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DEVIATION OF THE ACOUSTIC PRESSURE TO PARTICLE VELOCITY RATIO FROM THE ϱc VALUE IN LIQUIDS AND SOLIDS AT HIGH PRESSURES

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Basing on fundamental equations of nonlinear acoustics the authors determined the ratio of the acoustic pressure to the particle velocity p_a/u for a travelling plane wave as a function of condensation. Nonlinear effects in the medium depend on the nonlinearity parameter B/A and on the maximum pressure. As the measure of those nonlinearities, the deviation of the p_a/u ratio from the $\varrho_0 c_0$ value was introduced and computed for water, fat tissue, steel and aluminium alloy, up to pressures of 100 MPa. The B/A value for steel and aluminium could be determined basing on acousto-elastic properties of these metals.

In this way it could be shown that the capacitance hydrophone, used for measurements in lithotripsy, does not introduce nonlinearities caused by its steel front plate when measuring nonlinear acoustic pressure fields. For, the mentioned deviation is two orders of magnitude lower for steel than for water and soft tissues.

Key words: Hydrophone, nonlinearity, lithotripsy.

Opierając się na podstawowych związkach akustyki nieliniowej autorzy wyznaczyli stosunek akustycznego ciśnienia do prędkości cząstkowej p_a/u dla płaskiej fali bieżącej jako funkcję kondensacji. Nieliniowe efekty zależą od parametru nieliniowości B/A i od maksymalnego ciśnienia akustycznego w ośrodku. Jako miarę nieliniowości autorzy wprowadzili odchylenie stosunku p_a/u od wielkości $\varrho_0 c_0$ i wyznaczyli wartość tego odchylenia dla wody, tkanki tłuszczowej, stali i stopu aluminiowego dla ciśnień dochodzących do 100 MPa. Wyznaczono wartość B/A dla stali i stopu aluminiowego w oparciu o akusto-sprężyste własności tych metali.

Autorzy wykazali, że hydrofony pojemnościowe, stosowane w pomiarach w litotrypsji, nie wnoszą nieliniowości powodowanych przez ich stalową czołową płytę podczas pomiarów nieliniowych pól o dużych ciśnieniach akustycznych. Albowiem wspomniane odchylenie jest dwa rzędy wielkości mniejsze w stali niż w wodzie i w tkance tłuszczowej.

1. Introduction

The authors proposed and described in a previous paper [6] capacitance hydrophones for measurements of high pressure shock wave pulses, occuring in lithotripsy. At the pressure values of 100 MPa (~1000 atm) there exist nonlinear effects in the medium penetrated by the shock wave. For the study of these effects the authors have developed an experimental lithotrypsy system with shock wave pressures of 40 MPa [6] and recently even higher, equal to 62 MPa [7]. In such a situation it is necessary to use in the measurement system a perfect linear hydrophone to eliminate any additional nonlinear effects which be caused by the measurement device. The solid and plastic piezoelectric hydrophones, which are used in measurements of acoustical fields in lithotripsy, do not assure the indispensable linearity and moreover, they are quickly damaged under the action of schock waves of such high pressures.

The purpose of this paper was to investigate the linearity of the proposed capacitance hydrophones, since they promissed to be linear due to elastic properties of metals. For, the main part of the capacitance hydrophone, its front plate, which is penetrated by the wave, is made of metal. It was also intended to introduce a practical nonlinearity measure connecting both, the medium properties and the pressure value.

2. Basic relations

The dependence between pressure p and density ϱ for the adiabatic process is nonlinear. Expanding the state equation $p = p(\varrho, S)$ around the constant values of ϱ_0 and entropy S_0 (since the acoustic wave is basically an isentropic process) one can write [1]

$$p = p_0 + \left(\frac{\partial p}{\partial \varrho}\right)_{\varrho_0 S_0} (\varrho - \varrho_0) + \frac{1}{2!}\left(\frac{\partial^2 p}{\partial \varrho^2}\right)_{\varrho_0 S_0} (\varrho - \varrho_0)^2 + \ldots, \tag{1}$$

where p is the instantaneous pressure, p_0 – hydrostatic pressure. Hence the acoustic pressure can be denoted as the difference

$$p_a = p - p_0. \tag{2}$$

The acoustic pressure can be expressed in an another form

$$p_a = As + Bs^2/2! + Cs^3/3! + Ds^4/4! + \ldots \tag{3}$$

where $s = (\varrho - \varrho_0)/\varrho_0$ denotes condensation and

$$A = \varrho_0 \left(\frac{\partial p_a}{\partial \varrho}\right)_{\varrho_0 S_0} = \varrho_0 c_0^2; \quad B = \varrho_0^2 \left(\frac{\partial^2 p_a}{\partial \varrho^2}\right)_{\varrho_0 S_0};$$
$$C = \varrho_0^3 \left(\frac{\partial^3 p_a}{\partial \varrho^3}\right)_{\varrho_0 S_0} \qquad D = \varrho_0^4 \left(\frac{\partial^4 p_a}{\partial \varrho^4}\right)_{\varrho_0 S_0}. \tag{4}$$

ϱ_0 and ϱ denote mean and instantaneous density of medium. c_0 – wave velocity for small amplitudes, S_0 – constant enthropy. It was found experimentaly for water [2] $B/A = 5.2$, $C/A = 42$, $D/A = 437$. For the pressure $p_a = 100$ MPa the value of $s = 0.04$ (in water) and contributions of the terms with coefficients C and D to the series (3) are equal 1% and 0.1% only. Therefore, the ratio B/A is sufficient to be taken into consideration when calculating the acoustic pressure p_a from Eq. (3). For various soft tissues the nonlinearity parameter B/A was determined experimentally by DUNN et al. [5]. In solids it was measured for some crystals only [3].

The particle velocity u can be found as a function of s from expression [1]

$$u = \pm \frac{2c_0}{B/A}[1-(1+s)^{B/(2A)}]. \tag{5}$$

3. Acoustic impedance and the ratio p_a/u

In linear acoustic for a plane, travelling wave the ratio between the acoustic pressure and particle velocity equals $\varrho_0 c_0$. This equality is no more valid for nonlinear waves. Let us determine the ratio p_a/u. From Eqs. (3) and (5) one obtains

$$p_a/u = \pm\varrho_0 c_0 s\left[1+\frac{B}{2A}s\right]\frac{B}{2A}[1-(1+s)^{B/(2A)}]^{-1}. \tag{6}$$

Expanding Eq. (5) into the Taylor series ($s << 1$)

$$u = \mp c_0 s\left[1+\frac{s}{2}\left(\frac{B}{2\mathrm{A}}-1\right)+\ldots\right]. \tag{7}$$

From Eqs. (3) and (7) one obtains p_a/u in a more simple form

$$p_a/u = \mp\varrho_0 c_0\left(1+\frac{B}{2A}s\right)\Big/\left[1+\frac{s}{2}\left(\frac{B}{2\mathrm{A}}-1\right)\right] \underset{s\to 0}{\to} \varrho_0 c_0 \tag{8}$$

The last expression tends to $\varrho_0 c_0$ when s approaches 0. This case corresponds to the basic relation of linear acoustics.

The ratio p_a/u should not be considered as the acoustic impedance, bacause it depends on the wave amplitude. Since the profile of the propagating wave changes with distance, its harmonic contents and the spectrum of the p_a/u ratio changes too. Because $s = s(\omega)$ then one could determine the ratio $p_a(n\omega)/u(n\omega)$ for every harmonic n. In such a case the ratio p_a/u becomes the impedance for the given harmonic. However, these impedances are not additive [8].

The condensation s can be determined directly from Eqs. (3) and (4) as the function of acoustic pressure p_a and B/A ratio (when $C = D = 0$)

$$s = \frac{A}{B}\left[\sqrt{1+2Bp_a/(A\varrho_0 c_0^2)}-1\right]. \tag{9}$$

The ratio p_a/u, expressed by Eqs. (6) or (8), depends finally on the acoustic pressure p_a and the B/A ratio. The deviation of this ratio from the acoustic impedance $\varrho_0 c_0$ characterizes the nonlinearity of the medium for a given acoustic pressure. The relative value of this deviation equals

$$\Delta = \left|\frac{p_a/u-\varrho_0 c_0}{\varrho_0 c_0}\right|_{s\ll 1} \approx \frac{1}{2}\left(1+\frac{B}{2A}\right)\frac{p_a}{\varrho_0 c_0^2}, \tag{10}$$

as can be found from Eqs. (8) and (9).

Figure 1 shows the values of Δ and s calculated from Eqs. (6), (8) and (9) for water and fat tissue ($B/A = 11$) as the function of acoustic pressure. Eq. (8) gives a little higher value for the fat issue than Eq. (6) for $p > 50$ MPa (see Fig. 1). Numerical values were obtained by means of the personal computer IBM AT using double precision.

For acoustic pressures of 100 MPa, which are typical in lithotrypsy, the deviation Δ for water and fat tissue equals 7% and 12%, respectively.

It should be noticed that the calculated p_a/u value corresponds to the maximum value of s which may occur in the propagating wave as a function of time.

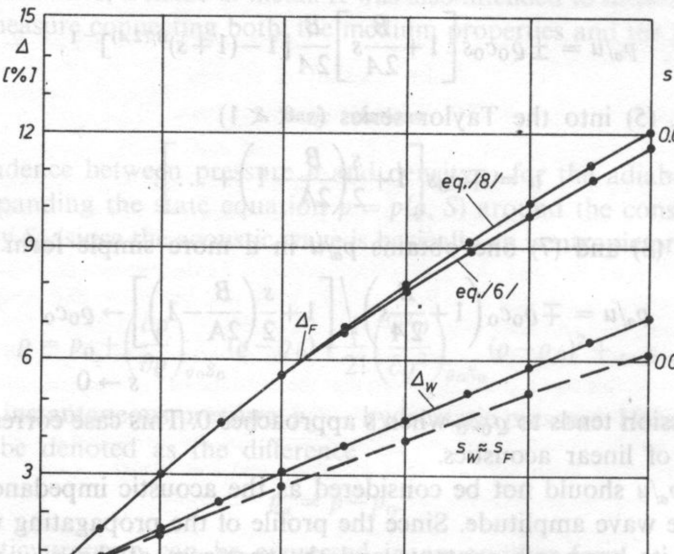

Fig. 1. The relative deviation Δ and condensation s (dashed line) versus acoustic pressure p_a, calculated for water and fat having the maximum value of B/A among soft tissues. Index W denotes water, F – fat tissue

4. Nonlinearity parameters for metals

Nonlinearity of typical metals is expressed by means of elasticity coefficients of third and higher orders. To compare the nonlinearity parameters of liquids and tissues with those of metals it was necessary to determine the B/A ratio for steel and aluminium, which are used for the construction of capacitance hydrophone.

Let us determine the ratio B/A for steel and aluminium basing on elasto-acoustic coefficients for those metals. Nonlinearity of solids causes the change of wave velocity under the action of stresses.

In such materials as steel and aluminium alloys the velocity changes can be described by the formula [4]

$$(c-c_0)/c_0 = \beta\sigma, \tag{11}$$

for stresses σ below the yield stress [9]. Equation (11) can be found from stress–velocity measurements usualy used for determination of third order elasticity coefficients in solids.

In Eq. (11) c denotes velocity of longitudinal wave, β is the acousto-elastic constant for longitudinal waves propagating in the direction of uniaxial stress σ. For carbon steel $\beta = -12.1\times10^{-11}$ Pa^{-1}, and for aluminium alloy (dural) $\beta = -77.5\times10^{-11}$ Pa^{-1} [4].

The rate at which a particular disturbance of the wave propagates through the medium equals [1]

$$c = \mp c_0(1+s)^{((B/2A)-1)}. \tag{12}$$

Expanding this equation one obtains

$$c = c_0\left[1+\left(\frac{B}{2A}+1\right)s+\frac{B}{2A}\left(\frac{B}{2A}+1\right)s^2+\ldots\right]. \tag{13}$$

Coefficients β are determined experimentally by measuring the flight time of the maximum amplitude of the ultrasonic pulse which propagates through a metal sample for various static stresses.

Equation (13) can be transformed into the expression

$$(c-c_0)/c_0 = [B/(2A)-1]s. \tag{14}$$

Expanding the root in Eq. (9), neglecting the terms of higher orders and putting into Eq. (14) one obtains

$$(c-c_0)/c_0 = [B/(2A)-1]p_a/(\varrho_0 c_0^2). \tag{15}$$

The last expression is analogeous to Eq. (11).

It was found experimentally [9] that β is independent on static stresses below the elasticity limit. Therefore it seems to be justified to assume that the relation (11) is valid for dynamic stresses σ too. In such a case, taking into account opposite signs of stresses and pressures one obtains from Eqs. (11) and (15)

$$B/A = 2(-\beta\varrho_0 c_0^2-1). \tag{16}$$

The corresponding values for steel and aluminium alloy (dural) determined from this formula are equal to $B/A = 3.5$ for steel and 13.5 for aluminium. Now the values of condensation s and deviation Δ could be determined for these materials basing on Eqs. (9), (10), (6) and (8) (Fig. 2). For the acoustic pressure of 100 MPa for steel and aluminium Δ equals 0.05% and 0.36%, respectively. The obtained value for steel is 2 orders of magnitude lower than for water and fat tissue (see Fig. 1 and 2). Therefore, the steel front plate of the capacitance hydrophone can be considered to be linear even for pressures as high as 100 MPa.

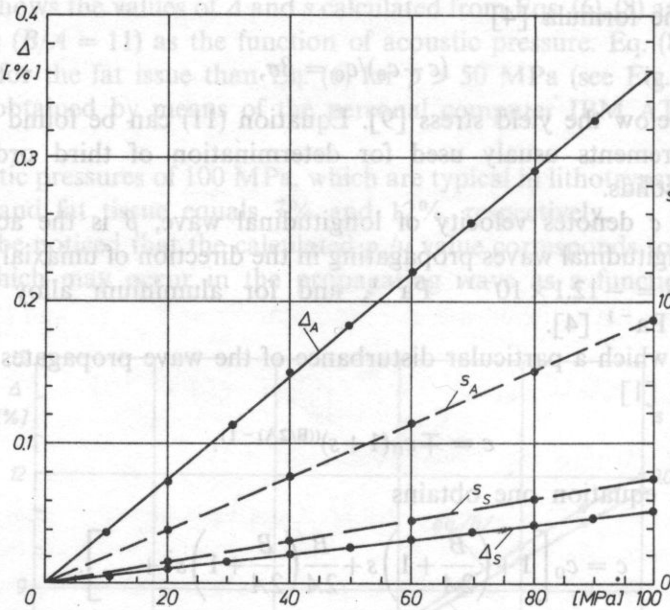

Fig. 2. The relative deviation Δ and condensation s (dashed line) versus acoustic pressure p_a, calculated for steel and for aluminium alloy. Index S denotes steel, A — aluminium alloy

For measurements of shock wave pressures used in lithotripsy the authors applied a capacitance hydrophone with the front plate made of a special steel (Polish mark 50HSA) which is characterized by a high elasticity limit of 1200 MPa. Its design and obtained measurement results were published elsewhere [6], [7].

5. Conclusions

a) Nonlinear acoustic effects occuring in various media are dependent on the ratio of nonlinearity parameters B/A and on the applied acoustic pressure. For the plane travelling wave the deviation Δ (see Eq. (10)) of the acoustic pressure to particle velocity ratio from the $\varrho_0 c_0$ value can be considered as a nonlinearity measure of these effects.

b) Basing on acousto-elastic properties of steel and aluminium alloy it was possible to determine for them B/A ratio (Eqs. (16) assuming stress to be lower than the elasticity limit.

c) It follows from deviations Δ, determined for water, fat tissue, steel and aluminium alloy, that steel is characterized by the lowest nonlinearity, 2 orders of magnitude lower than for water or fat tissue. Therefore, capacitance hydrophones do not introduce practically any additional nonlinearity caused by their front plates made of steel, into the measured nonlinear fields.

d) The calculated value of Δ makes it possible to estimate the error of pressure determination in the propagating plane wave from the approximate relation $p_a = \varrho_0 c_0 u$ when the particle velocity u was measured and vice versa.

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