INFLUENCE OF THE ACOUSTIC-STRUCTURAL COUPLINGS
UPON FREE VIBRATIONS OF MECHANICAL SYSTEMS

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The roles of acoustic-structural couplings in three entirely different aeroelastic systems are compared. The first system is a thin elastic cylindrical shell containing fluid in a coaxial annular duct, the second system is a thin plate vibrating in fluid near a rigid wall, and the third system is a simple single degree-of-freedom mechanical system, vibrating in contact with a confined volume of an acoustic medium. It is shown that the nature of the acoustic-structural coupling is identical in all the studied cases.

1. Introduction

From studies of the dynamical behaviour of cylindrical shells conveying fluid it is known that in the specific wave-number regions the strong acoustic-structural couplings exist between the vibration of a light fluid (e.g. a gas) and the shell [2, 3, 4]. Consequently, the spectrum of eigenfrequencies of the coupled shell-fluid system may be quite different from the spectra of the two separated subsystems (the shell in vacuo and the fluid in the rigid cylindrical duct).

The main idea of the paper is to point out some general and fundamental properties of coupled free vibrations of structures and acoustic media. Namely, the very similar dynamic phenomena as in the shell-fluid systems were also found for plate-fluid systems and even for systems having finite number of degrees of freedom.

2. Shell-fluid system

The natural vibration of the cylindrical shell (with the radius R and the wall thickness h) containing fluid (with the density ρ_t and characterized by the speed of sound c_0 in a coaxial duct of the radius R_d (see Fig. 1) was studied in detail in [4], and for $R_d = 0$ in [2, 3].
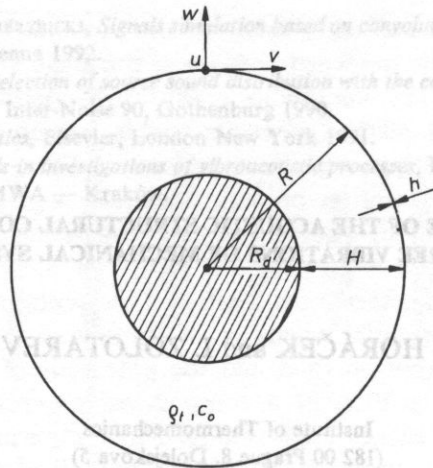
FIG. 1

The equations of motion of the thin shell can be written in the following form

$$\left[K + \frac{\rho_s R^2 (1-v^2)}{E} I \frac{\partial^2}{\partial t^2} \right] \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{R^2 (1-v^2)}{Eh} \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix}, \tag{1}$$

where K is the Goldenweizer-Novozhilov's structural operator, I is a 3×3 unit matrix; ρ_s, E and v are the density, Young's modulus and Poisson's ratio of the shell material; u, v and w are the axial, circumferential and radial displacements of the shell, respectively; and p is the acoustic perturbation pressure.

Following closely BOLOTIN's work [1], it is possible for a structural wave of the type

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} e^{i(\omega t - \bar{\alpha} x - n\theta)}, \tag{2}$$

to derive the perturbation pressure p for the potential flow field in the annulus in the form

$$p = \rho_t R \, \mu_n (\beta R, \beta R_d) \, \omega^2 \, W \, e^{i(\omega t - \bar{\alpha} x - n\theta)}, \tag{3}$$

where ω is the circular frequency, $\bar{\alpha}$ and n are the wave-numbers in the axial (x) and circumferential (θ) directions; U, V, W are the components of amplitude of vibration of the shell. The function μ_n, characterizes the added mass coefficient of a compressible fluid in the annulus and is given by the formula:

$$\mu_n (\beta R, \beta R_d) = \frac{1}{\beta R} \cdot \frac{{}^1\phi_n(\beta R) \, {}^2\phi_n'(\beta R_d) - {}^1\phi_n'(\beta R_d) \, {}^2\phi_n(\beta R)}{{}^1\phi_n'(\beta R) \, {}^2\phi_n'(\beta R_d) - {}^1\phi_n'(\beta R_d) \, {}^2\phi_n'(\beta R)}, \tag{4}$$

where

$$\beta = \left| \bar{\alpha}^2 - \left(\frac{\omega}{c_0}\right)^2 \right|^{1/2}, \tag{5}$$

$$^1\phi_n = I_n \quad \text{and} \quad ^2\phi_n = K_n \quad \text{for} \quad \bar{\alpha} > \frac{\omega}{c_0}, \tag{6}$$

$$^1\phi_n = J_n \quad \text{and} \quad ^2\phi_n = Y_n \quad \text{for} \quad \bar{\alpha} < \frac{\omega}{c_0}, \tag{7}$$

I_n, K_n, J_n, Y_n are the Bessel functions and the primes denote their derivatives. Substitution of the expressions (2) and (3) into Eq. (1) yields the characteristic equation for the natural frequencies of the system

$$\begin{vmatrix} f_{11}(\Omega^2) & f_{12} & f_{13} \\ f_{12} & f_{22}(\Omega^2) & f_{23} \\ f_{13} & f_{23} & f_{33}(\Omega^2) + \rho\psi(\Omega) \end{vmatrix} = 0, \tag{8}$$

where the following dimensionless variables were introduced

$$\Omega = \omega R \sqrt{\rho_s/E}, \quad \alpha = \bar{\alpha} R, \quad \rho = \rho_t R/\rho_s h, \quad c = c_0/\sqrt{E/\rho_s},$$

$$\xi = \beta R, \quad \xi_d = \beta R_d = \xi R_d/R; \tag{9}$$

f_{ij} are functions of $\alpha, n, v, h/R$ defining the vibration of the shell *in vacuo*, and ψ is the function, that defines the effects of fluid in the annulus

$$\psi = \pm (1 - v^2) \mu_n (\xi, \xi_d) \Omega^2. \tag{10}$$

The minus sign hold for fluid inside the thin shell $(R > R_d)$ and the plus sign for fluid outside $(R < R_d)$.

For the shell of finite length l simply supported at both ends the wavenumber α has only discrete values $\alpha_m = m\pi R/l$ $(m = 1, 2, ...)$, and for every studied mode of vibration, which is defined by the n waves in the circumferential direction and by the m halfwaves along the length of the shell, the Eq. (8) has an infinite number of solutions Ω.

The equation (8) can be rewritten in the form

$$[^1\phi_n'(\xi) \, ^2\phi_n'(\xi_d) - ^1\phi_n'(\xi_d) \, ^2\phi_n'(\xi)] F(\Omega^2) \pm \rho(1 - v^2) \Omega^2 \cdot$$

$$\cdot [^1\phi_n(\xi) \, ^2\phi_n'(\xi_d) - ^1\phi_n'(\xi_d) \, ^2\phi_n(\xi)] \cdot \frac{1}{\xi} [f_{11}(\Omega^2) f_{22}(\Omega^2) - f_{12}^2] = 0, \tag{11}$$

where $F(\Omega^2)$ is the determinant (8) for the thin shell *in vacuo*, i.e. for $\rho = 0$.

If the dimensionless fluid density ρ equals zero, i.e., for the shell *in vacuo* $(\rho_t = 0)$ or for fluid in the rigid annular duct $(h/R \rightarrow \infty)$, the shell-fluid system is uncoupled and we get from Eq. (11) three natural frequencies $\Omega_{0,i}$ $(i = 1, 2, 3)$ of the shell *in vacuo*, given

by the equation $F(\Omega^2)=0$, and from the first bracketed term of Eq. (11) the infinite number of acoustic resonant frequencies

$$\Omega_{a,k} = c\sqrt{\alpha^2 + z_{n,k}^2} \quad (k=1, 2,...),$$ (12)

where $z_{n,k}$ are the positive roots of the following equation:

$$J_n'(z)\, Y_n'(z\, R_d/R) - J_n'(z\, R_d/R)\, Y_n'(z) = 0.$$ (13)

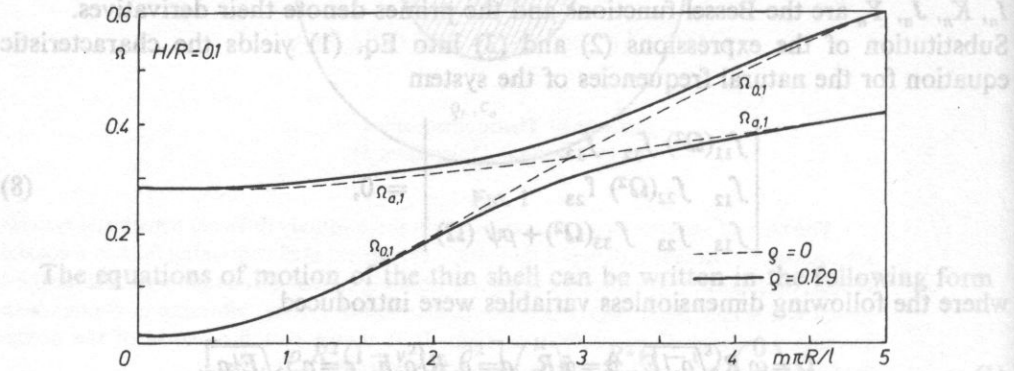For $\rho > 0$ the two subsystems (the shell in vacuo and the fluid in the rigid duct)

FIG. 2

are coupled, and from Eq. (11) one obtains an infinite number of frequencies Ω. The first two of them are shown, as an example, in Fig. 2 for the duralluminium shell-air system ($n=4$, $h/R=1/300$, $v=0.34$, $c=0.066$, $\rho=0.129$, $R_d/R=0.9$. The difference between the solutions for $\rho=0$ (dashed lines) and $\rho>0$ (solid lines), in Fig. 2 is essential near the wavenumber $m\pi R/l \approx 3$, where natural frequencies $\Omega_{0,1}$ and $\Omega_{a,1}$ of the both subsystems coincide, and where the strong acoustic-structural coupling exists. From the point of view of vibration of the shell *in vacuo* one natural frequency is here split up into the two different frequencies; the difference between them increases with the density ρ and by narrowing the annular gap width. In the regions of strong acoustic-structural coupling we cannot distinguish the acoustic modes from the structural ones. Out of this regions the vibrations are predominantly structural ($\Omega \to \Omega_{0,i}$) or acoustical ($\Omega \to \Omega_{a,k}$). The more accurate classification of the modes one gets by analysing the ratios of the perturbation flow and shell velocities [3].

3. Plate-fluid system

The second aeroelastic system studied is a thin elastic plate of thickness h, supported by an array of springs and placed near a parallel rigid wall at a distance H (see Fig. 3). Between the thin plate and the rigid wall there is an inviscid
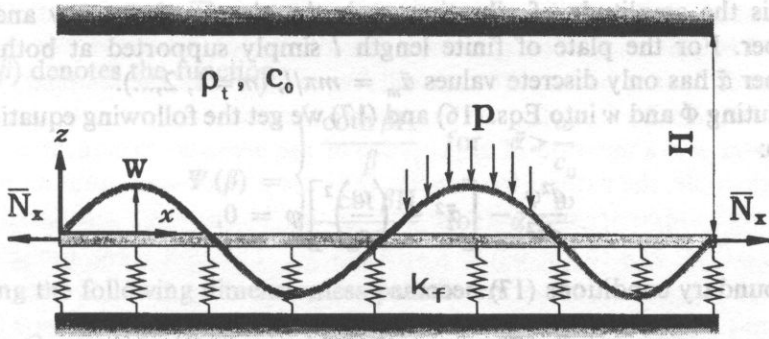
FIG. 3

compressible fluid of density ρ_t. The springs may be modelled by a continuous elastic foundatin of spring stiffness k_f per unit length of the plate. In general, the plate is subjected to a longitudinal tension $\bar{N}_x$ per unit width of the plate.

The displacement w of the plate is governed by the equation

$$D \frac{\partial^4 w}{\partial x^4} - \bar{N}_x \frac{\partial^2 w}{\partial x^2} + k_f w + \rho_s h \frac{\partial^2 w}{\partial t^2} + p = 0, \tag{14}$$

where $D = Eh^3/12\,(1-v^2)$ is the flexural rigidity of the plate; E and v are, respectively, the Young's modulus and Poisson's ratio of the plate material. The perturbation acoustic pressure p acting on the vibrating plate is given according to the linear potential 2D flow theory by the equation

$$p = -\rho_t \cdot \left(\frac{\partial \Phi}{\partial t}\right)\Big|_{z=0}, \tag{15}$$

where the perturbation flow velocity potential Φ must satisfy the wave equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{c_0^2} \frac{\partial^2 \Phi}{\partial t^2}, \tag{16}$$

and the boundary conditions

$$\frac{\partial \Phi}{\partial z}\Big|_{z=0} = \frac{\partial w}{\partial t}, \quad \frac{\partial \Phi}{\partial z}\Big|_{z=H} = 0 \tag{17}$$

on the lower vibrating and upper rigid channel walls, since no fluid can pass through the surface.

Assuming a solution in the form of travelling waves, we set

$$\begin{pmatrix} w \\ \Phi \end{pmatrix} = \begin{pmatrix} W \\ \varphi(z) \end{pmatrix} \cdot e^{i\,(\omega t - \bar{\alpha}x)}, \tag{18}$$

where W is the amplitude of vibration, ω is the circular frequency and $\bar{\alpha}$ is the wavenumber. For the plate of finite length l simply supported at both ends the wavenumber $\bar{\alpha}$ has only discrete values $\bar{\alpha}_m = m\pi/l$, $(m = 1, 2,...)$.

Substituting Φ and w into Eqs. (16) and (17) we get the following equation for the function φ:

$$\frac{d^2\varphi}{dz^2} - \left[\bar{\alpha}^2 - \left(\frac{\omega}{c_0} \right)^2 \right] \varphi = 0, \tag{19}$$

and the boundary conditions (17) become

$$\frac{\partial \varphi}{\partial z}\Big|_{z=0} = i \cdot W, \quad \frac{\partial \varphi}{\partial z}\Big|_{z=H} = 0. \tag{20}$$

Denoting

$$\beta = \sqrt{\left| \bar{\alpha}^2 - \left(\frac{\omega}{c_0} \right)^2 \right|}, \tag{21}$$

the general solution of Eq. (19) can be written in the form

$$\varphi = A \cdot e^{-\beta z} + B \cdot e^{+\beta z} \quad \text{for} \quad \bar{\alpha} > \frac{\omega}{c_0}; \tag{22}$$

$$\varphi = A \cdot \sin \beta z + B \cdot \cos \beta z \quad \text{for} \quad \bar{\alpha} < \frac{\omega}{c_0}. \tag{23}$$

Calculating the integration constants A and B from the boundary conditions (20) one obtains the following expression for the velocity potential:

$$\varphi = -i \frac{w}{\beta} \omega \cdot \frac{\cosh \beta (H-z)}{\sinh \beta H} \quad \text{for} \quad \bar{\alpha} > \frac{\omega}{c_0}; \tag{24}$$

$$\varphi = +i \frac{w}{\beta} \omega \cdot \frac{\cos \beta (H-z)}{\sin \beta H} \quad \text{for} \quad \bar{\alpha} < \frac{\omega}{c_0}, \tag{25}$$

and using Eqs. (18) and (15) we can write the solution for the perturbation pressure in the form

$$p = -\rho_t \omega^2 W \cdot \frac{\coth \beta H}{\beta} \cdot e^{i(\omega t - \bar{\alpha}x)} \quad \text{for} \quad \bar{\alpha} > \frac{\omega}{c_0};$$

$$p = +\rho_t \omega^2 W \cdot \frac{\cot \beta H}{\beta} \cdot e^{i(\omega t - \bar{\alpha}x)} \quad \text{for} \quad \bar{\alpha} < \frac{\omega}{c_0}. \tag{26}$$

Substituting p and w from Eqs. (26) and (18) into the equation of motion (14) we finally get the following characteristic equation for natural frequencies ω of the coupled plate-fluid system

$$D\tilde{\alpha}^4 + \overline{N}_x\tilde{\alpha}^2 + k_f - \rho_s h\omega^2 - \rho_t\omega^2 \cdot \Psi(\beta) = 0, \tag{27}$$

where $\Psi(\beta)$ denotes the function

$$\Psi(\beta) = \begin{cases} \dfrac{\coth\beta H}{\beta} & \text{for } \tilde{\alpha} > \dfrac{\omega}{c_0} \\[2mm] -\dfrac{\cot\beta H}{\beta} & \text{for } \tilde{\alpha} < \dfrac{\omega}{c_0} \end{cases} \tag{28}$$

Introducing the following dimensionless parameters

$$\Omega = \omega h\sqrt{\rho_s/E}, \qquad \alpha = h\tilde{\alpha},$$
$$N_x = \overline{N}_x/Eh, \qquad K = k_f h/E,$$
$$\rho = \rho_t/\rho_s, \qquad c = c_0/\sqrt{\rho_s/E}, \qquad \xi = \beta h = \sqrt{\left|\alpha^2 - \left(\dfrac{\Omega}{c}\right)^2\right|}, \tag{29}$$
$$\overline{H} = H/h,$$

the characteristic equation can be rewritten in the form

$$\frac{1}{\Psi(\xi)}\left[\frac{\alpha^4}{12(1-v^2)} + N\alpha^2 + K - \Omega^2\right] - \rho\Omega^2 = 0, \tag{30}$$

where

$$\Psi(\xi) = \begin{cases} \dfrac{\coth\xi\overline{H}}{\xi} & \text{for } \alpha > \dfrac{\Omega}{c} \\[2mm] -\dfrac{\cot\xi\overline{H}}{\xi} & \text{for } \alpha < \dfrac{\Omega}{c} \end{cases} \tag{31}$$

For the uncoupled plate and fluid systems ($\rho=0$) we get from Eq. (30) the natural frequency of the plate in vacuo ($\rho_t\to 0$)

$$\Omega_0 = \sqrt{\frac{\alpha^4}{12(1-v^2)} + N\alpha^2 + K}, \tag{32}$$

and the infinite number of acoustic resonant frequencies for the fluid in the rigid ($\rho_s\to\infty$) channel:

$$\Omega_{a,k} = c\sqrt{\alpha^2 + \left(\frac{k\pi}{\overline{H}}\right)^2}, \qquad (k=0, 1, 2,...) \tag{33}$$

because the equation

$$\frac{1}{\Psi(\xi)} \equiv \xi\,\mathrm{tg}\,\xi\overline{H} = 0 \tag{34}$$

must be satisfied.

For the coupled plate-fluid system ($\rho > 0$) we get from Eq. (30) the infinite number of natural frequencies Ω. The greater is the dimensionless density ρ, the greater becomes the difference between the natural frequencies Ω of the coupled system and the natural frequencies Ω_0 or $\Omega_{a,k}$ of the separated acoustic and structural subsystems. The parameter ρ is a measure of importance of the acoustic-structural coupling.

An an example, the natural frequencies Ω of the pre-stressed duraluminium plate interacting with the compressed air contained in the channel were calculated ($\rho = 0.01$, $c = 0.066$, $v = 0.34$, $\overline{H} = 10$, $N_x = 0.5$, $K = (0.0125)^2$). The first three branches of the solution Ω for the coupled system ($\rho = 0.01$) are shown in Fig. 4a (solid lines marked by the numbers 1, 2, 3). The solutions Ω_0 and $\Omega_{a,k}$ for the uncoupled systems ($\rho = 0$) are also plotted here thin dashed lines.
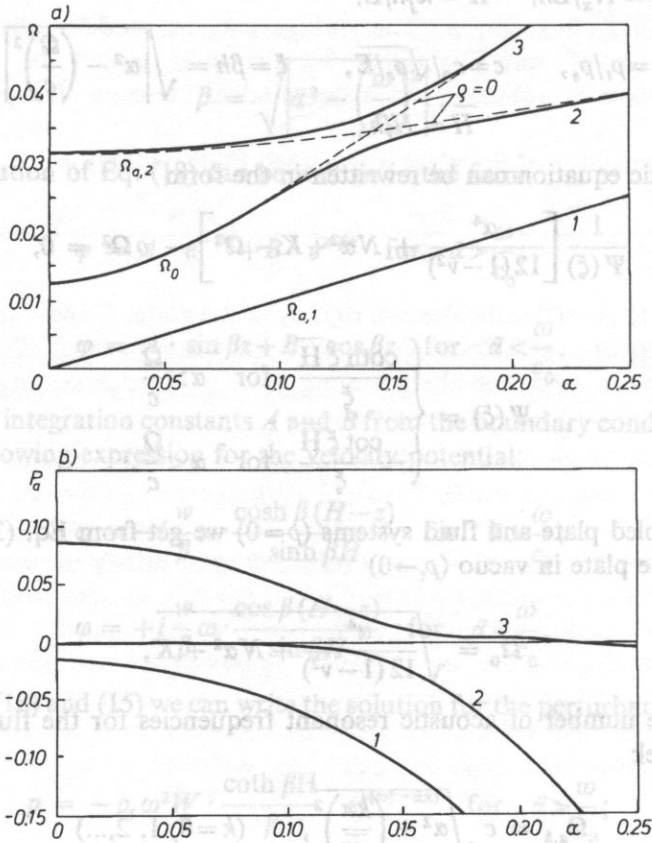

FIG. 4

The results of Figs. 2 and 4a show that the dynamical properties of the studied plate-fluid and shell-fluid systems are very close concerning the coupling of acoustic and structural natural frequencies.

The more detailed analysis of the acoustic-structural couplings gives the calculation of the amplitude p_a of the acoustic pressure on the vibrating plate surface. From Eq. (26) we get for the amplitude p_a the expression

$$p_a = -\rho E \frac{W}{h} P_a, \tag{35}$$

where $P_a = \Omega^2 \Psi(\xi)$ characterizes the ratio p_a/W of the amplitudes of the pressure and the displacement of the vibrating surface. The characteristics P_a for the studied plate-fluid system are plotted in Fig. 4b for the first three natural frequencies Ω depicted in Fig. 4a. The results displayed in Fig. 4 show, that if the mode is predominantly structural $(\Omega \to \Omega_0)$ the amplitude of the pressure is very small $(P_a \to 0)$ e.g. for the branch 2 and $\alpha > 0.1$. For predominantly acoustic modes $(\Omega \to \Omega_{a,k})$ the pressure P_a is much higher, e.g., for the branch 3 and $\alpha < 0.1$. If the acoustic and structural modes are strongly coupled (see the branches 2 and 3 near $\alpha = 0.14$), the pressure amplitudes P_a in absolute values for these two modes of vibration are comparable, and the difference between two modes is only in the opposite phase of the plate deflection and the pressure. We cannot distinguish the acoustic modes from the structural ones. From the point of view of free vibration of the plate *in vacuo* on mode of vibration is, in the region of strong acoustic structural coupling $(\alpha \approx 0.14)$, split in the two modes, similarly as for the shell-fluid system.

4. Simplified mechanical system

The single degree-of-freedom system, with mass M and stiffness k vibrating in contact of its surface S with a fluid near a rigid wall is depicted in Fig. 5a. This mechanical system can be, e.g., a very simplified model of the shell-fluid system solved in Sec. 2 of the paper. The analogy between the two systems is illustrated in Fig. 5b, where the mode $n = 4$ for the shell is schematically plotted.

The free vibration of the mechanical system is described by the equation

$$M \ddot{w}(t) + k w(t) + \int_S p(x,t) \, dS = 0, \tag{36}$$

where p is a perturbation pressure on the surface $S = \bar{h} L$. For simplicity it is supposed that p is a function of time t and the coordinate x $(0 \leqslant x \leqslant L)$; L and $\bar{h}$ are the length and the width of the surface of the mass M, respectively.

The pressure $p(x,t)$ is obtained from the 2D potential inviscid flow theory, as in Sec. 2 of the paper for the plate-fluid system. Equations (15), (16) and (17) for the fluid field are completed by the following boundary condition

$$\phi|_{x=0} = \phi|_{x=L} = 0 \tag{37}$$
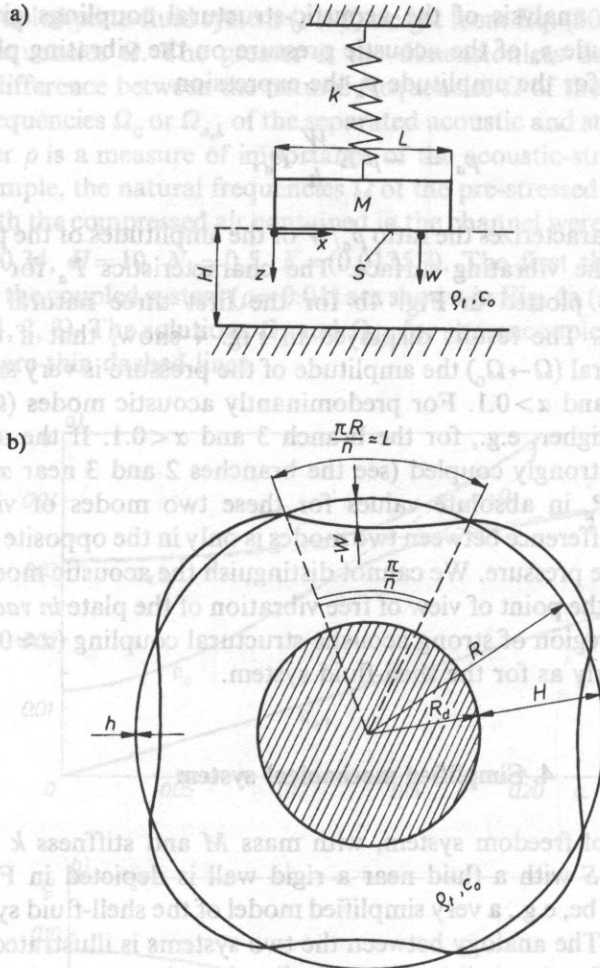
at the free edges of the volume of fluid.

a)

b)

FIG. 5

The harmonic motion of mass M with frequency and an amplitude W gives

$$w = W e^{i\omega t} \quad \text{and} \quad \phi(x, z, t) = \varphi(x, z) e^{i\omega t}, \tag{38}$$

where the potential ϕ is the solution of Eq. (16) with the boundary conditions (17) and (37). Solving this boundary problem we get for φ the solution

$$\varphi(x, z) = \sum_{k=1}^{+\infty} \sin \kappa_k x \, (A_k \cos \beta_k z + B_k \sin \beta_k z), \tag{39}$$

where

$$\beta_k = \sqrt{(\omega/c_0)^2 - \kappa_k^2}, \quad \kappa_k = k\pi/L. \tag{40}$$

Calculating the integration constants A_k, B_k from the conditions (17) and using Eq. (15), the perturbation pressure is obtained,

$$p(x,t) = \rho_t \frac{2}{L} \omega^2 W \sum_{k=1}^{+\infty} \frac{[1-(-1)^k]}{\kappa_k \beta_k} \cot(\beta_k H) \sin(\kappa_k x) e^{i\omega t}. \tag{41}$$

Substitution of the integral $F(t) = \int_0^L p(x,t)\,\bar{h}\,dx$ and w in Eq. (36), yields the characteristic equation for the natural frequencies of the coupled fluid-mechanical system

$$-\bar{\Omega}^2 + \lambda^2 + \rho\bar{\Omega}^2 \frac{8}{\pi^2} \sum_{j=0}^{+\infty} \frac{1}{(2j+1)^2} \frac{\cot(\bar{H}\sqrt{\bar{\Omega}^2-(2j+1)^2})}{\pi\bar{H}\sqrt{\bar{\Omega}^2-(2j+1)^2}} = 0, \tag{42}$$

where the following dimensionless quantities were introduced:

$$\bar{\Omega} = \omega L/\pi c_0, \quad \lambda = \sqrt{k/M}\,L/\pi c_0, \quad \rho = \rho_t SH/M, \quad \bar{H} = H/L. \tag{43}$$

The modes of vibration may be characterized by the ratio of amplitudes F_A of the aerodynamic force $F(t)$ and the amplitude of vibration

$$\bar{F} \equiv \frac{F_A/(\rho_t c_0^2 S)}{W/L} = \pi^2 \frac{\bar{H}}{\rho}(\bar{\Omega}^2 - \lambda^2). \tag{44}$$

The first two natural frequencies $\bar{\Omega}$ of the coupled fluid-mechanical system with the parameters $\bar{H}=0.13$ and $\rho=0.12$ which are the equivalents of the solved shell-fluid system ($H=R-R_d$, $L=\pi R/n$, $\bar{h}=l/m$, $M=\rho_s h\pi Rl/(m \cdot n)$), are depicted in Fig. 6 as
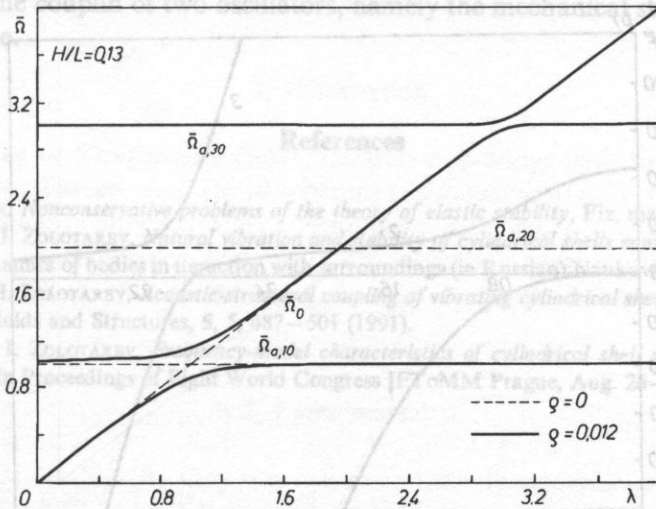

FIG. 6

functions of λ (full lines). If the dimensionless density is small ($\rho \to 0$), one obtains from Eq. (42) the natural frequency $\overline{\Omega}_0 = \lambda$ of the mechanical system *in vacuo*, and the acoustic resonant frequencies

$$\overline{\Omega}_{a,kj} = \sqrt{k^2 + (j/\overline{H})^2} \quad (k = 1, 2, ...; j = 0, 1, ...) \tag{45}$$

of the volume of fluid in the rectangular immovable slot dashed lines. Similarly to the shell-fluid system (see Fig. 2), in Fig. 6 appear the regions of λ where strong acoustic-structural couplings exist, or where the vibrations are predominantly structural ($\overline{\Omega} \to \overline{\Omega}_0$) or acoustical ($\overline{\Omega} \to \overline{\Omega}_{a,kj}$). The results of Fig. 6 are not only in
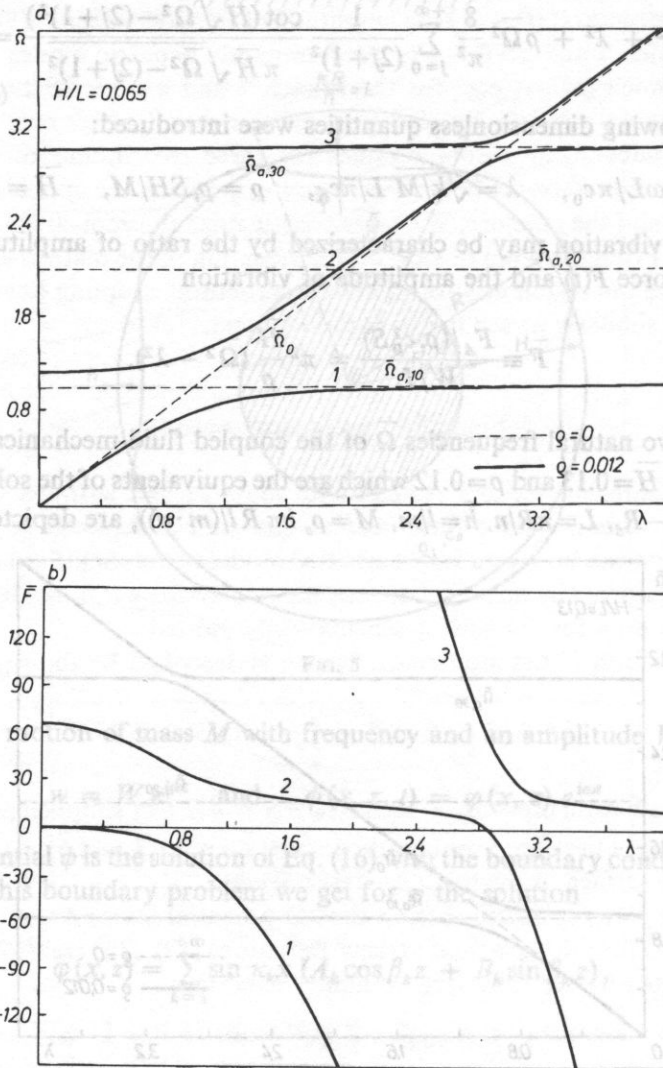

Fig. 7

qualitative agreement with the results of Fig. 2. Concerning, for example, the difference between the split natural frequencies Ω in the regions of strong coupling ($\lambda \approx 1$ in Fig. 6 and $\alpha_m = 2.8$ in Fig. 2), the difference reaches approximately 24% in both the cases studied.

By narrowing the gap width in the fluid-mechanical system (see Fig. 7a, where $\overline{H} = 0.13/2$), the regions of strong couplings substantially increase. The modes of vibration can also assorted according to the values of the force $\overline{F}$ (see Fig. 7b). If the mode is predominantly structural (see, e.g. branches 1 in Fig. 7 for $\lambda < 0.4$), then $\overline{F} \to 0$. If the mode is predominantly acoustical, then $\overline{F} \to \pm \infty$ (see, e.g., branches 1 for $\lambda > 2.4$). In the regions of strong acoustic-structural coupling (e.g. for branches 1 and 2 near $\lambda = 1$) we cannot distinguish the structural modes from the acoustic ones, because the absolute values of the aerodynamic force $\overline{F}$ for both the coupled natural frequencies Ω are practically identical.

5. Conclusion

It was shown that in fluid-elastic systems the strongest acoustic-structural couplings exist if the resonances of acoustic and mechanical subsystems are close to each other. In this case the coupled fluid-mechanical system has two different natural frequencies which are neither purely acoustic nor purely structural, and the dynamic properties of this system can not be studied separately for the structural and acoustical subsystems. Even a light medium can significantly change the spectrum of natural frequencies of a structure.

In all examples studied it was proved that the nature of the acoustic-structural couplings is the couplin of two oscillators, namely the mechanical structural and the acoustical one.

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