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PROPAGATION VELOCITY OF A VIBRATION VELOCITY WAVE, I.E. ACOUSTIC VELOCITY WAVE

ROMAN WYRZYKOWSKI

Institute of Physics, Pedagogic High School in Rzeszów
(35–310 Rzeszów, ul. Rejtana 16a)

In this paper the author proves that the propagation velocity of an acoustic velocity wave in the near field differs from the velocity of a pressure wave, while both differ from c_0 in d'Alambert's equation. The velocity of an acoustic velocity wave was calculated for a point source, for a cylindrical source of zero order and for a circular piston and ring in a baffle.

1. Introduction

In accordance with papers [8, 9], an arbitrary tensor physical quantity $p_{ijk...}$ which propagates in the form of a harmonic wave, can be noted as follows

$$p_{ijk...} = A_{ijk...}(x_i)e^{i[\omega t - f(x_i)]} \tag{1}$$

where $A_{ijk...}(x_i)$ is the amplitude in terms of position, and $f(x_i)$ represents the so-called wave front. The wave propagation condition is [4, 5, 7]:

$$\omega t - f(x_i) = \text{const} \tag{2}$$

what leads to an expression for the local velocity (velocity dependent on the position of the point in the acoustic field)

$$c = \omega/|\text{grad} f|. \tag{3}$$

If we write (1) for an acoustic pressure wave:

$$p = P_0(x_i)e^{i[\omega t - f(x_i)]} \tag{4}$$

which propagates with a velocity given in formula (3), then we can easily prove that in a general case a vibration velocity wave propagates with a different velocity. As we know [3–6], the vibration velocity, called also the acoustic velocity, is related to the acoustic pressure by Euler's equation, which has the following form the a harmonic wave

$$u = \frac{i}{\omega \varrho_0} \operatorname{grad} p \tag{5}$$

where ϱ_0 is the rest desity of the medium.

Both, the amplitude and the phase, are differentiated when the gradient of expression (4) is calculated. If we convert the obtained result into a form analogic to formula (1) we have:

$$u = U_0(x_i) e^{i[\omega t - f_1(x_i)]}. \tag{6}$$

Hence, when a pressure wave propagates with velocity (3) (where $f(x_i)$ will be the value of the wave front function from formula (4)), the velocity wave propagates with velocity

$$c_u = \omega / |\operatorname{grad} f_1| \tag{7}$$

where $f_1(x_i)$ is a different function — the function of the wave front of an acoustic velocity wave from formula (6).

It will be shown below that c_u differs from c for all waves (except plane waves) only at relatively small distances, when the local velocity of a pressure wave c differs from the material velocity c_0.

M. Kwiek [3] calculated the propagation velocity of a velocity wave for a point source, considering this as a special case and neglecting the generality of the problem. His calculation procedure is given in paragraph 2.

2. Propagation velocity of a velocity wave in the field of a point source

This example has been chosen purposely, because as we know the acoustic pressure wave of a point source is an elementary spherical wave, which propagates with a constant velocity, in paper [7] called the material velocity. The behaviour of the propagation velocity of a velocity wave in such a case is very interesting. The acoustic pressure at a distance r from the point source [6] equals

$$p = (A/r) e^{i(\omega t - k_0 r)} \qquad k_0 = \omega / c_0 \tag{8}$$

of course a point source is an abstract source, but it can be replaced in practice by a very small pulsating sphere.

Applying Euler's equation (5) in (8) we obtain the acoustic velocity

$$u = \frac{1}{\varrho c_0}\left(1 + \frac{1}{ik_0 r}\right)\frac{A}{r} e^{i(\omega t - k_0 r)}. \tag{9}$$

By separating the absolute value and the phase we bring formula (9) to a form analogic to (1)

$$u = \frac{A}{\varrho c_0 r}\sqrt{1 + \frac{1}{(k_0 r)^2}}\, e^{i\left(\omega t - k_0 r - \operatorname{tg}^{-1}\frac{1}{k_0 r}\right)} \tag{10}$$

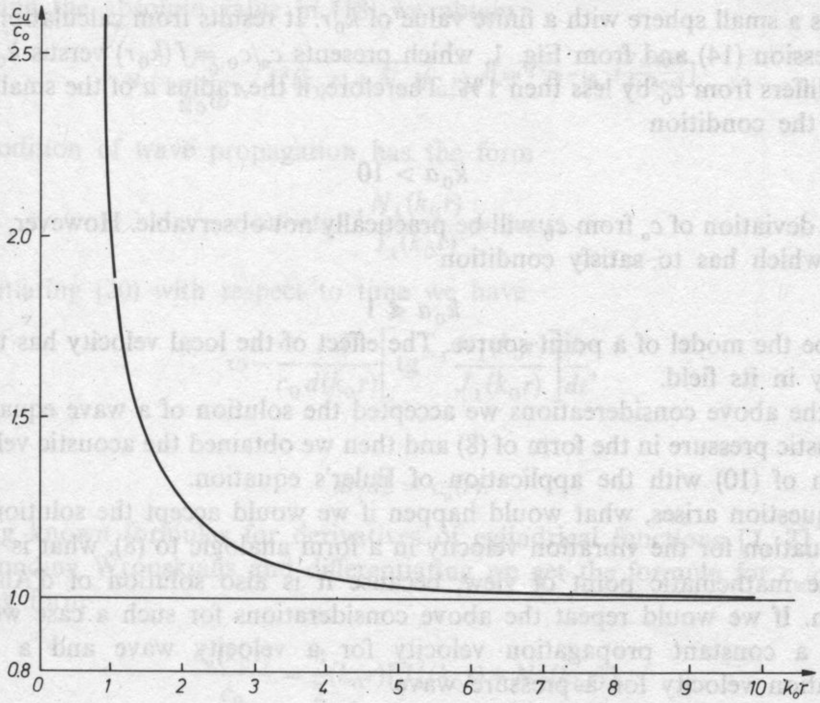

Fig. 1. c_u/c_0 versus $k_0 r$ for a point source

where tg^{-1} denotes arctg. The condition of wave propagation requires r and t to change in such a manner so the total phase remains constant, thus

$$\omega t - k_0 r - \mathrm{tg}^{-1}\frac{1}{k_0 r} = \mathrm{const}. \tag{11}$$

Differentiating both sides of (11) with respect to time we have

$$\omega - k_0 \cdot \frac{dr}{dt} + \frac{k_0}{1+\left(\frac{1}{k_0 r}\right)^2} \cdot \frac{1}{(k_o r)^2} \cdot \frac{dr}{dt} = 0. \tag{12}$$

Since

$$dr/dt = c_u \tag{13}$$

thus finally

$$c_u(k_0 r)/c_0 = [1+(k_0 r)^2]/(k_0 r)^2, \tag{14}$$

when $k_0 r \to 0$, $c_u/c_0 \to \infty$: This is not surprising, because the acoustic pressure p (8) and the vibration velocity (10) exhibits singularity at $r = 0$. Therefore, it is also not surprising that the propagation velocity c_u exhibits singularity in this point as well. Whereas, when $k_0 r \to \infty$ we have according to expactations $c_u \to c_0$. In practice there is always a small sphere with a finite value of $k_0 r$. It results from calculations based on expression (14) and from Fig. 1, which presents $c_u/c_0 = f(k_0 r)$ versus $k_0 r > 10$, that c_u differs from c_0 by less then 1%. Therefore, if the radius a of the small sphere satisfies the condition

$$k_0 a > 10 \tag{15}$$

then the deviation of c_u from c_0 will be practically not observable. However, a small sphere, which has to satisfy condition

$$k_0 a \ll 1 \tag{16}$$

should be the model of a point source. The effect of the local velocity has to occur distinctly in its field.

In the above considereations we accepted the solution of a wave equation for the acoustic pressure in the form of (8) and then we obtained the acoustic velocity in the form of (10) with the application of Euler's equation.

A question arises, what would happen if we would accept the solution of the wave equation for the vibration velocity in a form analogic to (8), what is possible from the mathematic point of view, because it is also solution of d'Alambert's equation. If we would repeat the above considerations for such a case we would achieve a constant propagation velocity for a velocity wave and a variable propagation velocity for a pressure wave.

It should be mentioned that this problem can not be solved with d'Alambert's and Euler's equations solely. Formula (8) for pressure was also obtained with the application of a different method, by differentiating Green's function, which has a definite physical interpretation.

3. Propagation velocity of an acoustic velocity wave in the field of a cylinder for a zero order wave

The acoustic pressure for a cylindrical wave of zero order is expressed by formula [4–6]

$$p = P_0[J_0(k_0 r) - iN_0(k_0 r)]e^{i\omega t}, \tag{17}$$

where P_0 denotes the pressure amplitude, which can be determined from the boundary condition on the surface of the cylinder (source); $J_0(\,)$ and $N_0(\,)$ are zero order Bessel and Neuman functions, respectively; $k_0 = \omega/c_0$. According to Euler's equation (5) the acoustic velocity equals [6, 2]

$$u = \frac{P_0}{\varrho_0 \omega} e^{i(\omega t + \frac{3}{2}\pi)} [J_1(k_0 r) - iN_1(k_0 r)], \tag{18}$$

where $J_1(\,)$ and $N_1(\,)$ are first order Bessel and Neuman functions, respectively.

Separating the absolute value in (18) we obtain

$$u = \frac{P_0}{\varrho_0 \omega}\sqrt{J_1^2(k_0 r)+N_1(k_0 r)}e^{i\left[\omega t+\frac{3}{2}\pi-\mathrm{tg}^{-1}\frac{N_1(k_0 r)}{J_1(k_0 r)}\right]}. \tag{19}$$

The condition of wave propagation has the form

$$\omega t-\mathrm{tg}^{-1}\frac{N_1(k_0 r)}{\mathrm{J}_1(k_0 r)} = \text{const.} \tag{20}$$

Differentiating (20) with respect to time we have

$$\omega-\frac{\omega}{c_0}\frac{d}{d(k_0 r)}\left[\mathrm{tg}^{-1}\frac{N_1(k_0 r)}{J_1(k_0 r)}\right]\frac{dr}{dt}, \tag{21}$$

where

$$dr/dt = c_u(r). \tag{22}$$

Applying known formulas for derivatives of cylindrical functions [1, 2] and for corresponding Wronskians and differentiating we get the formula for c_u/c_0 in the following form

$$\frac{c_u(k_0 r)}{c_0} = \frac{\pi}{2}(k_0 r)[J_1^2(k_0 r)+N_1^2(k_0 r)]. \tag{23}$$

When $k_0 r\to\infty$, in accordance with asymptotic formulae for cylindrical functions [1, 2] we have

$$J_1^2(k_0 r)+N_1^2(k_0 r) = 2/(\pi k_0 r) \qquad k_0 \gg 1 \tag{24}$$

and then we get from (23)

$$c_u/c_0 = 1 \qquad k_0 r \gg 1. \tag{25}$$

Whereas for small $k_0 r \ll 1$ we have

$$J_1(k_0 r) = 0 \qquad k_0 r \ll 1, \tag{26}$$

$$N_1(k_0 r) = \frac{1}{\pi}\frac{2}{k_0 r} \tag{27}$$

and from (23)

$$c_u/c_0 \to \infty \qquad k_0 r \to 0. \tag{28}$$

Of course $k_0 r = 0$ is only a mathematical limit without physical sense and (28) proves inly that the local velocity c_u exhibits singularity for $k_0 r = 0$.

Fig. 2 presents the function $c_u/c_0 = f(k_0 r) c_u$ differs from c_0 by less than 1% for values of $k_0 r$ higher than $k_0 r = 7$. Therefore, if a cylinder which radiates a zero order wave has a radius, a, which satisfies the condition $k_0 a > 7$, then the local velocity effect of a velocity wave practically will not occur. Whereas, for $k_0 a < 7$ this velocity

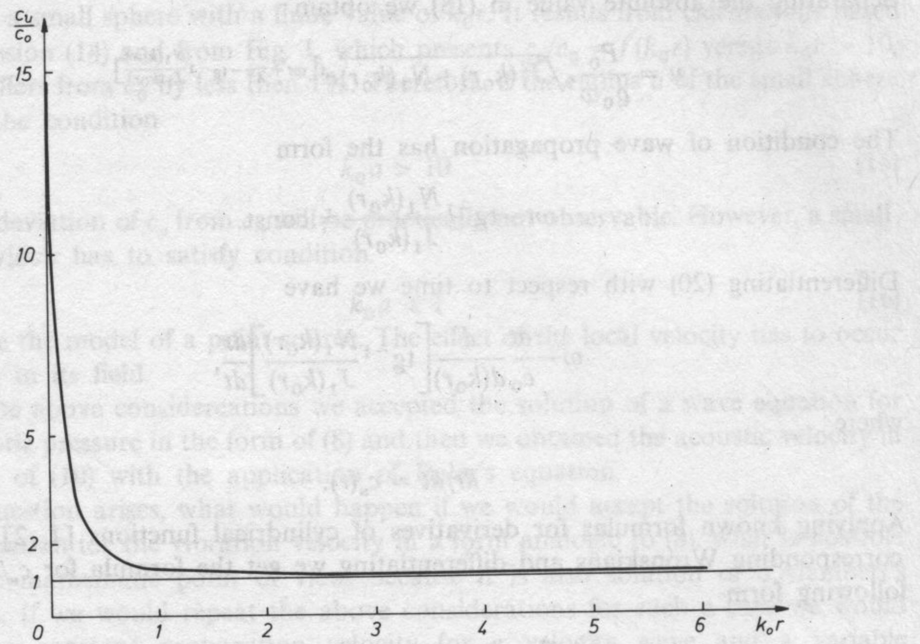

Fig. 2. c_u/c_0 versus $k_0 r$ for a cylindrical wave of zero order

will be reduced with the distance from the source to the value c_0, but this velocity will differ from the propagation velocity of a pressure wave. It has been shown in paper [7] that the velocity of a pressure wave increases from 0 to c_0.

4. Phase velocity of an acoustic velocity wave on the axis of symmetry of the field produced by a circular piston in a rigid baffle

As in papers [7, 8], we will consider a circular piston with radius a vibrating with a constant amplitude of vibration velocity u_0, and situated in an infinite plane rigid baffle. Axis z drawn from the center of the piston perpendicularily to its surface is identical with the axis of symmetry of the obtained acoustic field. The propagation velocity of a velocity wave (c_u) was calculated with the application of a formula for the acoustic pressure on the axis z for the field, given in a compact form by STENZEL [5, 6].

It to simplify the notation we will accept the formula given by STENZEL [5, 6] for the relative acoustic pressure

$$p_w = \frac{p}{\varrho_0 c_0 u_0} \doteq 2\sin\left[\frac{k_0}{2}(\sqrt{a^2+z^2}-z)\right] e^{i\left[\omega t+\frac{\pi}{2}-\frac{k_0}{2}(\sqrt{a^2+z^2}+z)\right]}. \tag{29}$$

where ϱ_0 is the rest density of the medium, u_0 is the amplitude of vibration velocity, which is constant on the source, $k_0 = \omega/c_0$.

In accordance to Euler's equation (5) we have

$$u = \frac{1}{\omega\varrho} e^{i\frac{\pi}{2}} \frac{dp}{dz} = U_0 \frac{dp_w}{dz}. \tag{30}$$

Further calculations are simplified by the fact that we only need the expression for the phase of u in order to determine the formula for the propagation velocity c_u.

After differentiating (29) with respect of z, we see that the total velocity phase can be expressed by

$$\varphi(z, t) = \omega t - \frac{k_0}{2}(\sqrt{a^2+z^2}+z) + \mathrm{tg}^{-1}\left\{\mathrm{tg}\left[\frac{k_0}{2}(\sqrt{a^2+z^2}-z)\right]\frac{\sqrt{a^2+z^2}+z}{\sqrt{a^2+z^2}-z}\right\} \tag{31}$$

and the condition for the propagation of a velocity wave has the following form

$$\varphi(z, t) = \text{const}. \tag{32}$$

In order to simplify formulas used in the further part of the paper we will denote

$$\varphi(z, t) = \omega t - F(z), \tag{33}$$

where

$$F(z) = \frac{k_0}{2}(\sqrt{a^2+z^2}-z) - \mathrm{tg}^{-1}\left\{\mathrm{tg}\left[\frac{k_0}{2}\sqrt{a^2+z^2}-z\right]\frac{\sqrt{a^2+z^2}+z}{\sqrt{a^2+z^2}-z}\right\}. \tag{34}$$

Differentiating both sides of (32) with respet to time and substituting φ (z, t) in the form given in (33), we obtain

$$\omega - (dF(z)/dz)(dz/dt) = 0, \tag{35}$$

where

$$dz/dt = c_u. \tag{36}$$

Therefore, from (35) we have

$$c_u = \frac{\omega}{dF(z)/dz}. \tag{37}$$

The following notation simplifications were introduced

$$dF/dz = (k_0/2)F_1(z), \tag{38}$$

hence

$$c_u(z)/c_0 = 2/F_1(z), \tag{39}$$

where $F_1(z)$ equals

$$F_1(z) = \frac{\frac{z}{a} + \sqrt{1+\left(\frac{z}{a}\right)^2}}{\sqrt{1+\left(\frac{z}{a}\right)^2}} - \frac{\left[\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right]^2}{\left[\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right]^2 + \mathrm{tg}^2\left[\frac{k_0 a}{2}\left(\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right)\right]\left(\sqrt{1+\left(\frac{z}{a}\right)^2} + \frac{z}{a}\right)^2} \times$$

$$\times \left\{ \frac{2\,\mathrm{tg}\left[\frac{k_0 a}{2}\left(\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right)\right]}{\frac{k_0 a}{2}\left[\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right]^2 \sqrt{1+\left(\frac{z}{a}\right)^2}} - \left\{1 + \mathrm{tg}^2\left[\frac{k_0 a}{2}\left(\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right)\right]\right\} \frac{\sqrt{1+\left(\frac{z}{a}\right)^2} + \frac{z}{a}}{\sqrt{1+\left(\frac{z}{a}\right)^2}} \right\}. \tag{40}$$

For a specific case, when $z/a = 0$, we achieve the following expression

$$F_1(0) = 2\left\{1 - \frac{\mathrm{tg}\left(\frac{k_0 a}{2}\right)}{\frac{k_0 a}{2}\left[1 + \mathrm{tg}^2\left(\frac{k_0 a}{2}\right)\right]}\right\} = 2\left[1 - \frac{\sin(k_0 a)}{k_0 a}\right] \tag{41}$$

and from (27) we have

$$\frac{c_u(0)}{c_u} = \frac{1 + \mathrm{tg}^2\left(\frac{k_0 a}{2}\right)}{1 + \mathrm{tg}^2\left(\frac{k_0 a}{2}\right) - \mathrm{tg}\left(\frac{k_0 a}{2}\right)\Big/\frac{k_0 a}{2}}. \tag{42}$$

At $z = 0$ the value of $c_u(0)$ depends on $k_0 a$. For $k_0 a = 0$ (limiting case without physical sense) we would have $c_u \to \infty$. When $k_0 a \to \infty$, $c_u \to c_0$, in spite of the periodicity of function $\mathrm{tg}(k_0 a/2)$. Fig. 3 presents the dependence $c_u(0)/c_0 = f(k_0 a)$.

As for the full expression (40) we can see that when $z/a \to \infty$, then from formula (27) we obtain at the limit $F_1(\infty) = 2$ and from (27).

$$c_u(\infty)/c_0 = 1. \tag{43}$$

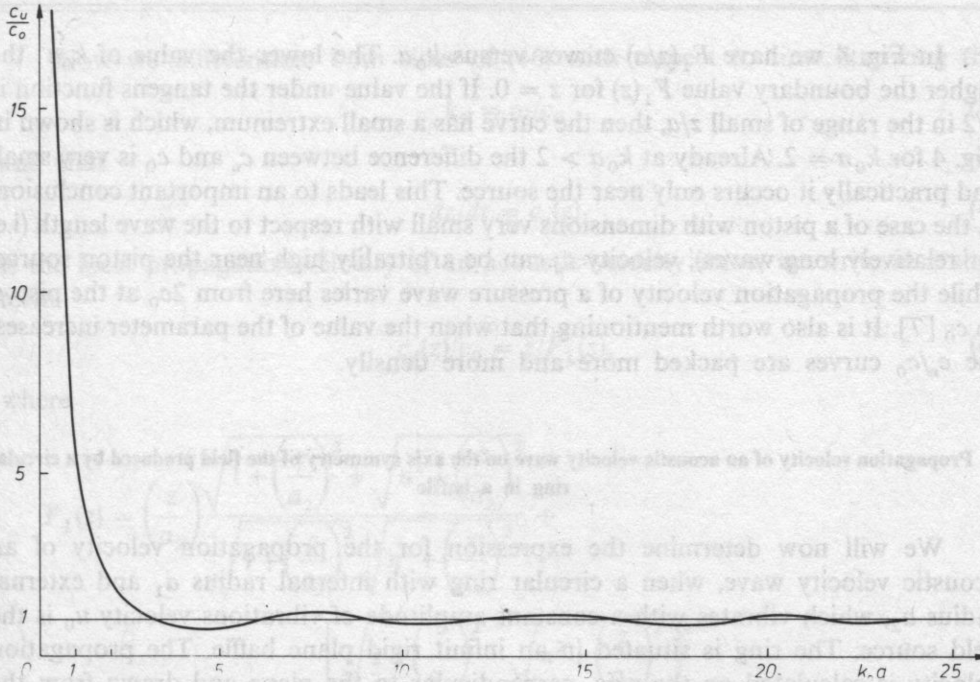

Fig. 3. c_u/c_0 versus k_0a at the center of a circular piston

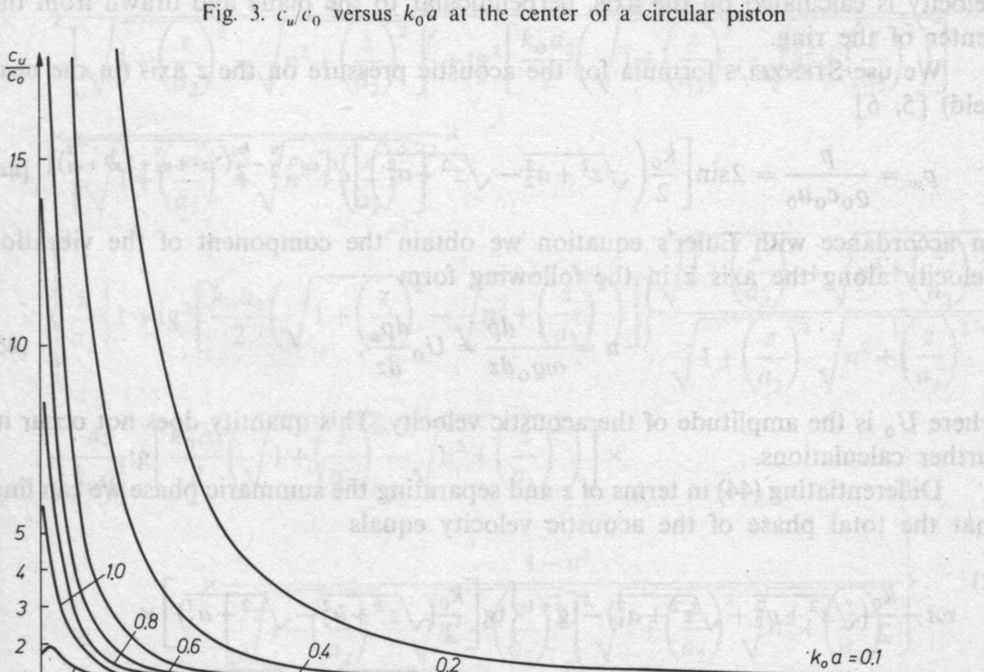

Fig. 4. c_u/c_0 versus z/a for a circular piston for different values of k_0a

In Fig. 4 we have $F_1(z/a)$ curves versus k_0a. The lower the value of k_0a the higher the boundary value $F_1(z)$ for $z = 0$. If the value under the tangens function is $\pi/2$ in the range of small z/a, then the curve has a small extremum, which is shown in Fig. 4 for $k_0a = 2$. Already at $k_0a > 2$ the difference between c_u and c_0 is very small and practically it occurs only near the source. This leads to an important conclusion; in the case of a piston with dimensions very small with respect to the wave length (i.e. for relatively long waves), velocity c_u can be arbitralily high near the piston source, while the propagation velocity of a pressure wave varies here from $2c_0$ at the piston to c_0 [7]. It is also worth mentioning that when the value of the parameter increases, the c_u/c_0 curves are packed more and more densily.

5. Propagation velocity of an acoustic velocity wave on the axis symmetry of the field produced by a circular ring in a baffle

We will now determine the expression for the propagation velocity of an acoustic velocity wave, when a circular ring with internal radius a_1 and external radius a_2, which vibrates with a constant amplitude of vibrations velocity u_0 is the field source. The ring is situated in an infinit rigid plane baffle. The propagation velocity is calculated on the axis, perpendicular to the plane and drawn from the center of the ring.

We use STENZEL'S formula for the acoustic pressure on the z axis (in the near field) [5, 6]

$$p_w = \frac{p}{\varrho_0 c_0 u_0} = 2\sin\left[\frac{k_0}{2}\left(\sqrt{z^2+a_2^2}-\sqrt{z^2+a_1^2}\right)\right]e^{i\left[\omega t+\frac{\pi}{2}-\frac{k_0}{2}\left(\sqrt{z^2+a_2^2}+\sqrt{z^2+a_1^2}\right)\right]} \quad (44)$$

In accordance with Euler's equation we obtain the component of the vibration velocity along the axis z in the following form

$$u = \frac{i}{\omega\varrho_0}\frac{dp}{dz} = U_0\frac{dp_w}{dz}, \quad (45)$$

where U_0 is the amplitude of the acoustic velocity. This quantity does not occur in further calculations.

Differentiating (44) in terms of z and separating the summaric phase we can find that the total phase of the acoustic velocity equals

$$\omega t-\frac{k_0}{2}\left(\sqrt{z^2+a_2^2}+\sqrt{z^2+a_1^2}\right)-\mathrm{tg}^{-1}\left\{\mathrm{tg}\left[\frac{k_0}{2}\left(\sqrt{z^2+a_2^2}-\sqrt{z^2+a_1^2}\right)\right]\times\right.$$
$$\left.\times\frac{\sqrt{z^2+a_2^2}+\sqrt{z^2+a_1^2}}{\sqrt{z^2+a_2^2}+\sqrt{z^2+a_1^2}}\right\} = \text{const.} \quad (46)$$

The constancy of the total phase of acoustic velocity is the condition for wave propagation.

Now we differentiate both sides of (46) with respect to time, knowing that

$$k_0 = \omega/c_0 \tag{47}$$

and that

$$dz/dt = c_u(z) \tag{48}$$

is the local propagation velocity of an acoustic velocity wave, we write in a short form:

$$c_u(z)/c_0 = z/F_1(z), \tag{49}$$

where

$$F_1(z) = \left(\frac{z}{a_2}\right)\frac{\sqrt{1+\left(\frac{z}{a_2}\right)^2}+\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}}{\sqrt{1+\left(\frac{z}{a_2}\right)^2}\sqrt{n^2+\left(\frac{z}{a_1}\right)^2}}+$$

$$+\frac{\left[\sqrt{1+\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right]^2}{\left[\sqrt{1+\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right]^2+\mathrm{tg}^2\left[\frac{k_0 a_2}{2}\left(\sqrt{1+\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right)\right]}\times$$

$$\times\frac{}{\left[\sqrt{1+\left(\frac{z}{a_2}\right)^2}+\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right]^2}\times$$

$$\times\left\{\frac{z}{a_2}\left\{1+\mathrm{tg}^2\left[\frac{k_0 a_2}{2}\left(\sqrt{1+\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right)\right]\right\}\frac{\sqrt{1+\left(\frac{z}{a_2}\right)^2}+\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}}{\sqrt{1+\left(\frac{z}{a_2}\right)^2}\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}}+\right.$$

$$+\frac{4z}{k_0 a_2^2}\mathrm{tg}\left[\frac{k_0 a_2}{2}\left(\sqrt{1+\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_1}\right)^2}\right)\right]\times$$

$$\left.\times\frac{1-n^2}{\left[\sqrt{1-\left(\frac{z}{a_2}\right)^2}-\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\right]^2\sqrt{1+\left(\frac{z}{a_2}\right)^2}\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}}\right\}. \tag{50}$$

In formula (50) we denoted

$$a_1 = n\,a_2 \quad n < 1. \tag{51}$$

For $n = 0$ the ring changes into a circular piston and formula (50) is transformed into formula (40).

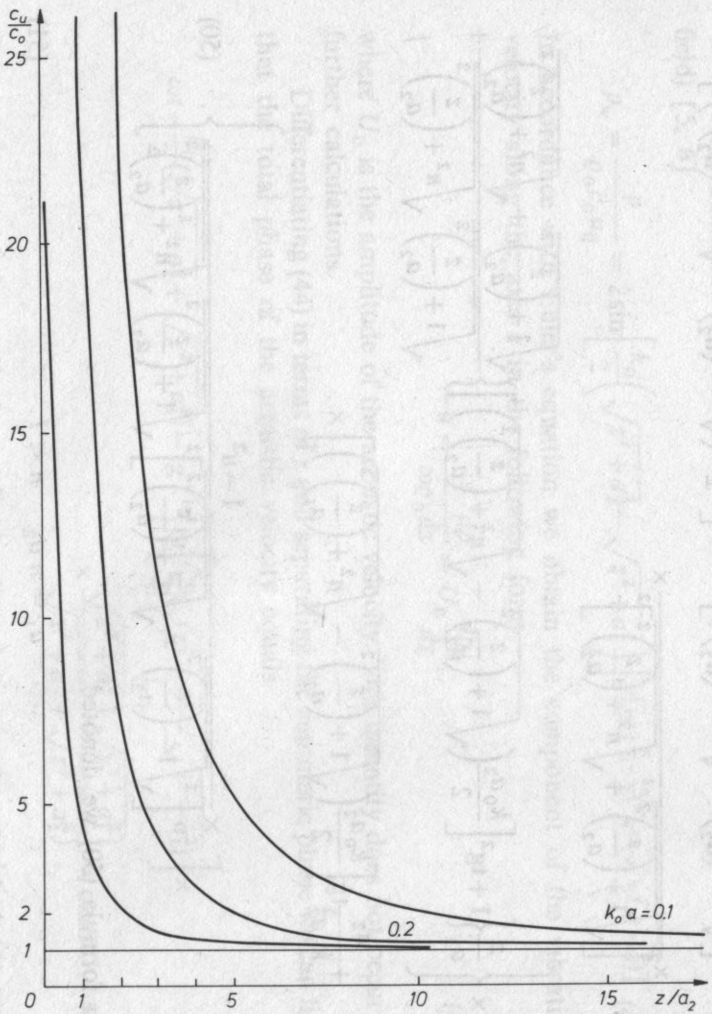

Fig. 5. c_u/c_0 versus z/a_2 for a circular ring, for $k_0 a = 0.1$ and 0.2

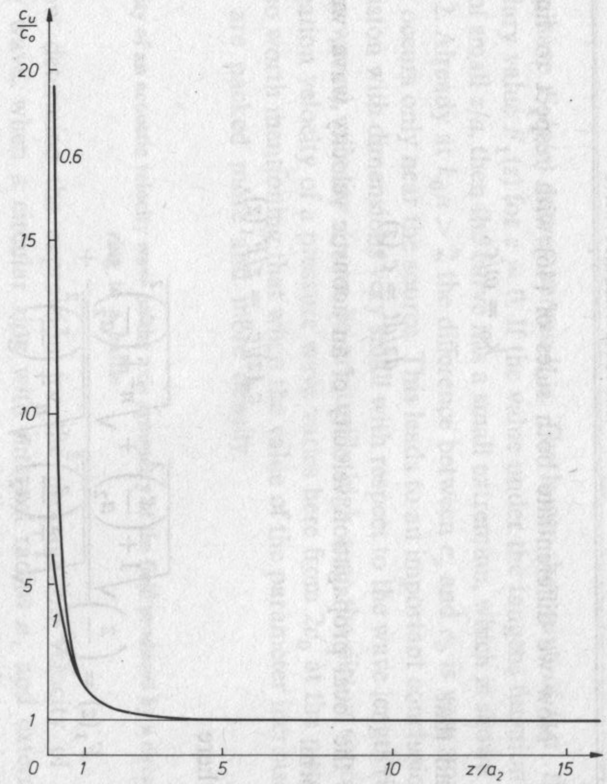

Fig. 6. c_u/c_0 versus z/a_2 for a circular ring for $k_0 a = 1$

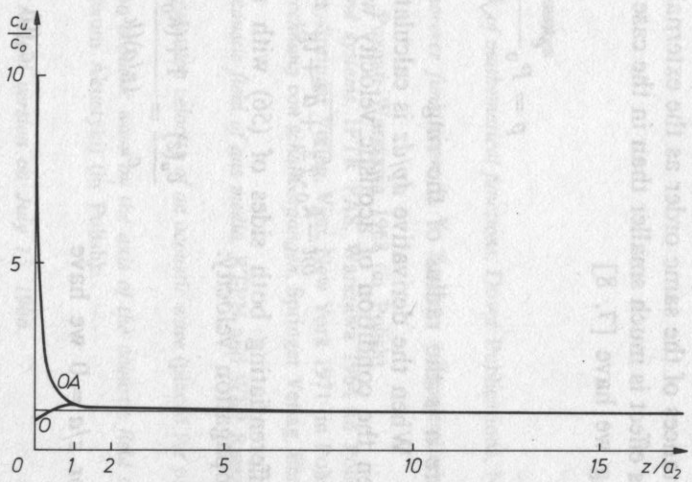

Fig. 7. c_u/c_0 versus z/a_2 for a circular ring for $k_0a = 5$

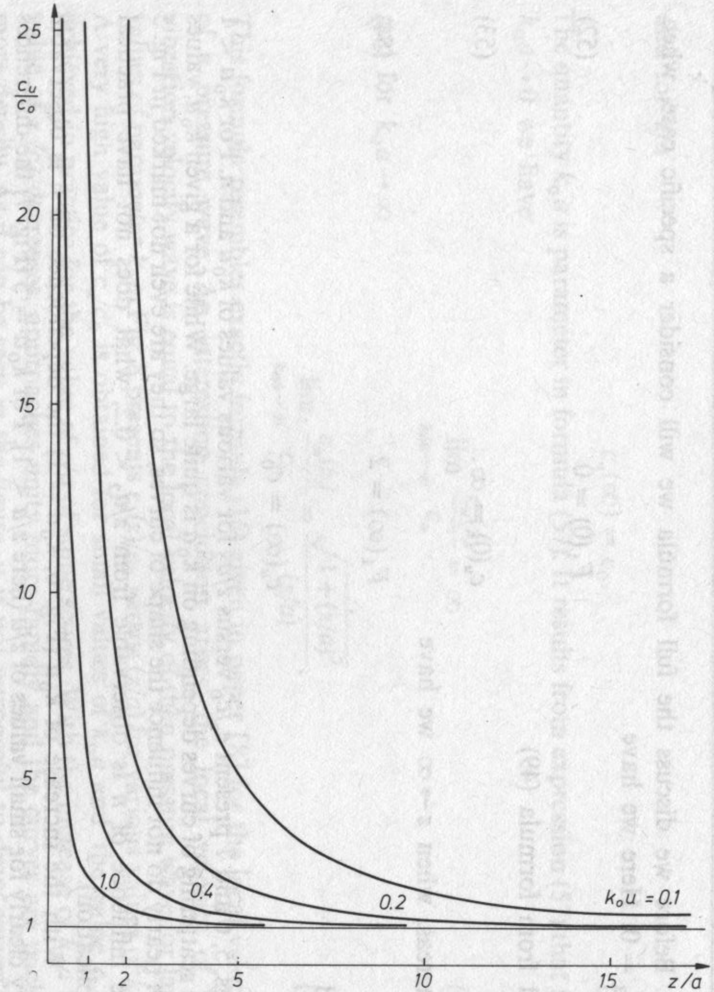

Fig. 8. c_u/c_0 versus z/a for a ring densly packed with point sources

Before we discuss the full formula we will consider a specific case, when $z/a_2 = 0$. Here we have

$$F_1(0) = 0 \tag{52}$$

and from formula (49)

$$c_u(0) = \infty. \tag{53}$$

Whereas, when $z \to \infty$ we have

$$F_1(\infty) = 2 \tag{54}$$

and

$$c_u(\infty) = c_0.$$

Figs. 5, 6 and 7 present c_u/c_0 versus z/a_2 for various values of $k_0 a$ and n. For $k_0 a < 1$ the scattering of curves depending on $k_0 a$ is quite large. While for a given $k_0 a$, values of n nearly do not influence the shape of curves, so they are even not marked in Fig. 5 (the influence of n is observable from $z/a_2 < 0.2$, what does not have practical application).

With the increase of $k_0 a$ (Fig. 6, $k_0 a = 1$) the dependence on n is observable very clearly for small values of z/a (here $z/a < 1$). For $k_0 a = 5$ (Fig. 7) the differences between the oridinates of curves for $n = 0$ and $n = 0.4$ are even greater, but also only up to $z/a = 1$. Hence, the effect of the local velocity of a wave occurs here only at distances of the same order as the external radius of the ring, from the surface. But this effect is much smaller than in the case of a pressure wave. For an infinitely thin ring we have [7, 8]

$$p = P_0 \frac{e^{i(\omega t - k_0\sqrt{z^2+a^2})}}{\sqrt{z^2+a^2}}, \tag{55}$$

were a is the radius of the ring.

When the derivative dp/dz is calculated and the full phase angle is separated, then the condition of acoustic velocity wave propagation has the following form

$$\omega t - k_0\sqrt{z^2+a^2} + \mathrm{tg}^{-1}\left(k_0\sqrt{z^2+a^2}\right) = \mathrm{const}. \tag{56}$$

Differentiating both sides of (56) with respect to time we achieve the relative propagation velocity:

$$\frac{c_u(z)}{c_u} = \frac{1+(k_0 a)^2[1+(z/a)^2}{(z/a)(k_0 a)^2\sqrt{1+(z/a)^2}}. \tag{57}$$

For $z/a = 0$ we have

$$c_u(0) = \infty \tag{58}$$

and for z/a

$$c_u(\infty) = c_0. \tag{59}$$

The quantity $k_0 a$ is parameter in formula (57). It results from expression (57) that for $k_0 a \to 0$ we have

$$\lim_{k_0 a \to \infty} \frac{c_u(z)}{c_u} = \infty \tag{60}$$

and for $k_0 a \to \infty$

$$\lim_{k_0 a \to \infty} \frac{c_u(z)}{c_0} = \frac{\sqrt{1+(z/a)^2}}{(z/a)}. \tag{61}$$

This formula resembles exactly formula (16) given in paper [8] for the local velocity of a pressure wave. An interesting conclusion arises: the local velocity of the vibration velocity wave is equal to the local velocity of then pressure wave for large values of parameter $k_0 a$. Fig. 8 presents $c_u(z)/c_0$ versus (z/a) for various values of $k_0 a$. A very high value of c_u/c_0 is obtained for small values of $k_0 a$ and the value c_0 is achieved in practice for the value of z/a of some scores. With the increase of $k_0 a$ the value of c_u/c_0 decreases, approaches unity more rapidly and the curves are packed more densily. As it can be seen in the figure, it is not purposeful to draw curves for $k_0 a > 1$, because it would only complicate the picture and large differences between c_u and c_0 would only for $z/a \ll 1$.

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