{zm}, so that all terms with a power greater than two can be neglected in the expansion of the root of expression (3) into a power series. In this case, at $r_o \leqslant r_{om}$ and $r_z \leqslant r_{zm}$ it can be accepted in approximation that [4, 10]

$$d = z + \frac{\xi^2+\eta^2}{2z} + \frac{x^2+y^2}{2z} - \frac{x\xi+y\eta}{z}, \tag{4}$$

while

$$1/d = 1/z, \tag{5}$$

if $\lambda \ll r_{om}$, $r_z \ll z$.

The zone of the acoustic field of a sound source σ_0, in which mentioned above relations are valid, is called the Fresnel zone. Therefore, it results from what was previously said and from (2), that in an area on plane S_z

$$\xi^2+\eta^2 \leqslant r_{zm}^2 \tag{6}$$

which fulfills condition $r_{zm}/z \ll 1$, the distribution of the complex amplitude of the acoustic potential can be described as follows

$$\Phi(\xi, \eta) = \Phi_0(\xi, \eta) R(\xi, \eta), \tag{7}$$

where

$$\Phi_0(\xi, \eta) = \frac{1}{2\pi z}\exp(j2\pi\nu z)\exp\left[j\frac{\pi\nu}{z}(\xi^2+\eta^2)\right], \tag{8}$$

while

$$R(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varkappa(x, y)\exp\left[j\frac{\pi\nu}{z}(x^2+y^2)\right]\exp\left[-j\frac{2\pi\nu}{z}(x\xi+y\eta)\right]dxdy. \tag{9}$$

3. Focusing plane phase sound sources

A plane sound source σ_0 will focus the radiated vibration energy in the focal point F located on the main axis $0z$ of the source at a distance f from its surface (Fig. 2), if mutual phase shifts of the acoustic potential of partial spherical waves produced

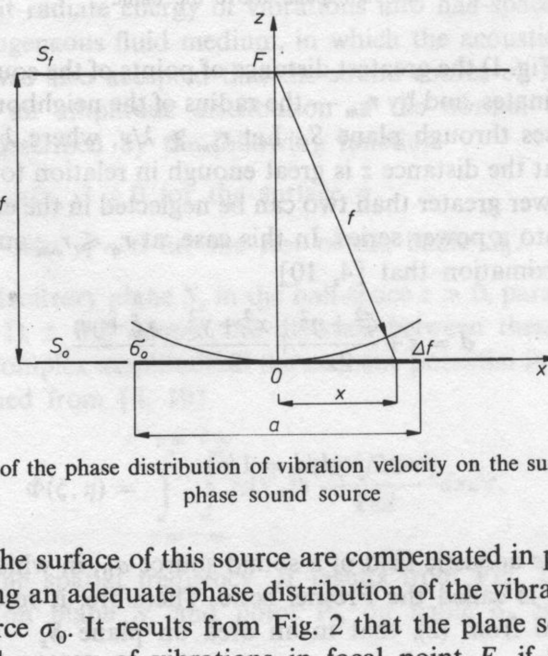

Fig. 2. Determination of the phase distribution of vibration velocity on the surface of a plane focusing phase sound source

by every point of the surface of this source are compensated in point F. This can be obtained by chosing an adequate phase distribution of the vibration velocity on the surface of the source σ_0. It results from Fig. 2 that the plane sound sporce σ_0 will focus the radiated energy of vibrations in focal point F, if the produced by it distribution of the complex amplitude of vibration velocity in the baffle S_0 will be determined as follows

$$\varkappa(x, y) = f(x, y)\exp[j\alpha(x, y)] \tag{10}$$

where function

$$f(x, y) \neq 0 \quad \text{for surface source } \sigma_0; \qquad f(x, y) = 0 \quad \text{for the rest of the surface of the baffle } S_0 \tag{11}$$

describes the amplitude distribution, while function

$$\alpha(x, y) = -2\pi\nu(\sqrt{f^2+x^2+y^2}-f) \tag{12}$$

describes the distribution of the phase of the vibration velocity in the baffle S_0. Let us accept that the focal point F is sufficiently distant from source σ_0, so $r_{om} \ll f$. In such a case when the root of expression (12) is expanded into a power series, all terms which have higher powers than second can be neglected. Hence we have

$$\varkappa(x, y) = f(x, y)\exp\left[-j\frac{\pi\nu}{f}(x^2+y^2)\right] \tag{13}$$

According to this and (7), the distribution of the complex amplitude of the acoustic potential in area (6) of plane S_z, produced by a focusing plane sound source σ_0 can be written as follows

$$\Phi(\xi, \eta) = \Phi_0(\xi, \eta)R(\xi, \eta), \tag{14}$$

where

$$\Phi_0(\xi, \eta) = \frac{1}{2\pi z}\exp(j2\pi\nu z)\exp\left[j\frac{\pi\nu}{z}(\xi^2+\eta^2)\right], \tag{15}$$

while

$$R(\xi, \eta) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f(x, y)\exp\left[-j\frac{\pi\nu}{w}(x^2+y^2)\right]\exp\left[-j\frac{2\pi\nu}{z}(x\xi+y\eta)\right]dxdy. \tag{16}$$

and

$$1/w = 1/f - 1/z. \tag{17}$$

4. The distribution of the amplitude of the acoustic potential in the focal plane

Let us determine the distribution of the amplitude of the acoustic potential in the region (6) of the focal plane (Fig. 2). In accordance with (9) and (13) we have

$$|\Phi(\xi, \eta)| = |\Phi_0(\xi, \eta)||R(\xi, \eta)|, \tag{18}$$

where

$$\Phi_0(\xi, \eta) = \frac{1}{2\pi f}\exp(j2\pi\nu f)\exp\left[j\frac{\pi\nu}{f}(\xi^2+\eta^2)\right] \tag{19}$$

and

$$R(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\exp\left[-j\frac{2\pi v}{f}(x\xi + y\eta)\right] dxdy. \tag{20}$$

It should be noticed that the right side of the above expression is analogic to a simple two-dimensional Fourier transform. Taking into consideration the spatial spectrum of the amplitude distribution of the vibration velocity in the baffle S_0 we have

$$F(v_x, v_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\exp[-j2\pi(xv_x + yv_y)]\, dxdy. \tag{21}$$

Hence, as a result of this and (20) we achieve

$$R(\xi, \eta) = F(v\xi/f,\ v\eta/f). \tag{22}$$

It results that the amplitude distribution of the acoustic potential in region (6) of the focal plane S_f determines the spatial spectrum of the amplitude distribution of vibration velocity in the baffle S_0. Phase sound source σ_0 under consideration produces (20) in focal point F an acoustic potential with an amplitude equal to

$$|\Phi_F| = F_0/(2\pi f), \tag{23}$$

where

$$F_0 = F(0, 0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\, dx\, dy \tag{24}$$

is the bulk efficiency of this source which is situated in baffle S_0.

Let us now assume that the focusing phase sound source σ_0 is of rectangular shape with sides: a and b (Fig. 1) and then let us determine the influence of the amplitude distribution of vibration velocity in the baffle S_0, which contains this source, on the amplitude distribution of the acoustic potential in region (6) of the focal plane S_f. The following distributions will be considered: unform, Hamming's, Hanning's and Blackman's. The influence of these distributions on the far field of a rectangular sound source has been analysed in paper [9]. For comparison let us also take into consideration the Gaussian distribution, which was analysed in papers [2, 3].

a) Uniform distribution. Let us accept that the amplitude distribution of vibration velocity in the baffle is expressed as follows (Fig. 3)

$$f(x, y) = \varkappa_0 f(x) f(y), \tag{25}$$

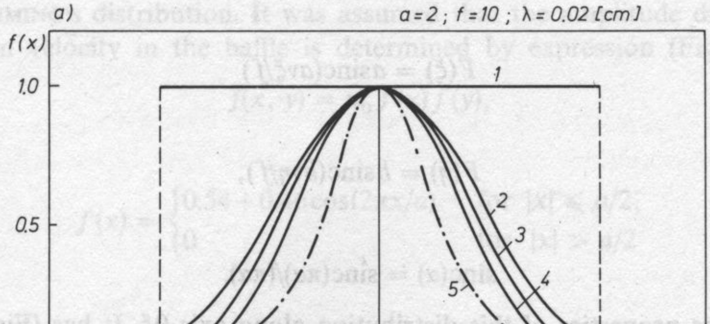

Fig. 3. Amplitude distributions a) and phase distributions b) of the vibration velocity on the surface of a focusing rectangular phase sound source: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S, 5 — Gaussian distribution

where

$$f(x) = \begin{cases} 1 & \text{for } |x| \leqslant a/2, \\ 0 & \text{for } |x| > a/2 \end{cases} \tag{26}$$

and

$$f(y) = \begin{cases} 1 & \text{for } |y| \leqslant b/2, \\ 0 & \text{for } |y| > b/2. \end{cases} \tag{27}$$

In this case the amplitude distribution of the acoustic potential in region (6) of the focal plane S_f has the following form

$$|\Phi(\xi, \eta)| = \frac{\varkappa_0}{2\pi f}|F(\xi)||F(\eta)|, \tag{28}$$

where

$$F(\xi) = a\,\mathrm{sinc}(av\xi/f) \tag{29}$$

and

$$F(\eta) = b\,\mathrm{sinc}(bv\eta/f), \tag{30}$$

while

$$\mathrm{sinc}(\alpha) = \mathrm{sinc}(\pi\alpha)/(\pi\alpha). \tag{31}$$

Let us analyse properties of this distribution along axis 0ξ. It has (Fig. 4) *a* main

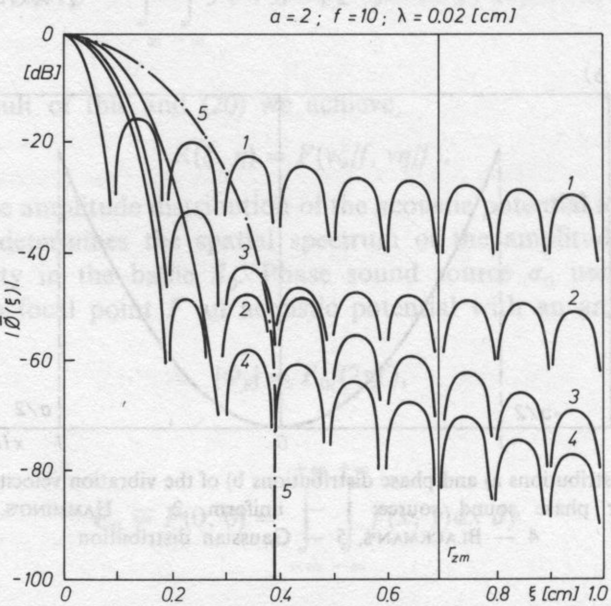

Fig. 4. Amplitude distribution of the acoustic potential in the focal point plane, produced by a rectangular focusing phase sound source with the following distributions: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S, 5 — GAUSSIAN

maximum in focal F equal to

$$|\Phi_F| = \varkappa_0 ab/(2\pi f) \tag{32}$$

and side maxima, which decrease with the increase of $|\xi|$ with the rate of 20 dB/decade. The width of the main maximum on level -3 dB is equal to

$$\Delta\xi = 0.9f/(av). \tag{33}$$

The highest side maximum is by 13 dB lower from the main maximum.

b) HAMMING'S distribution. It was assumed that the amplitude distribution of the vibration velocity in the baffle is determined by expression (Fig. 3)

$$f(x, y) = \varkappa_0 f(x) f(y), \tag{34}$$

where

$$f(x) = \begin{cases} 0.54 + 0.46\cos(2\pi x/a) & \text{for } |x| \leqslant a/2, \\ 0 & \text{for } |x| > a/2 \end{cases} \tag{35}$$

and

$$f(y) = \begin{cases} 0.54 + 0.46\cos(2\pi y/b) & \text{for } |y| \leqslant b/2, \\ 0 & \text{for } |y| > b/2. \end{cases} \tag{36}$$

In this case the amplitude distribution of the acoustic potential in the region (6) of the focal plane has the following form

$$|\Phi(\xi, \eta)| = \frac{\varkappa_0}{2\pi f}|F(\xi)||F(\eta)|, \tag{37}$$

$$F(\xi) = a[0.54\,\mathrm{sinc}(av\xi/f) + 0.23\,\mathrm{sinc}(av\xi/f - 1) + 0.23\,\mathrm{sinc}(av\xi/f + 1)] \tag{38}$$

and

$$F(\eta) = b[0.54\,\mathrm{sinc}(bv\eta/f) + 0.23\,\mathrm{sinc}(bv\eta/f - 1) + 0.23\,\mathrm{sinc}(bv\eta/f + 1)]. \tag{39}$$

This distribution has a main maximum in focal point F along the 0ξ axis (Fig. 4), equal to

$$|\Phi_F| = 0.54^2 \frac{\varkappa_0 ab}{2\pi f} \tag{40}$$

and side maxima, which decrease with the increase of $|\xi|$ with the rate of 20 dB/decade. The width of the main maximum on level -3 dB is equal to

$$\Delta\xi = 1.3f/(av). \tag{41}$$

The highest side maximum is by 42 dB lower than the main maximum.

c) HANNING'S distribution. Let us assume that the amplitude distribution of the vibration velocity in the baffle is determined as follows (Fig. 3)

$$f(x, y) = \varkappa_0 f(x) f(y), \tag{42}$$

where

$$f(x) = \begin{cases} 0.5 + 0.5\cos(2\pi x/a) & \text{for } |x| \leqslant a/2, \\ 0 & \text{for } |x| > a/2 \end{cases} \tag{43}$$

and

$$f(y) = \begin{cases} 0.5+0.5\cos(2\pi y/b) & \text{for } |y| \leqslant b/2, \\ 0 & \text{for } |y| > b/2. \end{cases} \tag{44}$$

In this case the amplitude distribution of the acoustic potential in region (6) of the focal plane has the following form

$$|\Phi(\xi, \eta)| = \frac{\varkappa_0}{2\pi f}|F(\xi)||F(\eta)|, \tag{45}$$

where

$$F(\xi) = a[0.5\operatorname{sinc}(av\xi/f)+0.25\operatorname{sinc}(av\xi/f-1)+0.25\operatorname{sinc}(av\xi/f+1)] \tag{46}$$

and

$$F(\eta) = b[0.5\operatorname{sinc}(bv\eta/f)+0.25\operatorname{sinc}(bv\eta/f-1)+0.25\operatorname{sinc}(bv\eta/f+1)]. \tag{47}$$

This distribution has a main maximum in focal point F along axis 0ξ (Fig. 4), equal to

$$|\Phi_F| = 0.5^2\frac{\varkappa_0 ab}{2\pi f} \tag{48}$$

and side maxima, which decrease with the increase of $|\xi|$ with the rate of 60 dB/decade. The width of the main maximum on the level of -3 dB is equal to

$$\Delta\xi = 1.4f/(av). \tag{49}$$

The highest side maximum is by 32 dB smaller from the main maximum.

d) BLACKMAN'S distribution. Let us assume that the amplitude distribution of te vibration velocity in the baffle is determined as follows (Fig. 3)

$$f(x, y) = \varkappa_0 f(x) f(y), \tag{50}$$

where

$$f(x) = \begin{cases} 0.42+0.5\cos(2\pi x/a)+0.08\cos(4\pi x/a) & \text{for } |x| \leqslant a/2, \\ 0 & \text{for } |x| > a/2 \end{cases} \tag{51}$$

and

$$f(y) = \begin{cases} 0.42+0.5\cos(2\pi x/b)+0.08\cos(4\pi y/b) & \text{for } |y| \leqslant b/2, \\ 0 & \text{for } |y| > b/2. \end{cases} \tag{52}$$

In this case the amplitude distribution of the acoustic potential in region (6) of the focal plane has the following form

$$|\Phi(\xi, \eta)| = \frac{\varkappa_0}{2\pi f}|F(\xi)||F(\eta)|, \tag{53}$$

where

$$F(\xi) = a[0.42\,\mathrm{sinc}(av\xi/f) + 0.25\,\mathrm{sinc}(av\xi/f - 1) + 0.25\,\mathrm{sinc}(av\xi/f + 1) + $$
$$+ 0.04\sin(av\xi/f - 2) + 0.04\sin(av\xi/f + 2)] \quad (54)$$

and

$$F(\eta) = b[0.42\,\mathrm{sinc}(bv\eta/f) + 0.25\,\mathrm{sinc}(bv\eta/f - 1) + 0.25\,\mathrm{sinc}(bv\eta/f + 1) + $$
$$+ 0.04\,\mathrm{sinc}(bv\eta/f - 2) + 0.04\,\mathrm{sinc}(bv\eta/f + 2)]. \quad (55)$$

This distribution has a main maximum in focal point F along the 0ξ axis (Fig. 4), equal to

$$|\Phi_F| = 0.42^2 \frac{\varkappa_0 ab}{2\pi f} \quad (56)$$

and side maxima, which decrease with the increase of $|\xi|$ with the rate of 34 dB/decade. The width of the main maximum on the level of -3 dB is equals

$$\Delta\xi = 2f/(av). \quad (57)$$

The highest side maximum is by 57 dB lower from the main maximum.

e) Gaussian distribution. Let us assume that the amplitude distribution of the vibration velocity in the baffle is determined as follows (Fig. 3)

$$f(x, y) = \varkappa_0 f(x) f(y), \quad (58)$$

where

$$f(x) = \exp(-x^2/2\sigma_x^2) \quad (59)$$

and

$$f(y) = \exp(-y^2/2\sigma_y^2). \quad (60)$$

For comparison, Fig. 4 presents the amplitude distribution of the acoustic along the 0ξ axis, produced by a focusing rectangular phase source with a Gaussian amplitude distribution of the vibration velocity. It was accepted that $a = 7.06\sigma_x$ and $b = 7.06\sigma_y$. For these values of parameters σ_x and σ_y the Gaussian distribution assumes neglectable values along the edge of the source [11].

From among all analysed distributions, the main maximum of the amplitude distribution of the acoustic potential in region (6) of the focal plane S_f has the smallest width for the uniform distribution and the highest for BLACKMAN'S amplitude distribution of vibration velocity. The width of the main maximum for a given distribution increases with the increase of the focal lenght f of source σ_0 and with the increase of wave length λ, while it decreases with the increase of the a and b dimensions of the source. Side maxima are damped to the greatest extent for BLACKMAN'S distribution, and to the smallest extent for the uniform distribution. The

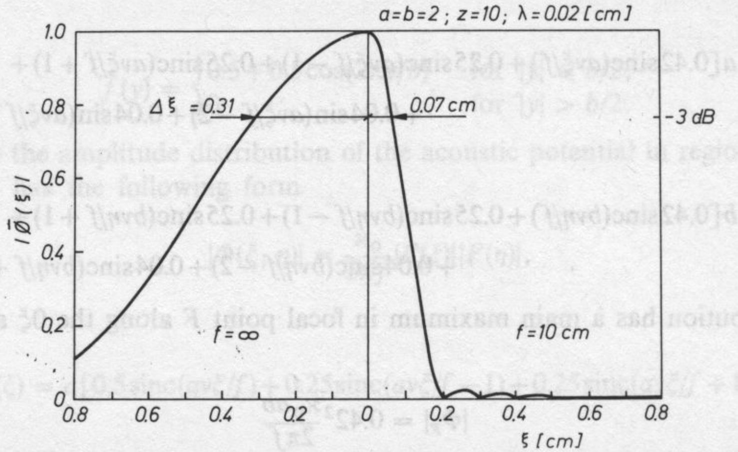

Fig. 5. A comparison between amplitude distribution of the acoustic potential at the same distance from a focusing and non-focusing rectangular sound source with HANNING'S amplitude distribution of the vibration velocity

HANNING'S distribution seems to be of greatest practical application. It makes it possible to locate sources in casings along its edge. For this amplitude distribution of vibration velocity, the amplitude distribution of the acoustic potential in the focal plane S_f has a relatively narrow main maximum and sufficiently strongly damped side maxima. For comarative reasons amplitude distributions of the acoustic potential produced in region (6) of the same plane S_z by a focusing and non-focusing rectangular sound with HANNING'S amplitude distribution of vibration velocity have been presented in Fig. 5.

5. Amplitude distribution of the acoustic potential along the main axis

Let us determine the amplitude distribution of the acoustic potential along the main axis $0z$ of a focusing rectangular phase source σ_0. Accepting in (14) that $\xi, \eta = 0$ we obtain

$$|\Phi(z)| = \frac{1}{2\pi z}|R(z)|, \tag{61}$$

where

$$R(z) = \int_{a/2}^{+a/2}\int_{b/2}^{+b/2} f(x, y)\exp\left[-j\frac{\pi\nu}{w}(x^2+y^2)\right]dxdy. \tag{62}$$

Amplitude distributions of the acoustic potential along the main axis of a focusing square phase source σ_0 with the following distributions of the amplitude of vibration velocity: uniform, HAMMING'S, HANNING'S and BLACKMAN'S were

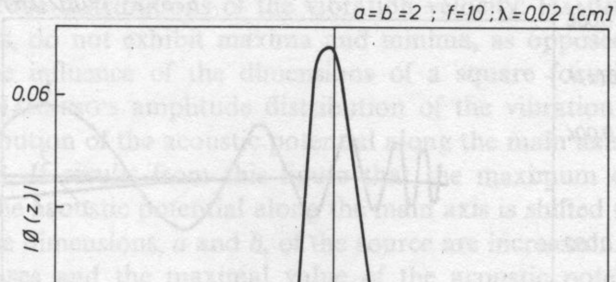

Fig. 6. Amplitude distributions of the acoustic potential along the main axis 0z of a focusing square phase sound source with the following distributions: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S

calculated on a minicomputer with the application of the trapezoid method of calculating the values of definite integrals. Results are presented in Fig. 6. It can be seen that at a uniform amplitude distribution of vibration velocity, the amplitude distribution of the acoustic potential along the 0z axis has a main maximum located at a small distance before the focal point F and is preceeded by a series of initial maxima and minima. As for HAMMING'S, HANNING'S and BLACKMAN'S distributions, the amplitude distribution of the acoustic potential along the main axis 0z has only one maximum shifted slightly further towards the source. The amplitude of the acoustic potential in this maximum acquires the highest value for HAMMING'S distribution and the lowest value for BLACKMAN'S distribution. The main maximum is shifted towards the source the most for BLACKMAN'S distribution. For comparison, Fig. 7 presents amplitude distributions of the acoustic potential along the main axis 0z for a non-focusing square sound source with the following amplitude distribution of the vibration velocity: uniform, HAMMING'S, HANNING'S and BLACKMAN'S. In this case amplitude distributions of the acoustic potential along the main axis for the

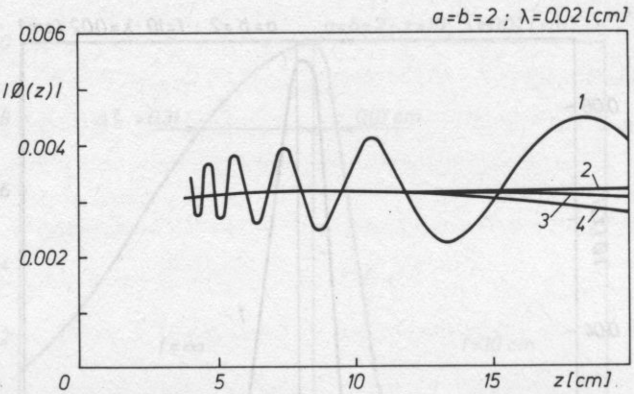

Fig. 7. Amplitude distribution of the acoustic potential along the main axis 0*z* of a non-focusing square sound source with the following distributions: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S

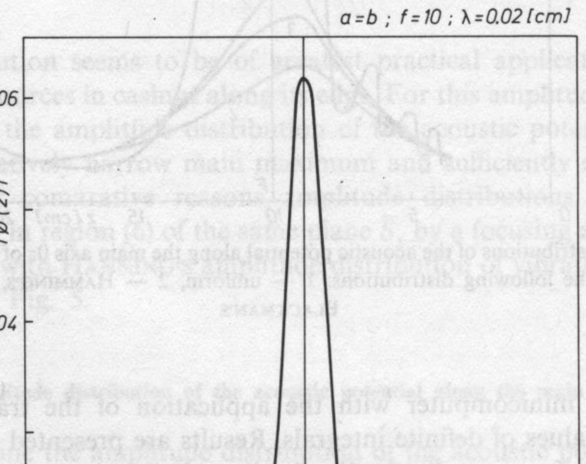

Fig. 8. The influence of dimensions of a focusing square phase sound source with HAMMING'S amplitude distribution of vibration velocity on the amplitude distribution of the acoustic potential along the main axis 0*z*

following amplitude distributions of the vibration velocity: HAMMING'S, HANNING'S and BLACKMAN'S, do not exhibit maxima and minima, as opposed to the uniform distribution. The influence of the dimensions of a square focusing phase sound source with a HANNING'S amplitude distribution of the vibration velocity, on the amplitude distribution of the acoustic potential along the main axis of this source is shown in Fig. 8. It results from this figure that the maximum of the amplitude distribution of the acoustic potential along the main axis is shifted towards the focal point F when the dimensions, a and b, of the source are increased. At the same time its width decreases and the maximal value of the acoustic potential increases.

6. Amplitude distributions of the acoustic potential in planes perpendicular to the main axis

The amplitude distribution of the acoustic potential in region (6) of an arbitrary plane $z \neq f$, perpendicular to the main axis $0z$ of a focusing rectangular phase sound source σ_0 according to (14) can be determined from expression

$$|\Phi(\xi, \eta)| = \frac{1}{2\pi z}|R(\xi, \eta)|, \tag{63}$$

where

$$R(\xi, \eta) = \int_{-a/2}^{+a/2}\int_{-b/2}^{+b/2} f(x, y)\exp\left[-j\frac{\pi v}{w}(x^2+y^2)\right]\exp\left[-j\frac{2\pi v}{z}(x\xi+\eta)\right]dxdy. \tag{64}$$

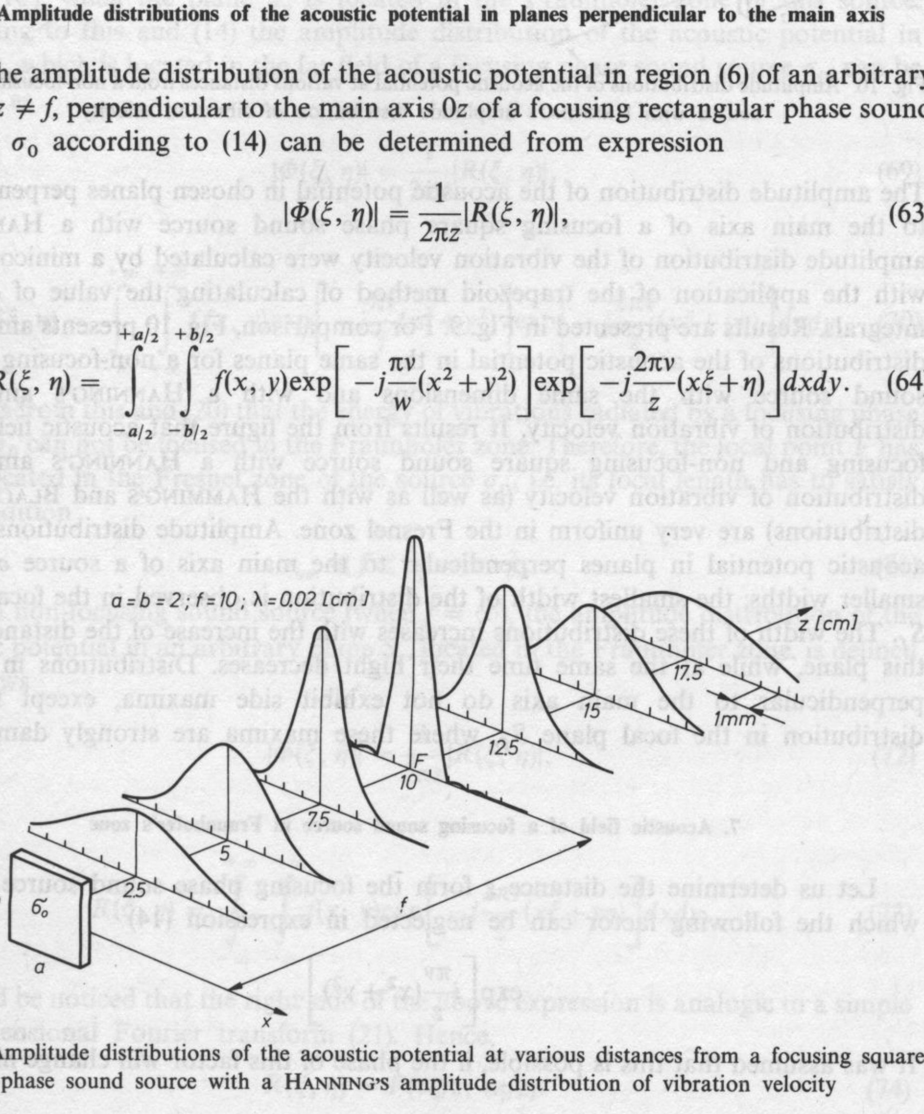

Fig. 9. Amplitude distributions of the acoustic potential at various distances from a focusing square phase sound source with a HANNING'S amplitude distribution of vibration velocity

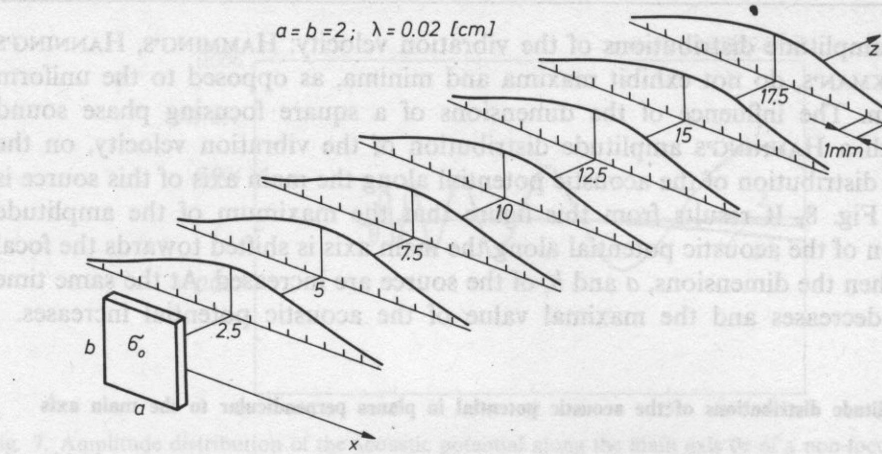

Fig. 10. Amplitude distributions of the acoustic potential at various distances from a non-focusing square source with HANNING'S amplitude distribution of vibration velocity

The amplitude distribution of the acoustic potential in chosen planes perpendicular to the main axis of a focusing square phase sound source with a HANNING'S amplitude distribution of the vibration velocity were calculated by a minicomputer with the application of the trapezoid method of calculating the value of definite integrals. Results are presented in Fig. 9. For comparison, Fig. 10 presents amplitude distributions of the acoustic potential in the same planes for a non-focusing square sound source with the same dimensions and with a HANNING'S amplitude distribution of vibration velocity. It results from the figure that acoustic fields of a focusing and non-focusing square sound source with a HANNING'S amplitude distribution of vibration velocity (as well as with the HAMMING'S and BLACKMAN'S distributions) are very uniform in the Fresnel zone. Amplitude distributions of the acoustic potential in planes perpendicular to the main axis of a source σ_0 have smaller widths; the smallest width of the distribution is observed in the focal plane S_f. The width of these distributions increases with the increase of the distance from this plane, while at the same time their hight decreases. Distributions in planes perpendicular to the main axis do not exhibit side maxima, except for the distribution in the focal plane S_f, where these maxima are strongly damped.

7. Acoustic field of a focusing sound source in Fraunhofer's zone

Let us determine the distance z form the focusing phase sound source σ_0, for which the following factor can be neglected in expression (14)

$$\exp\left[j\frac{\pi\nu}{z}(x^2+y^2)\right]. \tag{65}$$

It was assumed that this is possible, if the phase of this factor will change more less than one radian in the region of the source σ_0. Because

$$x^2+y^2 \leqslant r_{om}^2, \tag{66}$$

where r_{om} is the greatest distance between the points of the contour of the source and the origin of coordinates (Fig. 1); then factor (65) can be neglected when

$$\pi v r_{om}^2/z \ll 1. \tag{67}$$

Hence, at a distance from source σ_0 we have

$$z \gg z_g = \pi v r_{om}^2, \tag{68}$$

i.e. [4, 10], when the plane S_z is located in the Fraunhofer zone of this source. According to this and (14) the amplitude distribution of the acoustic potential in plane S_z, which is located in the far field of a focusing phase sound source σ_0, can be defined as

$$|\Phi(\xi, \eta)| = \frac{1}{2\pi z}|R(\xi, \eta)|, \tag{69}$$

where

$$R(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\exp\left[-j\frac{\pi v}{f}(x^2+y^2)\right]\exp\left[-j\frac{2\pi v}{z}(x\xi+y\eta)\right]dxdy. \tag{70}$$

It results from this and (20) that the energy of vibrations radiated by a focusing phase source σ_0 can not be focused in the Fraunhofer zone. Therefore, the focal point F has to be located in the Fresnel zone of the source σ_0, i.e. its focal length has to satisfy the condition

$$r_{om} \ll f \leqslant z_g = \pi v r_{om}^2. \tag{71}$$

As for a non-focusing sound source (when $f = \infty$), the amplitude distribution of the acoustic potential in an arbitrary plane S_z, located in the Fraunhofer zone, is defined as follows

$$|\Phi(\xi, \eta)| = \frac{1}{2\pi z}|R(\xi, \eta)|, \tag{72}$$

where

$$R(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\exp\left[-j\frac{2\pi v}{z}(x\xi+y\eta)\right]dxdy. \tag{73}$$

It should be noticed that the right side of the above expression is analogic to a simple two-dimensional Fourier transform (21). Hence,

$$R(\xi, \eta) = F(v\xi/z, v\eta/z). \tag{74}$$

Therefore, the amplitude distribution of the acoustic potential in region (6) of an arbitrary plane S_z, located in the Fraunhofer zone of a non-focusing sound source σ_0, is defined by the spatial spectrum of the amplitude distribution of the vibration velocity in the baffle S_0, produced by this source. Focusing properties of a source become negligible with the increase of the distance from the source; the complex character of radiation of the source vanishes and the field produced by it becomes more similar to the field produced by plane sound wave.

8. Conclusions

On the basis of the above discussion, it can be stated that in the Fresnel zone acoustic fields of focusing and non-focusing rectangular sound sources with the amplitude distributions of vibration velocity: HAMMING'S, HANNING'S and BLACKMAN'S, are very uniform, very much like acoustic fields in this zone of focusing and non-focusing circular sound sources with Gaussian amplitude distributions of vibration velocity [2, 3]. Amplitude distributions of the acoustic potential along the main axis of a focusing rectangular phase sound source with HAMMING'S, HANNING'S and BLACKMAN'S amplitude distributions of vibration velocity, have only one maximum located near the focal point. This maximum has the greatest value in the case of HAMMING'S distribution. The value of the maximum increases and is shifted towards the focal point when the dimensions of the source are increased. Maxima of amplitude distributions of the acoustic potential in planes perpendicular to the main axis of a focusing and non-focusing rectangular sound source are located along this axis and in the Fresnel zone of a focusing sound source these maxima are much narrower. The main maximum in the focal plane has the smallest width and the highest value. This maximum is accompanied by side maxima, strongly damped in the case of HAMMING'S, HANNING'S and BLACKMAN'S distribution. From among all analysed distributions, the width of the main maximum was smallest for the uniform distribution and largest for BLACKMAN'S distribution. However, they are much more narrow than the width of the main maximum of the amplitude distribution of the acoustic potential in the focal plane, produced by a focusing sound source with a Gaussian amplitude distribution of the vibration velocity. It seems possible to use a focusing square phase sound source with adequately great dimensions with respect to the wave length (e.g. $a/\lambda = 200$) and HANNING'S amplitude distribution of the vibration velocity, in ultrasonic diagnostic systems in order to increase their transverse resolving power. Such a source can be realized in practice in the form of a phase system of piezoelectric transducers with a discrete HANNING'S distribution of their bulk efficiencies. The desired efficiency distribution of individual transducers and adequate phases of vibrations can be obtained by a suitable selection of amplitudes and reciprocal phase shifts of input signals. Yet a more detailed analysis of this problem requires a separate paper.

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