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DIRECTIONAL CHARACTERISTIC OF A CIRCULAR PLATE VIBRATING UNDER THE EXTERNAL PRESSURE

W. RDZANEK

Institute of Physics, Pedagogical University
(35-310 Rzeszów, Rejtana 16)

In this paper, the problem of acoustic wave radiated and received by a lip restrained circular plate vibrating in a rigid baffle is distributed. The vibratory system is found in the lossless and homogeneous liquid medium. The dynamics influence of the wave emitted by plate on its vibration form has been omitted as well as the losses in plate have been neglected. The axially symmetric vibration induced by the sinusoidal varried in time external pressure has been considered. By assuming the known distribution of pressure forcing vibration, a simple expression has been found allows to determine the directional characteristic.

W pracy rozwiązano zagadnienie promieniowania i odbioru fal akustycznych przez utwierdzoną na obrzeżu w sztywnej odgrodzie płytę kołową. Układ drgający znajduje się w bezstratnym i jednorodnym ośrodku płynnym. Pominięto dynamiczne oddziaływanie promieniowanej przez płytę fali na jej postać drgań oraz zaniedbano straty w płycie. Rozpatrzono osiowosymetryczne drgania wymuszone sinusoidalnie zmiennym w czasie ciśnieniem zewnętrznym. Zakładając znany rozkład ciśnienia wymuszającego drgania, ustalono wyrażenie mające elementarną postać, pozwalającą na wyznaczenie charakterystyki kierunkowości.

1. Introduction

A utilization of circular plates and membranes as the vibratory systems for design of acoustic devices which satisfy a function of sender or receiver of acoustic waves, requires identyfying, between the others, directional characteristic. A relative big attention to be payed to this problem by using the approximation methods as well as the equivalent schemes is however not sufficient in case of an exact analysis [3, 6].

The precisely mathematically formulated basis magnitudes characterizing the circular membrane as a source or a receiver of acoustic energy can be found in [1], the main point of which is focussed on the frequency response of input impedance for the circular membrane stimulated to the forced vibration.

A directional characteristic of a circular membrane excited to a resonance vibration is described, for example in [6, 7] – but in more general case – of nonresonance vibration, in [4].

Describing the problem of radiation of circular plate only the following boundary cases have been analyzed – a focused forced vibration and a vibrated plate modelled by a system of concentrated constants with the equivalent force [3, 6].

The most precisely considerations needed to find acoustic properties of a vibrated plate require undergoing of investigations for the nonresonance vibration, taking into account a time – and position-depended factor forcing the vibration.

In this paper, the problem of acoustic wave emission by a lip restrained circular plate placed in a rigid, planer baffle is considered. A plate is enough thin as well as the effect of medium forcing the vibration is enough large to omitt the influence of losses including the forced vibration created by a self-acoustics field. It has been assumed that the vibratory system is found in the lossless and homogeneous liquid medium with the small value of self-resistance.

Assuming the known sinusoidal varied in time surface distribution of factor forcing the vibration, the directional characteristic has been established.

Expressions, to be here obtained for the resonance frequencies are reduced to the known formulae, formely established in [5].

Results of numerical calculations have been presented graphically.

Based on the results here obtained, the continuation of analysis will be possible in emission of energy vibration, the exact application of which is useful for the sound emission problems of the vibrated plates.

2. Assumptions of analysis

A forced transversal vibration of a homogeneous circular plate for which the energy losses don't occur as well as the local of plate stimulated by the surrounded environment is neglected, can be described by the following equation, [3]

$$B\nabla^4 \eta(r, t) + \varrho h \frac{\partial^2 \eta(r, t)}{\partial t^2} = f(r, t), \tag{1}$$

where

$$B = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

denotes the bending stiffness of plate, η denotes transversely displacement of point located on the plate surface, h – denotes the width of plate, E denotes Young modulus, ν denotes Poisson coefficient, ϱ denotes volumen density of plate. A medium, for which the influence of a radiated field of plate on the form of vibration can be omitted in Eq. (1), is identified, for example with the air possesing small value of the self-resistance. The theory of bending of thin plates is applied for the plate thickness h, fullfield the inequality, [2].

$$h \leqslant 0.1\, D, \tag{2a}$$

where D denotes the diameter of plate.

Assuming that a factor forcing the vibration is identified with the external pressure

$$f(r,\, t) = f(r) \exp(i\omega t), \tag{3}$$

an amplitude of which has the following form

$$f(r) = \begin{cases} f_0 & \text{for} \quad 0 < r < a_0 \\ 0 & \text{for} \quad a_0 < r < a \end{cases} \tag{4}$$

where $f_0 = \text{const}$.

From the practical point of view, this kind of extorsion can be realized for example by two surface circular electrodes parallel to the plate surface with radius $a_0 < a$, where denotes radius of plate.

Equation of vibration (1) for the extortion induced by external factor Eqs. (3), (4) has the following form

$$\begin{aligned} \eta_1(r)/\eta_0 = 1 - \frac{\gamma_0}{2S(\gamma)} \Big\{ \frac{1}{\gamma} I_1(\gamma_0) + \frac{\pi}{2} I_1(\gamma) [J_1(\gamma_0) N_0(\gamma) + \\ - J_0(\gamma) N_1(\gamma_0)] - \frac{\pi}{2} I_0(\gamma) [J_1(\gamma) N_1(\gamma_0) - J_1(\gamma_0) N_1(\gamma)] \Big\} \\ \times J_0(kr) + \frac{\gamma_0}{2S(\gamma)} \Big\{ \frac{1}{\gamma} J_1(\gamma_0) + J_1(\gamma) [I_1(\gamma_0) K_0(\gamma) + I_0(\gamma) \\ \times K_1(\gamma_0)] - J_0(\gamma) [I_1(\gamma_0) K_1(\gamma) - I_1(\gamma) K_1(\gamma_0)] \Big\} I_0(kr) \end{aligned} \tag{5}$$

for $0 < r < a_0$,

$$\begin{aligned} \eta_2(r)/\eta_0 = -\frac{\gamma_0}{2S(\gamma)} \Big\{ \frac{1}{\gamma} I_1(\gamma_0) + \frac{\pi}{2} J_1(\gamma_0) [N_0(\gamma) I_1(\gamma) + N_1(\gamma) I_0(\gamma)] \Big\} J_0(kr) \\ - \frac{\gamma_0}{2S(\gamma)} \Big\{ \frac{1}{\gamma} J_1(\gamma_0) + I_1(\gamma_0) [K_0(\gamma) \times J_1(\gamma) - K_1(\gamma) J_0(\gamma)] \Big\} I_0(kr) \\ + \frac{\gamma_0}{2} \Big[I_1(\gamma_0) K_0(kr) + \frac{\pi}{2} J_1(\gamma_0) N_0(kr) \Big] \end{aligned} \tag{6}$$

for $a_0 < r < a$, furthermore the following notations have been derived

$$\gamma = ka, \gamma_0 = ka_0, S(\gamma) = J_0(\gamma) I_1(\gamma) + I_0(\gamma) J_1(\gamma) \tag{7}$$

and

$$\eta_0 = -\frac{f_0}{Bk^4}, \quad k^2 = \omega \left[\frac{M}{B}\right]^{\frac{1}{2}}, \tag{8}$$

where $M = \varrho h$ denotes a mass of plate divided by the unit of area. The time term, defined by the notation of $\exp(i\omega t)$ is omitted in text begining from Eq. (5).

In the particular case for which the whole surface of plate is excited to vibration by a factor different from zero ($a_0 = a$), Eq. (5) and Eq. (6) are reduced to the following form

$$\eta_1(r)/\eta_0 = 1 - \frac{1}{S(\gamma)} [I_1(\gamma) J_0(kr) + J_1(\gamma) I_0(kr)] \tag{9}$$

and $\eta_2(r)/\eta_0 = 0$

3. Acoustic pressure in the Fraunhofer zone

A distribution of acoustic pressure in the Fraunhofer zone of the source vibrated in a rigid, planer baffle is calculated based on the relation, [3]

$$p(R, \theta, \varphi) = \frac{i\varrho_0 \omega \exp(-ik_0 R)}{2\pi} \frac{}{R} \int_{\delta_0} v(r_0, \varphi_0) \times \exp[ik_0 r_0 \sin\theta \cos(\varphi - \varphi_0)] \, d\delta_0 \tag{10}$$

for $\frac{1}{2} k_0 r_0 \left(\frac{r_0}{R}\right) \ll 1$, furthermore R, θ, φ denote the spherical coordinates of a point of field, r_0, φ_0 denote the polar coordinates of a point of the source, ϱ_0 denotes a density of fluid medium, $k_0 = \frac{2\pi}{\lambda}$, $\delta_0 = \pi a^2$.

In the case of circular plate excited to axially symmetric vibration, Eqs. (3), (4) the rate of vibration is independent of the angular variable φ_0. Assuming also relation

$$v(r_0) = i\omega\eta(r_0) \tag{11}$$

Eq. (10) defined for acoustic pressure is reduced to the form

$$p(R, \theta) = -\varrho_0 \omega^2 \frac{\exp(-ik_0 R)}{R} \Big[\int_0^{a_0} \eta_1(r_0) J_0(k_0 r_0 \sin\theta) r_0 \, dr_0 + \int_{a_0}^{a} \eta_2(r_0) J_0(k_0 r_0 \sin\theta) r_0 \, dr_0 \Big]. \tag{12}$$

After integration we obtain relation

$$p(R,\theta)=\frac{\varrho_0 f_0 \varepsilon^2 a^2}{2M}\frac{\exp(-ik_0 R)}{R} \times \frac{2J_1(\varepsilon x)/(\varepsilon x)-U(\varepsilon,\gamma)J_1(x)x/\gamma - W(\varepsilon,\gamma)J_0(x)}{1-(x/\gamma)^4\sin^4\theta}, \tag{13}$$

where $\varepsilon_0 = a_0/a$, $x = k_0 a \sin\theta$ and

$$U(\varepsilon,\gamma)=\frac{2}{\varepsilon\gamma S}[J_1(\varepsilon\gamma)I_0(\gamma)-I_1(\varepsilon\gamma)J_0(\gamma)], \tag{14}$$

$$W(\varepsilon,\gamma)=\frac{2}{\varepsilon\gamma S}[J_1(\varepsilon\gamma)I_1(\gamma)+I_1(\varepsilon\gamma)J_1(\gamma)]. \tag{15}$$

For the principal direction i.e. $\theta = 0$, we obtain the following relation for acoustic pressure

$$p_0=\frac{\varrho_0 f_0 \varepsilon^2 a^2[1-W(\varepsilon,\gamma)]}{2M}\frac{\exp(-ik_0 R)}{R}, \tag{16}$$

the form of which we apply for calculation of the directionality coefficient, [3]

$$K(\theta)=\frac{|p|}{|p_0|} \tag{17}$$

if $p_0 \neq 0$. But for

$$1-W(\varepsilon,\gamma)=0, \tag{18}$$

the state of vibratory system is such, that a volumen displacement of the vibrated plate is equal to zero (compare with [1]), then a directionality coefficient must be defined in another way. It can be achieved by referring the value of pressure $p(R,\theta)$ to that of pressure $p'(R,\theta_0)$ in such direction θ_0 for which this value reaches maximum.

In the case of coincidence between a frequency factor forcing vibration and a frequency of proper vibration, the damping effects arr negligible, i.e.

$$\gamma=\gamma_n, \quad S(\gamma_n)=0 \tag{19}$$

we obtain relation

$$K_n(\theta)=\lim_{\gamma\to\gamma_n} K(\theta)=\frac{|J_0(x)-xJ_1(x)J_1(\gamma_n)/(\gamma_n J_1(\gamma_n))|}{|1-(x/\gamma_n)^4|}, \tag{20}$$

identical with this published in [5].

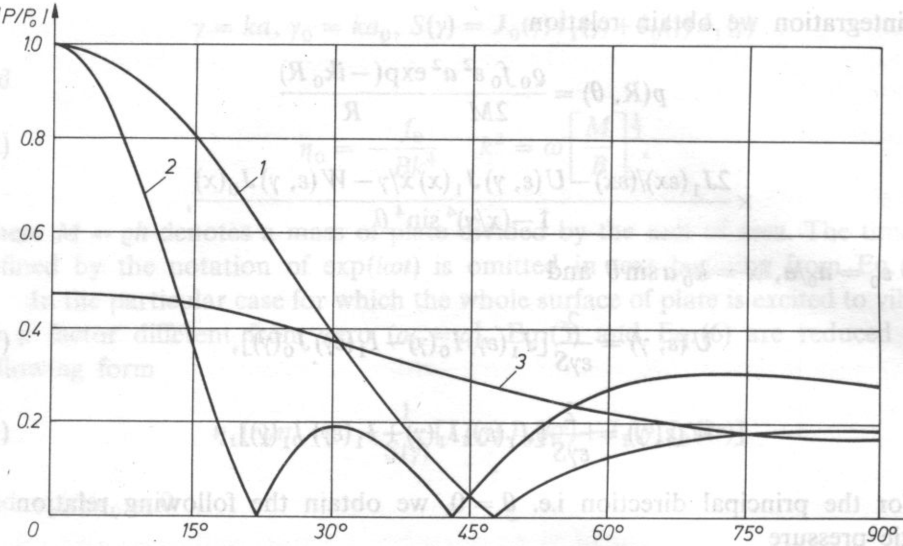

FIG. 1. The directionality coefficient of the radiation of a circular plate with different values of *ka*. Curve 1 — $ka = 10$, $k_0/k = 0.5$; curve 2 — $ka = 10$, $k_0/k = 1$, curve 3 — $ka = 5$, $k_0/k = 1$. It has been assumed that p_0 denotes the pressure on the principal direction with $ka = 10$, $a_0/a = 1$

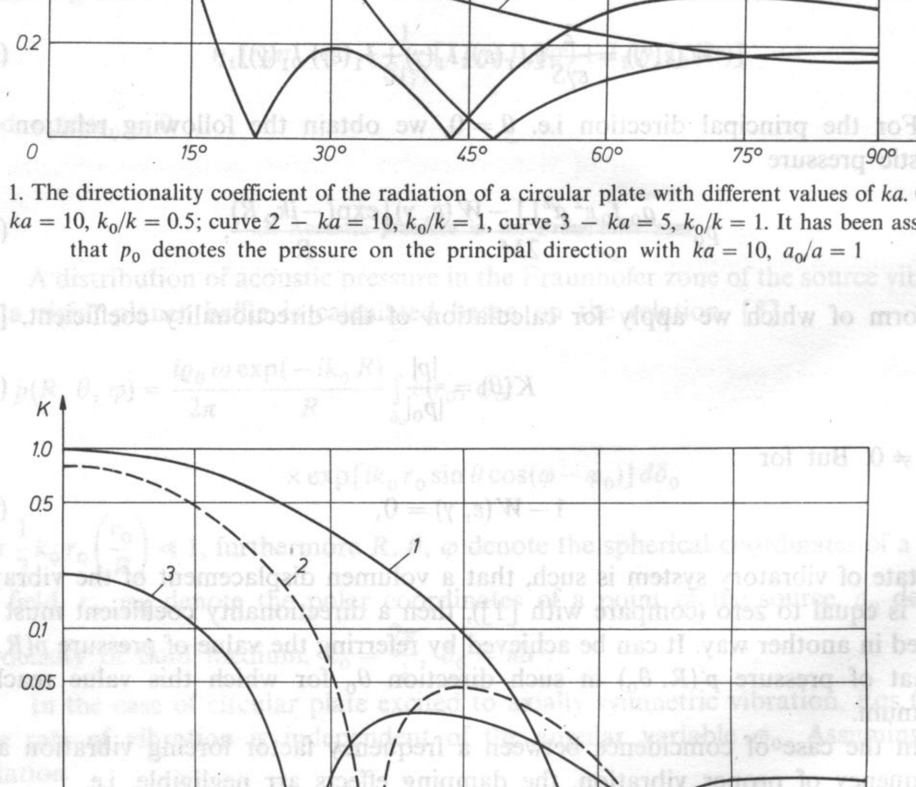

FIG. 2. The directionality coefficient of the radiation of a circular plate with different values of a_0/a. Curve 1 — $a_0/a = 1$, curve 2 — $a_0/a = 0.5$, curve 3 — $a_0/a = 0.2$. It has been assumed that $ka = 5$, $k_0/k = 2$

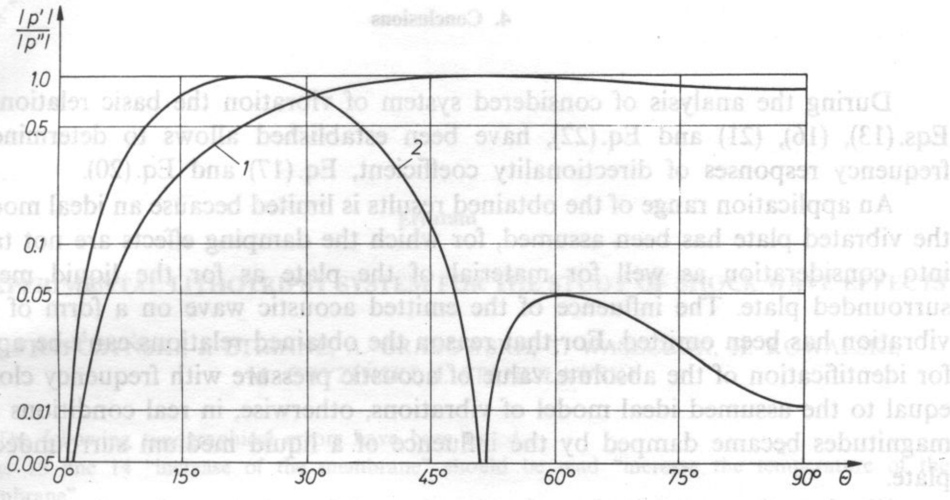

FIG. 3. The relative acoustic pressure p'/p_{max} depended on the direction of radiation, θ with $ka = 6.3064$. Curve 1 – $k_0/k = 1$, curve 2 – $k_0/k = 2$. It has been assumed that p_{max} denotes the pressure on the direction of the maximal radiation, θ_0

If a volumen displacement of plate is equal to zero, then

$$p'(R,\theta) = \frac{\varrho_0 f_0 \varepsilon^2 a \exp(-ik_0 R)}{2M\; R} \times$$
$$\times \frac{2J_1(\varepsilon x)/(\varepsilon x) + xJ_1(x)\,[J_0(\gamma) - 2J_1(\varepsilon\gamma)/(\varepsilon\gamma)]/(\gamma) - J_0(x)}{1-(x/\gamma)^4}, \tag{21}$$

moreover, if we assume $\gamma = \gamma_n$, then Eq. (21) is adequate to the description of vibration state, for which the resonance coincides with antiresonance [1]

Diagrams of directionality coefficient of the radiation of a plate stimulated to the forced vibration are shown in Fig. 1, 2 and 3. In Fig. 2, the value $p(R, \theta)$ of the acoustics pressure, Eq. (13), was referred to the value p_0 on the principal axis, moreover it was assumed that $a_0 = a$, i.e.

$$p_0 = \frac{\varrho_0 f_0 a^2 [1 - W(\gamma)]}{2M}, \quad W_0(\gamma) = \frac{4J_1(\gamma) I_1(\gamma)}{\gamma S} \tag{22}$$

In Fig. 1 the pressure p_0 has been determined for $\gamma = 10$ in the principal direction, but the value of p_{max} shown in Fig. 3, has been defined for $\gamma = \gamma_2 = 6.3064\ldots$ in direction of maximal radiation θ_0

4. Conclusions

During the analysis of considered system of vibration the basic relations, i.e. Eqs. (13), (16), (21) and Eq. (22), have been established allows to determine the frequency responses of directionality coefficient, Eq. (17) and Eq. (20).

An application range of the obtained results is limited because an ideal model of the vibrated plate has been assumed, for which the damping effects are not taking into consideration as well for material of the plate as for the liquid medium surrounded plate. The influence of the emitted acoustic wave on a form of plate vibration has been omitted. For that reason the obtained relations can't be applied for identification of the absolute value of acoustic pressure with frequency close or equal to the assumed ideal model of vibrations, otherwise, in real conditions these magnitudes became damped by the influence of a liquid medium surrounded the plate.

The loss rigorous criterion of estimation of the relative value of pressure are considered for example for the directionality coefficient.

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