ARCHIVES OF ACOUSTICS
15, 3-4, 531-542 (1990)

WAVES WITH FINITE AMPLITUDE IN BESSEL HORNS

T. ZAMORSKI

Institute of Physics, Pedagogical University in Rzeszów Department of Acoustics
(35-310 Rzeszów, ul. Rejtana 16 a)

The equation of the propagation a one-dimensional wave with finite amplitude in Bessel horns filled with nondissipative fluid is formulated in this paper. The solution to the equation of the propagation is analysed with the small parameter method. It was proved that a sinusoidal wave with finite amplitude, generated on the entry of the waveguide, is deformed when it moves along the horn. This manifests itself with the generation of higher harmonics with amplitudes dependent on position and frequency. The case of a conical waveguide filled with air is considered on the basis of general formulae achieved for the family of Bessel horns. Particularily the second harmonic was taken into account.

W pracy sformułowano równanie propagacji jednowymiarowej fali o skończonej amplitudzie w tubach Bessela wypełnionych bezstratnym ośrodkiem płynnym. Przeanalizowano rozwiązanie równania propagacji stosując metodę małego parametru. Wykazano, że fala sinusoidalna o skończonej amplitudzie generowana na wlocie falowodu ulega zniekształceniu przy przesuwaniu się w głąb tuby, co objawia się powstawaniem wyższych harmonicznych o amplitudzie zależnej od położenia i od częstości. Na podstawie ogólnych wzorów uzyskanych dla rodziny tub Bessela rozważono przypadek falowodu stożkowego wypełnionego powietrzem, ze szczególnym uwzględnieniem drugiej harmonicznej.

1. Introduction

The problem of propagation of elastic waves with finite amplitude in waveguides with regularity changing cross-sections (horns) has been relatively rarely considered in acoustic literature, as opposed to the linear theory of horns. This paper is based on the equation of propagation of a wave with finite amplitude in a horn with arbitrary shape. This equation has been formulated in papers [5, 6, 8] in Lagrange's coordinates on the assumption that the wave is one dimensional and that the gas medium in the horn is nondissipative.

Figure 1 presents a layer of the medium in the waveguide. Before the transition of the wave disturbance this layer is contained between surfaces $S_{(a)}$ and $S_{(a+ds)}$, where

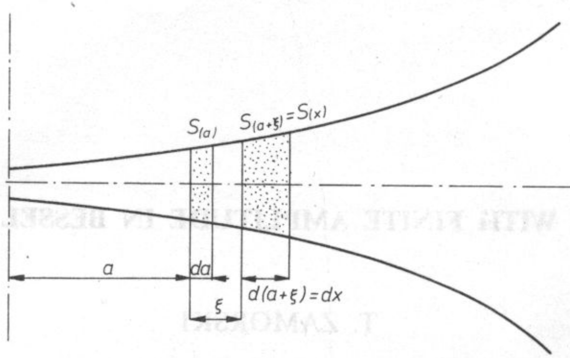

FIG. 1. Displacement of a layer of the medium under the influence of an acoustic wave

a is the Lagrange coordinate. In the wave's presence the layer moves to the position $a+\xi$ and it has thickness equal to $d(a+\xi)=dx=\left(1+\frac{\partial\xi}{\partial a}\right)da$. In such a case variable a determines the particle's position in the medium at rest and is independent of time t; while ξ is the displacement of the particle and depends on both, a and t. Variable $x=a+\xi$ is an Euler's coordinate. In Euler's coordinates ξ is the displacement of an arbitrary particle in point x; ξ depends on x and t here. After several transformations of the equation of continuity, equation of motion and the adiabate equation for considered layer, we can reach the equation of propagation for a wave with finite amplitude for displacement ξ [5, 6, 8]

$$\frac{1}{\left[\frac{S_{(a+\xi)}}{S_{(a)}}\right]^{\gamma-1}(1+\xi')^{\gamma}}\left\{\frac{\frac{\partial}{\partial a}\left[\frac{S_{(a+\xi)}}{S_{(a)}}\right]}{\frac{S_{(a+\xi)}}{S_{(a)}}}+\frac{\frac{\partial^2\xi}{\partial a^2}}{1+\xi'}\right\}=\frac{1}{c^2}\frac{\partial^2\xi}{\partial t^2}, \tag{1}$$

where $\xi'=\frac{\partial\xi}{\partial a}$ is the so-called deformation of the medium, and $\gamma=\frac{C_p}{C_v}$ is the adiabatic exponent. The quantity $c^2=\frac{\gamma p_0}{\varrho_0}$ is the square of sound velocity for small amplitudes while $\varrho_0=\varrho_{(a)}=\text{const}$ is the static density, and $P_0=P_{(a)}=\text{const}$ is the static pressure in the layer of the medium.

Equation (1) was derivated on the assumption that the horn is filled with gas. It can be also applied for liquid media, if the liquid satisfies empirical equation [11]:

$$P=\bar{P}_0\left(\frac{\varrho}{\varrho_0}\right)^{\Gamma}, \tag{2}$$

where $\bar{P}_0$ and Γ are constants determined for a given liquid from experiment [11], with adequate approximation. Equation (1) greatly simplifies itself for exponential horns and such a case has been discussed in literature [2, 5, 8]. This paper tries to consider the transmission of waves with finite amplitudes in Bessel horns, with special interest in the conical waveguide which is rather frequently applied, because of it's simple construction.

2. Discussion of the propagation equation of waves with finite amplitudes in Bessel horns

The following dependence between the cross-sectional area and position of the horn's axis determines the family of Bessel horns [7]:

$$S = B_0(x_0 + \bar{x})^{\mu} \tag{3}$$

where x_0 is the distance between the throat of a horn and fictitious vertex (Fig. 2), B_0 is a so chosen constant that the area at the throat is equal to $S_0 = B_0 x_0^{\mu}$, while μ is the coefficient of flare of the waveguide and is a positive real number.

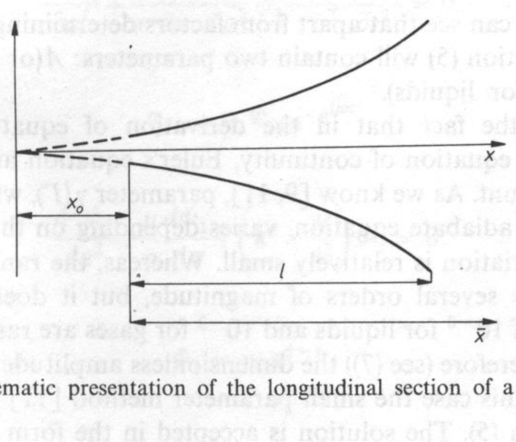

FIG. 2. A schematic presentation of the longitudinal section of a Bessel horn

From (3) it results that

$$\frac{S_{(a+\xi)}}{S_{(a)}} = \left(1 + \frac{\xi}{a}\right)^{\mu}. \tag{4}$$

Including (4) in (1) we achieve the propagation equation of a wave with finite amplitude in a Bessel horn

$$\frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2}\left[\left(1+\frac{\xi}{a}\right)^{\mu(\gamma-1)} + (1+\xi')^{\gamma}\right] = \mu\frac{a\xi' - \xi}{a^2 + a\xi} + \frac{\dfrac{\partial^2 \xi}{\partial a^2}}{1+\xi'}. \tag{5}$$

Let us assume that there are no reflections at the mouth of a horn and that a hypothetical piston vibrating with harmonical motion is the source of waves at the throat. Lagrange's coordinate of the piston, a_0, is a constant. The piston displacement is time-dependent and is equal to

$$\xi_{(a_0,t)} = \bar{A} \cdot e^{i\omega t}, \tag{6}$$

where ω is the pulsation.
The amplitude of vibration in formula (6) can be expressed in dimensionless notation by relating it to the inverse of the wave number $k = \omega/c = 2\pi/\lambda$:

$$A = \frac{\bar{A}}{k^{-1}} = 2\pi\frac{\bar{A}}{\lambda} = 2\pi M, \tag{7}$$

where M is the ratio of piston's deflection amplitude and wave length λ.
Including (7) expression (6), which defines the boundary condition for a wave in a horn, has the following form

$$\xi_{(a_0,t)} = \frac{c}{\omega} A \cdot e^{i\omega t}. \tag{8}$$

From (5) and (8) we can see that apart from factors determining the horn's geometry the solution of equation (5) will contain two parameters: A(or M) and the adiabatic exponent γ (Γ — for liquids).
This results from the fact that in the derivation of equations (1) and (5) the non-linearity of the equation of continuity, Euler's equation and adiabate equation was taken into account. As we know [9,11], parameter γ (Γ), which characterizes the non-linearity of the adiabate equation, varies depending on the type of gas (liquid). But it's range of variation is relatively small. Whereas, the range of variation of the M number includes several orders of magnitude, but it does not exceed one. In practice M values of 10^{-3} for liquids and 10^{-2} for gases are rarely exceeded even for great intensities. Therefore (see (7)) the dimensionless amplitude A in formula (8) does not exceed one. In this case the small parameter method [11] can be applied in the solution of equation (5). The solution is accepted in the form of a power series for amplitude A:

$$\xi_{(a,t)} = \frac{c}{\omega}[A\varphi_{1(a,t)} + A^2\varphi_{2(a,t)} + A^3\varphi_{3(a,t)} + \ldots], \tag{9}$$

where functions $\varphi_1, \varphi_2, \varphi_3$ have to fulfil the boundary condition (8) at the throat of a horn:

$$\varphi_{1(a_0,t)} = e^{i\omega t}; \quad \varphi_{2(a_0,t)} = \varphi_{3(a_0,t)} \cdots = 0. \tag{10}$$

Further considerations will be performed in the second approximation, i.e. with the designed of terms with higher order than A^2. Including (9) in (5) so that the equation of propagation contains only terms with A and A^2, we have

$$\frac{1}{\omega c}(A\ddot{\varphi}_1+A^2\ddot{\varphi}_2)+\frac{\gamma A^2}{\omega^2}\varphi_1'\ddot{\varphi}_1+\frac{\mu(\gamma-1)}{a}\frac{A^2}{\omega^2}\ddot{\varphi}_1\varphi_1=\frac{Ac}{\omega}\varphi_1''+\frac{A^2 c}{\omega}\varphi_2''+$$
$$+\mu\frac{a\left(\frac{Ac}{\omega}\varphi_1'+\frac{A^2 c}{\omega}\varphi_2'\right)-\left(\frac{Ac}{\omega}\varphi_1+\frac{A^2 c}{\omega}\varphi_2\right)}{a^2+a\left(\frac{Ac}{\omega}\varphi_1+\frac{A^2 c}{\omega}\varphi_2\right)}-\frac{A^2 c^2}{\omega^2}\varphi_1'\varphi_1'', \tag{11}$$

where dots mark differentiation in terms of time, commas-in terms of coordinate a. Expression (11) can be noted in the form of a sum of two equations – one includes factor A, the second A^2

$$A\cdot F_{1(a,t)}+A^2\cdot F_{2(a,t)}=0. \tag{12}$$

This equation is fulfilled for an arbitrary $A\neq 0$, it equalities $F_{1(a,t)}=0$ and $F_{2(a,t)}=0$ occur independently. It results from (11) that only function $\varphi_{1(a,t)}$ occurs in equation $F_{1(a,t)}=0$:

$$\frac{\partial^2\varphi_1}{\partial a^2}+\frac{\mu}{a}\frac{\partial\varphi_1}{\partial a}-\frac{\mu}{a^2}\varphi_1-\frac{1}{c^2}\frac{\partial^2\varphi_1}{\partial t^2}=0. \tag{13}$$

Accepting

$$\varphi_{1(a,t)}=\Phi_{1(a)}\cdot e^{i\omega t} \tag{14}$$

we have

$$\frac{d^2\Phi_1}{da^2}+\frac{\mu}{a}\frac{d\Phi_1}{da}+\left(k^2-\frac{\mu}{a^2}\right)\Phi_1=0. \tag{15}$$

Substituting then

$$\Phi_1=y\cdot a^{1-\nu}, \tag{16}$$

where

$$\nu=\frac{\mu+1}{2} \tag{17}$$

we obtain Bessel equation for y [4, 7]

$$\frac{d^2 y}{da^2}+\frac{1}{a}\frac{dy}{da}+\left(k^2-\frac{\nu^2}{a^2}\right)y=0. \tag{18}$$

Equation (15) describes wave motion in a Bessel horn in the first approximation. Selecting the solution to equation (18) in the form of a Hankel function [7] and including (16) we have

$$\Phi_{1(a)}=a^{1-\nu}\left[C\cdot H^{(1)}_{\nu(ka)}+B\cdot H^{(2)}_{\nu(ka)}\right]. \tag{19}$$

$H^{(1)}_{\nu(ka)}$ and $H^{(2)}_{\nu(ka)}$ are ν — order Hankel functions [7]. It results from the definition of these functions that $H^{(2)}_{\nu(ka)}$ describes a wave propagating in the direction of increasing "a" values, while $H^{(1)}_{\nu(ka)}$ — a wave propagating in the opposite direction. Because we previously assumed that there are no reflections at the mouth of the horn, only a progressive wave occurs and we should accept $C=0$. Finally, including (14), we have

$$\varphi_1 = a^{1-\nu}\cdot B\cdot H^{(2)}_{\nu(ka)}\cdot e^{i\omega t}. \tag{20}$$

Constant B can be determined from boundary condition (10)

$$B = \frac{a_0^{\nu-1}}{H^{(2)}_{\nu(ka_0)}}. \tag{21}$$

Eventually, the first approximation:

$$\varphi_1 = \left(\frac{a}{a_0}\right)^{1-\nu}\cdot\frac{H^{(2)}_{\nu(ka)}}{H^{(2)}_{\nu(ka_0)}}\cdot e^{i\omega t} \tag{22}$$

A more detailed discussion of equation (22) is not necessary, because the linear theory of Bessel horns is well known [1, 6, 7]. Equation $F_{2(a,t)}=0$ with terms containing factor A^2 has the following form (see (11));

$$\frac{\partial^2\varphi_2}{\partial a^2}+\frac{\mu}{a}\frac{\partial\varphi_2}{\partial a}-\frac{\mu}{a^2}\varphi_2-\frac{1}{c^2}\frac{\partial^2\varphi_2}{\partial t^2}=\varepsilon_{(a,t)}, \tag{23}$$

where

$$\varepsilon_{(a,t)}=\frac{c}{\omega}\varphi_1'\varphi_2''+\frac{1}{\omega ca}(\mu\gamma-\mu+1)\,\varphi_1\ddot{\varphi}_1+\frac{\gamma}{\omega c}\varphi_1'\ddot{\varphi}_1-\frac{c}{\omega a}\varphi_1''\varphi_1. \tag{24}$$

Equation (23) is similar to equation (13), but on the right side it has term $\varepsilon_{(a,t)}$ defined by the solution achieved in the first approximation, $\varphi_{1(a,t)}$. We can see from expression (22) that φ_1 is a periodical function with pulsation ω. Also derivatives cf φ_1 in terms of time and position are periodical functions with pulsation ω. Because products of these derivatives occur in (24), ε has to be a periodical function with pulsation $2\,\omega$. Expression $\varepsilon_{(a,t)}$ determines the form of the solution of the heterogeneous equation (23), so [4] that the solution can be a function with pulsation $2\,\omega$

$$\varphi_{2(a,t)}=\Phi_{2(a)}\cdot e^{2i\omega t}. \tag{25}$$

Hence, function $\Phi_{2(a)}$ which defines the second harmonic, has to satisfy equation

$$\frac{d^2\Phi_2}{da^2}+\frac{\mu}{a}\frac{d\Phi_2}{da}-\frac{\mu}{a^2}\Phi_2+k_1^2\Phi_2=\Psi_{(a)}, \tag{26}$$

where

$$\Psi_{(a)}=k^{-1}\Phi_1'\Phi_1''-ka^{-1}(\mu\gamma-\mu+1)\,\Phi_1^2-\gamma k\Phi_1'\Phi_1-(ka)^{-1}\Phi_1''\Phi_1, \tag{27}$$

$$k_1=2k. \tag{28}$$

The fact that function $\Psi_{(a)}$ in equation (26) is determined by the first approximation of the solution can be explained by the wave's second harmonic being excited due to a disturbance of the medium in the horn by the first harmonic. The solution to equation (26) can be presented in the following form [4]:

$$\Phi_2 = f_2 \int \frac{f_1 \Psi_{(a)}}{W} da - f_1 \int \frac{f_2 \Psi_{(a)}}{W} da + B_1 f_1 + B_2 f_2, \tag{29}$$

where

$$W = \frac{2}{\pi a^2} \tag{30}$$

$$f_1 = a^{1-\nu} \cdot I_{\nu(k_1 a)}, \tag{31}$$

$$f_2 = a^{1-\nu} \cdot Y_{\nu(k_1 a)}. \tag{32}$$

and $I_{\nu(k_1 a)}$, $Y_{\nu(k_1 a)}$ are ν-order Bessel functions of the first and second type respectively [7]. In accordance with condition (10) and expression (25) has to be equal to 0 for a particle at the throat of the horn ($a = a_0$). This condition can be fulfilled for adequately selected constants B_1, B_2 in expression (29).

Further approximations can be calculated in an analogous manner by including terms with higher powers of A from expression (9) in the equation of propagation (5) and formulating linear differential equations for functions φ_n, corresponding with successive harmonics. Condition (10), i.e. $\varphi_{n(a_0,t)} = 0$ in every time moment t for $n > 1$ is valid for higher harmonics on the entry of the waveguide. Higher harmonics for $a > a_0$ are formed, because of non-linear properties of the medium in the horn. They are a sign of a deformation of the wave front as the wave disturbance moves along the horn. The total power on the entry of the waveguide is the power of the first harmonic. Because the total power remains constant in a nondissipative medium, the formation of higher harmonics is related with a decrease of the acoustic power of a wave with fundamental frequency. In practice most information for a waveguide with definite geometry should be contributed by the analysis of the second harmonic. Higher harmonics are of less significance, because $A < 1$ (see (9)). We will now analyse the second harmonic in a waveguide with conical shape.

3. Conical horn

A conical horn has worse transmission properties than Bessel horns of higher order [1, 7] exponential [6] of catenoidal [10] waveguides, but it is often applied for the simplicity of its shape. In this case the coefficient of flare is equal $\mu = 2$. Therefore, $\nu = 3/2$ (see (17)) and the solution of the equation of propagation for the first harmonic (20) has the following form

$$\varphi_1 = a^{-\frac{1}{2}} B \cdot H^{(2)}_{\frac{3}{2}(ka)} \cdot e^{i\omega t}, \tag{33}$$

where constant B, defined by expression (21), is equal to:

$$B = \frac{\sqrt{a_0}}{H^{(2)}_{\frac{3}{2}(ka_0)}}. \tag{34}$$

In order to determine the second harmonic, equation (26) has to be solved for function Φ_2. The integral of this equation for $\mu = 2$ can be determined from (29) with the utilization of relationships between Bessel functions $I_{\frac{3}{2}(k_1 a)}$, $Y_{(\frac{3}{2}k_1 a)}$ and trigonometric functions [7]

$$I_{\frac{3}{2}(k_1 a)} = \sqrt{\frac{2}{\pi k_1 a}}\left[\frac{\sin(k_1 a)}{k_1 a} - \cos(k_1 a)\right], \tag{35}$$

$$Y_{\frac{3}{2}(k_1 a)} = -\sqrt{\frac{2}{\pi k_1 a}}\left[\sin(k_1 a) + \frac{\cos(k_1 a)}{k_1 a}\right]. \tag{36}$$

Calculations result in:

$$\begin{aligned}\mathrm{Re}[\Phi_2] = \frac{a_0}{a}\left[D_1(\Sigma-\chi)+D_2(\Theta+\Lambda)+\frac{L_1}{k_1 a}-N_2\right]\sin(k_1 a)\\ +\frac{a_0}{a}\left[D_1(\Theta-\Lambda)-D_2(\Sigma+\chi)-L_1-\frac{N_2}{k_1 a}\right]\cos(k_1 a),\end{aligned} \tag{37 a}$$

$$\begin{aligned}\mathrm{Im}[\Phi_2] = \frac{a_0}{a}\left[D_2(\Sigma-\chi)-D_1(\Theta+\Lambda)+\frac{L_2}{k_1 a}-N_1\right]\sin(k_1 a)\\ +\frac{a_0}{a}\left[D_2(\Theta-\Lambda)+D_1(\Sigma+\chi)-L_2-\frac{N_1}{ka}\right]\cos(k_1 a),\end{aligned} \tag{37 b}$$

where D_1, D_2, L_1, L_2, N_1, N_2 are constants, while Σ, χ, Θ, Λ are functions with following forms:

$$\Theta = 2\left(\frac{2}{k_1 a}\right)^4 - 5\left(\frac{2}{k_1 a}\right)^2 - \frac{\gamma+1}{2}\ln(k_1 a), \tag{38}$$

$$\Lambda = \frac{\gamma+1}{2k_1 a}Si(2k_1 a) + \frac{\gamma+1}{2}[Ci(2k_1 a) - C], \tag{39}$$

$$\Sigma = 4\left(\frac{2}{k_1 a}\right)^3 - 3\frac{2}{k_1 a} + \frac{\gamma+1}{2k_1 a}\ln(k_1 a). \tag{40}$$

$$\chi = \frac{\gamma+1}{2}\mathrm{Si}(2k_1 a) + \frac{\gamma+1}{2k_1 a}[Ci(2k_1 a) - C]. \tag{41}$$

Symbols $Si(2k_1 a)$, $Ci(2k_1 a)$ in expressions (39), (41) denote tabelarized integral sine and cosine functions, while C is Euler's constant [3].

We can calculate constants D_1 and D_2 from (34)

$$D_1 = (\pi a_0)^{-1} \cdot \mathrm{Re}[B^2]; \quad D_2 = (\pi a_0)^{-1} \cdot \mathrm{Im}[B^2]. \tag{42}$$

Other constants were derived from the boundary condition (10) and have the following form:

$$L_1 = \frac{k_1 a_0}{1+(k_1 a_0)^2} \{k_1 a_0 [D_1(\Theta_0 - \Lambda_0) - D_2(\Sigma_0 + \chi_0)] - D_1(\Sigma_0 - \chi_0) + \\ - D_2(\Theta_0 + \Lambda_0)\} \tag{43}$$

$$L_2 = \frac{k_1 a_0}{1+(k_1 a_0)^2} \{k_1 a_0 [D_2(\Theta_0 - \Lambda_0) + D_1(\Sigma_0 + \chi_0)] + D_1(\Theta_0 + \Lambda_0) + \\ - D_2(\Sigma_0 - \chi_0)\}, \tag{44}$$

$$N_1 = \frac{k_1 a_0}{1+(k_1 a_0)^2} \{k_1 a_0 [D_1(\Theta_0 + \Lambda_0) - D_2(\Sigma_0 + \chi_0)] - D_2(\Theta - \Lambda_0) + \\ - D_1(\Sigma_0 - \chi_0)\}, \tag{45}$$

$$N_2 = \frac{k_1 a_0}{1+(k_1 a_0)^2} \{k_1 a_0 [D_1(\Sigma_0 - \chi_0) + D_2(\Theta_0 + \Lambda_0)] + D_1(\Theta_0 - \Lambda_0) + \\ - D_2(\Sigma_0 + \chi_0)\}, \tag{46}$$

where $\Theta_0, \Lambda_0, \Sigma_0, \chi_0$ are values of functions defined by formulae (38–41) for $a = a_0$. Having calculated Φ_2 we can now determine φ_2 from (25). Then, knowing φ_1 and φ_2 we can find the vibration velocity of a particle from (9)

$$v_{(a,t)} = iAc\Phi_{1(a)} \cdot e^{i\omega t} + i2A^2 c\Phi_{2(a)} \cdot e^{2i\omega t}. \tag{47}$$

Formula (47) determines the vibration velocity in Lagrange's coordinates. If we want to know the vibration velocity in a point with abscissa x (see Fig. 1), then we have to apply the relationship between Lagrange's and Euler's coordinates [11]

$$v_{(x,t)} = v_{(a,t)} - \frac{\partial v}{\partial a} \cdot \xi_{(a,t)} + \dots \tag{48}$$

Applying formulae (9) and (47) we obtain the second approximation

$$v_{(x,t)} = iAc\Phi_{1(a)} \cdot e^{i\omega t} + icA^2 \left[2\Phi_{2(a)} - \frac{1}{k}\frac{\partial \Phi_1}{\partial a} \cdot \Phi_{1(a)} \right] \cdot e^{2i\omega t}. \tag{49}$$

To end the case of a conical horn ($\mu = 2$) we will present a numerical example. Amplitudes of vibration velocity of the first and second harmonic of a wave in a waveguide with the following dimensions:

distance between the throat of the horn and fictitious of the cone $x_0 = 10^{-1}$ m,
horn throat diameter $d_0 = 2 \cdot 10^{-2}$ m,
horn mouth diameter $d_w = 2 \cdot 10^{-1}$ m,
length $l = 9 \cdot 10^{-1}$ m,

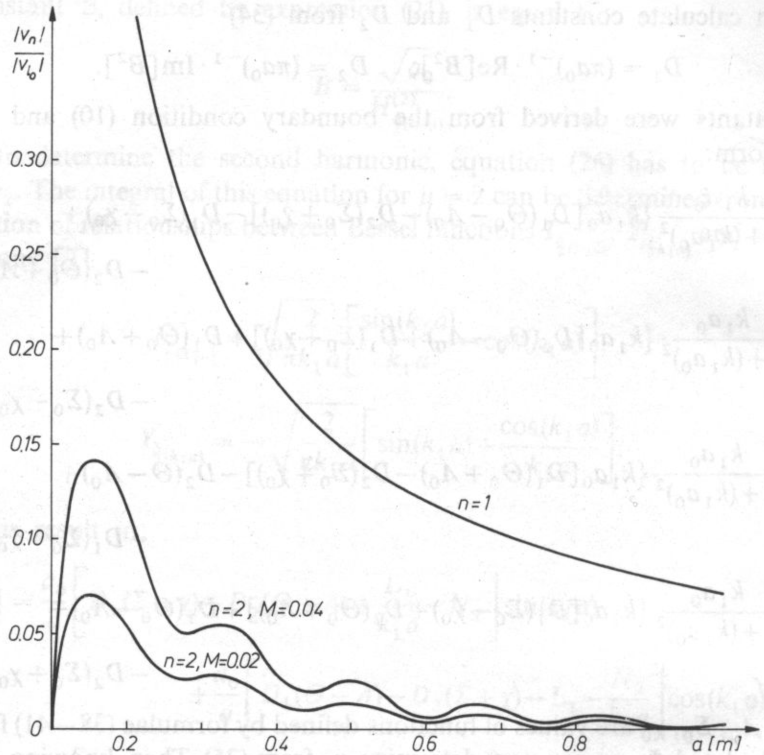

FIG. 3. Amplitudes of vibration velocities of particles of the medium in a horn for the first and second harmonic. Frequency of sine wave at the throat of a horn $f = 540$ Hz ($k = 10\ \text{m}^{-1}$)

were numerically calculated on the basis of expressions (7), (33), (34) and (37–47).

Figure 3 presents the relationship between amplitudes of vibration velocities of the first and second harmonic, and position in the horn of particles of the medium for a definite vibration frequency of the piston at the throat $f = 540\ \text{Hz}\ \left(k = 10\frac{1}{\text{m}}\right)$. Amplitudes of vibration velocities of both harmonics were related to the amplitude of vibration velocity of the first harmonic on the horn's throat $|v_{10}|$. We can see that the amplitude of vibration velocity of the first harmonic is a monotone function which decreases as it moves away from the throat of the horn. The second harmonic initially rapidly increases near the throat, but then it begins to decrease also, with characteristic oscillations. The amplitude decrease of the vibration velocity of the second harmonic, accompanying the growth of the distance from the throat, means that the flare of the waveguide's walls has a restraining effect on the development of distortions of the wave front during the propagation of a wave with high amplitude. This results from the fact that with the increase of the cross section of the horn, the

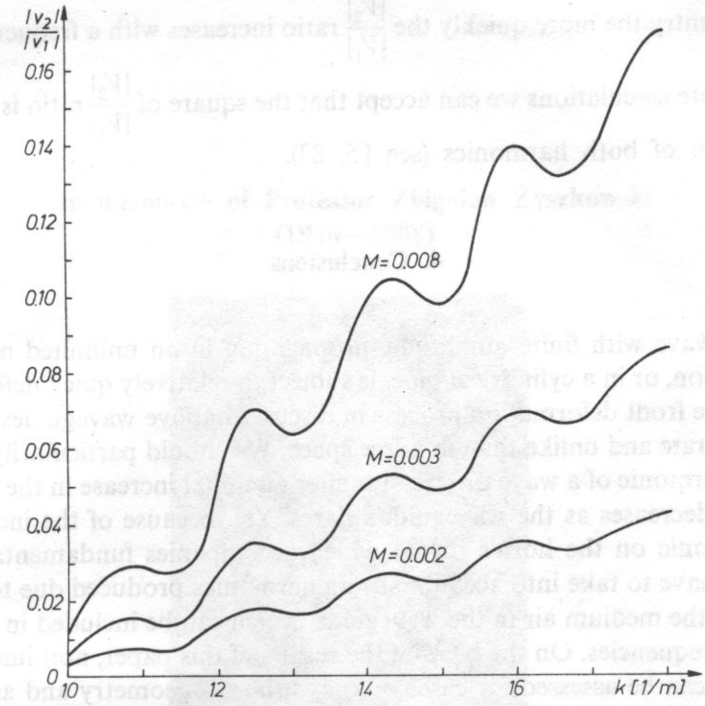

FIG. 4. Ratio of a particle velocities for the second harmonic and the fundamental at the horn mouth

acoustic energy per cross-section area unit decreases and the amplitude of the wave disturbance decreases.

As we know, pulsation of wave's higher harmonics in space (or in time) is caused by dispersion [9, 11]. Therefore, oscillations of the second harmonic, shown in Fig. 3, should prove that in terms of the non-linear theory, a nondissipative fluid contained in the horn is a dispersion medium. As it should have been expected the amplitude of vibration velocity of the second harmonic increases with the increase of the amplitude of the source's vibrations. This is shown in Fig. 3, where calculation results of $\frac{|v_2|}{|v_{10}|}$ for two values of the number M (see [7]) on the horn's throat $M = 0.02$ and $M = 0.04$, are taken into account.

The amplitude of vibration velocity of the second harmonic for a given particle of the medium in the horn increases in relation to the amplitude of vibration velocity of the first harmonic when frequency is increased. This relationship is shown in Fig. 4 where results of calculations of the ratio of amplitudes of vibration velocities of both harmonics are presented in terms of wave number k for particles on the horn's mouth. We can see that the higher the amplitude of vibrations of the piston on the waveguide's entry the more quickly the $\frac{|V_2|}{|V_1|}$ ratio increases with a frequency increase. In approximate calculations we can accept that the square of $\frac{|V_2|}{|V_1|}$ ratio is equal to the intensity ratio of both harmonics (see [5, 8]).

4. Conclusions

A plane wave with finite amplitude, propagating in an unlimited medium with small dispersion, or in a cylindrical pipe, is subject to relatively quick deformation [9, 11]. The wave front deformation process in discussed above waveguides takes place, at a different rate and unlike than in a free space. We should particularily notice that the second harmonic of a wave progressive after an initial increase in the narrow part of the horn, decreases as the waveguide's flares. Yet, because of the increase of the second harmonic on the horn's throat which accompanies fundamental frequency increase, we have to take into account strong harmonics produced due to non-linear properties of the medium air in the waveguide, which can be included in the range of transmitted frequencies. On the basis of the results of this paper, non-linear effects in Bessel horns can be assessed for cases of known horn's geometry and amplitude on the throat of the waveguide.

References

[1] S. Ballantine, *On the propagation of sound in the general Bessel horn of in finite length*, J. Franklin Inst. **203**, 85–102, 852–853 (1927).
[2] Р. Г. Галюлин, Л. В. Коркишко, *Стоячие волны конечной амплитуды в экспоненциальном канале*, Акуст. Журн. **31**, 4., 520–522 (1985).
[3] Е. Янке, Ф. Емде, Ф. Лош, *Специальные функции*, Изд. „Наука", Москва 1968.
[4] Е. Камке, *Справочник по обыкновенным дифференциальным уравнениям*, Изд. Иностранной Литературы, Москва 1951.
[5] M. Kwiek, *Laboratory acoustics*, Part 1 (in Polish) PWN, Poznań–Warszawa 1968.
[6] N. W. McLachlan, *Loud speakers*, Oxford University Press, London 1934.
[7] N. W. McLachlan, *Bessel functions for engineers* (in Polish) PWN, Warszawa 1964.
[8] Y. Rocard, *General dynamics of vibrations*, Ungar, New York 1960 p. 467–479
[9] О. В. Руденко, С. И. Солуян, *Теоретические основы нелинейной акустики*, Изд. „Наука", Москва 1975.
[10] R. Wyrzykowski, *Linear theory of acoustic field of gas media* (in Polish) RTPN, WSP Rzeszów 1972.
[11] Л. К. Зарембо, В. А. Красильников, *Введение в нелинейную акустику*, Изд. „Наука", Москва 1966.

Received on April 4, 1989