

## VIBRATIONS OF CIRCULAR PLATE INTERACTING WITH AN IDEAL COMPRESSIBLE FLUID

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In this work, numerical simulations describing the circular plate vibration suppression are presented. It was assumed that the vibrating plate is clamped at the circumference of a planar finite baffle and that it interacts with an ideal homogeneous compressible fluid. The formal solution of the fluid-plate-coupled equation is given for a plate driven by a harmonic surface force with constant density; the state-space realisation of the model is given. Three parameters that characterise fluid loading, internal damping of the plate material and the ratio of the plate radius to the baffle size are included in this model. The modern control theory is then applied to the system state-space equation. An optimal reduction of the plate vibrations was obtained for the point control force located centrally using a linear quadratic regulator (LQR). The simulations of the active attenuation of the plate vibrations were made with a *Simulink/Matlab*<sup>®</sup> computer program. The results indicate that it is possible to achieve a significant reduction of the vibration amplitude using only one control force.

### 1. Introduction

The determination of the real vibration source deflection is a very important element of the active control. It is well known that when a structure radiates into air, the radiation field generated by the structure does not contribute significantly to the surface velocity distribution and the interaction effect is negligible. This assumption is often referred to as the “uncoupled” assumption. However, for structures radiating into relatively dense fluids, such as water, the effect of the radiated sound field on the structural response cannot be ignored. In this case the acoustic pressure generated by the structure reacts with the source surface and changes its response. As a result of the fluid coupling, the response of the structure can be significantly different in fluid from those in vacuum or air [1].

Another problem arises from the fact that vibrating plates are usually characterised by low-frequency harmonic vibrations. This means that it can be very difficult to satisfy the condition of an “infinite baffle”, which is often applied for the calculation of acoustical quantities because of the constructional reasons. In such situations the lengths of the emitted waves are comparable with the geometric size of the source and the finite baffle

dimension has an influence on the system acoustic radiation [6, 11, 15]. To develop successfully an effective solution of the active vibration control solution it is necessary to take into account the phenomena described above in the mathematical model of the considered system.

There are many cases of practical interest to the industry and marine engineering in which the control of the sound and vibrations of plates interacting with a fluid is important. Active vibration control of a flexible structure is a subject that has been vastly researched and described in the recent years. Knyazew and Tartakovskii (1967) were the first who investigated the control of sound radiation using control forces on the vibrating structure [17]. A large number of studies on the active vibration control have been reported. In those studies the classical control, feedforward control, modern control and robust control have been used [17 and references cited inside]. The more recently published research works, dealing with the active control of harmonic sound radiation from planar vibrating structures situated in an infinite baffle, make use of either acoustic or vibration control sources [4, 5, 14]. The amplitudes of the control forces are achieved by applying point forces; the quadratic optimisation is used to calculate the optimal control gains that are necessary to minimise a performance index (cost function) proportional to the radiated acoustic power or to the acoustic pressure. However, reducing structural source vibrations can increase the life of underwater equipment as well as decrease the noise radiated into the surrounding medium. Another approach consists in the suppression of the vibrations of the structure. This work is aimed at this subject. The optimal reduction of plate vibrations is obtained for the point control force located centrally using a linear-quadratic regulator (LQR). The method presented does not assure, however, that the radiated sound will be properly reduced in each case.

To the author's knowledge, the problem of the cancellation of the active vibration of a plate located in a finite baffle and interacting with a fluid has not been treated in the literature, but different aspects of this problem are dealt with in various separate papers. The influence of the interaction fluid with the with the radiation of the circular plate has been considered by several authors [1, 7, 9, 12, 13]. In some works the radiation of sources vibrating in a finite baffle have been investigated. In most of them this problem has been solved by applying the properties of the oblate spheroidal coordinates system [6, 11, 15]. For a circular plate supplied with a finite rigid baffle, the oblate spheroid is also particularly suited for the study of sound radiation. Therefore the basic quantities that characterise the acoustic field were calculated by the author in a similar way [8, 9, 11].

In this paper, the problem of the active vibration control of a circular fluid-loaded plate is analysed. It was assumed that the plate excited harmonically at low frequencies is clamped at the circumference of a limited baffle and that it radiates into a moderately "heavy" fluid. The fluid-plate coupled partial differential equation had been solved previously [8]. The determination of the acoustic pressure is based on the admissible functions for the homogeneous plate in vacuo and on the properties of the oblate spheroidal coordinates. Modern control theory is applied to reduce the considered plate vibrations using a linear-quadratic regulator (LQR) with position and vibration velocity errors feedback signals. In order to design the optimal controller, the equation of motion of a fluid loaded

plate driven by a primary external force is expressed in the state-space form. The secondary control force is determined by solving the poles placement problem at the desired locations given by the Ackermann's formula. Graphical representations of the results are presented.

### 2. The fluid-plate coupled equation

A circular thin plate of radius  $a$  and thickness  $H$  is surrounded by a lossless liquid medium with static a density  $\rho_0$ . It is assumed that the plate clamped in a flat, rigid motionless and finite baffle of radius  $b$  is made of a homogeneous isotropic material with density  $\rho$ , Poisson's ratio  $\nu$ , Young's modulus  $E$  and has a Kelvin-Voigt internal damping.

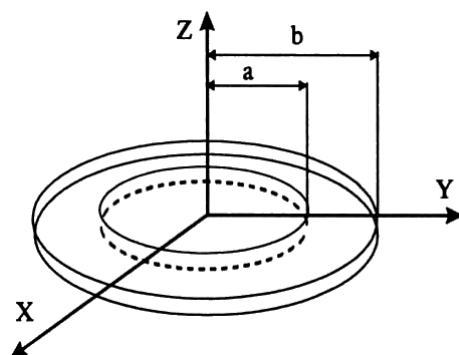


Fig. 1. A circular plate in a rigid baffle of radius  $b$ .

Under the time-harmonic external excitation with constant amplitude, which acts on the whole plate surface, the structural dynamics of the plate is reduced to a one-dimensional problem (axially symmetric vibrations) and the structural waves generate a two-dimensional fluid motion in the  $x - z$  plane. Taking into account the influence of radiated waves on the plate vibrations as well as internal damping inside the plate material, the plate differential equation of motion can be written as follows [13, 16]:

$$B\nabla^4 w(r, t) + 2\mu \frac{\partial}{\partial t} [\nabla^4 w(r, t)] + \rho H \frac{\partial^2}{\partial t^2} w(r, t) = f(r, t), \tag{2.1}$$

where  $\nabla^4 = \nabla^2 \nabla^2$ ,  $\nabla^2 = \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \right]^2$  is the Laplace operator,  $B = Eh^3/12(1 - \nu^2)$  is the bending stiffness,  $\mu$  is the coefficient of internal damping, and  $w(r, t)$  is the displacement at of points on the plate surface time  $t$  which satisfies the following boundary conditions

$$\begin{aligned} w(r, t)|_{r=a} &= 0, \\ \frac{\partial}{\partial r} w(r, t)|_{r=a} &= 0. \end{aligned} \tag{2..2}$$

The eigenvalue problem associated with above differential equation is [18]:

$$\nabla^4 w_m(r) = \lambda_m m w_m(r), \tag{2.3}$$

where  $\lambda_m$  is the  $m$ -th eigenvalue,  $m$  is the mass per unit area, and  $w_m(r)$  is the associated eigenfunction (modal shape function). For the clamped circular plate the eigenfunctions  $w_m(r)$  have the form [16]:

$$w_m(r) = u_{0m} \left[ J_0 \left( \gamma_m \frac{r}{a} \right) - \frac{J_0(\gamma_m)}{I_0(\gamma_m)} I_0 \left( \gamma_m \frac{r}{a} \right) \right], \quad (2.4)$$

where  $J_0(x)$ ,  $I_0(x)$  designate the cylinder functions,  $\gamma_m = k_m a$  is the  $m$ -th root of the frequency equation

$$J_0(\gamma_m)I_1(\gamma_m) + J_1(\gamma_m)I_0(\gamma_m) = 0, \quad m = 1, 2, \dots, \quad (2.5)$$

that describes the natural frequency of for  $m$ -th mode of the plate

$$\omega_m = \frac{1}{a^2} \gamma_m^2 \left( \frac{B}{\rho H} \right)^{1/2}. \quad (2.6)$$

The eigenfunctions have an orthonormal property if [7, 13]:

$$u_{0m} = 1/(aJ_0(k_m a)), \quad (2.7)$$

and the eigenvalues are related to natural plate frequencies by

$$\lambda_m = \omega_m^2. \quad (2.8)$$

For the analytical development being undertaken in this paper, the right hand side of Eq. (2.1) can be expressed as follows [12]:

$$f(r, t) = f_w(r, t) + f_s(r, t) + f_p(r, t). \quad (2.9)$$

It is assumed that the plate is excited to vibration by an external time-harmonic force:

$$f_w(r, t) = F_0 e^{-i\omega t} \quad (2.10)$$

and that the control force  $f_s(r, t)$ , which will minimise the radiated acoustic pressure, is a point force located at the origin:

$$f_s(r, t) = u(t) \delta(r - r_s)|_{r_s=0}, \quad (2.11)$$

where  $r_s$  is the location of the point force input on the plate surface.

The third component of the right hand side of equation (2.1),  $f_p(r, t)$ , represents the acoustic fluid-loading acting on the plate as an additional force. The value of this force exerted by the fluid on the plate surface can be calculated as follows [12]:

$$f_p(r, t) = -p(r, z, t)|_{z=0}, \quad (2.12)$$

where  $p(r, z = 0, t)$  is the acoustic pressure at the point on the surface of the plate.

The acoustic waves propagating through the fluid must satisfy the wave equation

$$\nabla^2 p(r, z, t) = \frac{1}{c^2} \frac{\partial^2 p(r, z, t)}{\partial t^2}, \quad (2.13)$$

where  $\nabla^2$  is the two-dimensional Laplace operator, and  $c$  is sound velocity in the fluid. At the fluid-structure interface, the pressure must satisfy the boundary condition [12]

$$\left. \frac{\partial p(r, z, t)}{\partial n} \right|_{r=a} = -\rho_0 \frac{\partial^2}{\partial t^2} w(r, t) = -\rho_0 \ddot{w}(r, t), \tag{2.14}$$

with  $n$  denoting the normal to the structure.

If the acoustic pressure radiates from the plate vibrating harmonically, the wave equation reduces to the Helmholtz equation

$$(\nabla^2 + k_0) p(r, z) = 0, \tag{2.15}$$

where  $p(r, z)$  is the pressure amplitude and  $k_0 = \omega/c$  is the acoustic wave number at frequency  $\omega$ .

### 3. Solution of the Helmholtz equation

The acoustic radiation from surfaces, that are separable for the Helmholtz wave equation, may be calculated by the eigenfunction expansions. For the circular plate located in a finite baffle, the problem of determining a distribution of the acoustic pressure has been obtain by using the method of separation of variables in the oblate spheroidal coordinate system (OSCS) [8]. The OSC system is particularly suitable for the study of radiation of circular sources because one limit of these shapes is approached.

Due to the symmetry of the radiated waves with respect to the  $z$  axis for  $p(r, z) = p(\eta, \xi)$ , where  $\eta, \xi$  are the spheroidal coordinates (Fig. 2), the following equation for the outgoing waves has been obtained [3, 8]:

$$p(\eta, \xi) = -i\omega\rho_0 \sum_l^\infty A_l S_{0l}^{(1)}(-ih, \eta) R_{0l}^{(3)}(-ih, i\xi), \tag{3.1}$$

where  $S_{0l}^{(1)}(-ih, \eta)$  denotes the angular spheroidal function of the first kind,  $R_{0l}^{(3)}(-ih, i\xi)$  is the radial spheroidal function of the third kind,  $A_l$  are the expansion coefficients and  $h = k_0 b$ .

The coefficients can be derived from the boundary condition (2.14), which in oblate spheroidal system has the form:

$$\left. \frac{\partial p}{\partial n} = \frac{1}{h_\xi} \frac{\partial p}{\partial \xi} \right|_{\xi=\xi_0} = \begin{cases} -\rho_0 \ddot{w}(\eta, 0, t) & \eta_0 \leq \eta \leq 1, \\ 0 & \eta_0 \geq \eta \geq -\eta_0, \\ \rho_0 \ddot{w}(\eta, 0, t), & -1 \leq \eta \leq -\eta_0. \end{cases} \tag{3.2}$$

Applying the orthogonal property of the angular spheroidal functions [3]

$$\int_{-1}^1 S_{mn}(-ih, \eta) S_{m'n'}(-ih, \eta) d\eta = \delta_{mn'} N_{nm}, \tag{3.3}$$

it is possible to determine an expression for the eigenfunction expansion coefficients

$$A_l = \frac{ibW_l(-ih, t)}{\omega \frac{\partial R_{0l}^{(3)}(-ih, 0)}{\partial \xi} N_{0l}}, \tag{3.4}$$

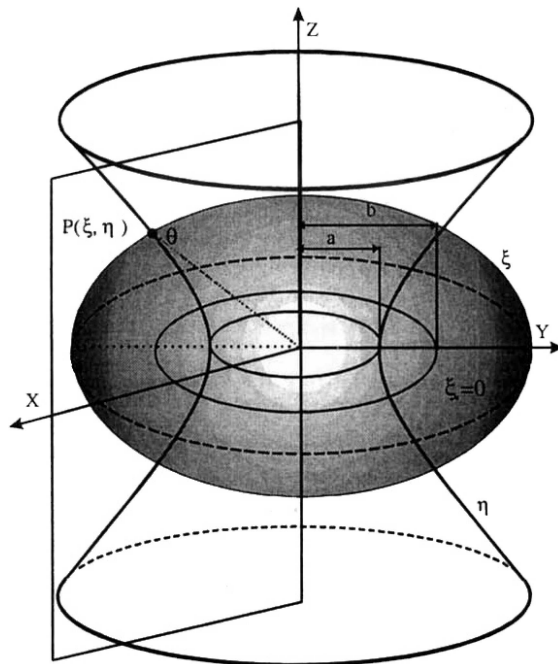


Fig. 2. Geometry of the system.

where  $N_{0l}$  denotes the norm factor and

$$W_l(-ih, t) = \int_{-1}^1 \ddot{w}(\eta, t) S_{0l}(-ih, \eta) \eta \, d\eta \tag{3.5}$$

is the characteristic function in the oblate spheroidal coordinate system. Finally, we obtain the expression for the acoustic pressure written as:

$$p(\eta, \xi) = -b\rho_0 \sum_l^\infty W_l S_{0l}^{(1)}(-ih, \eta) \frac{R_{0l}^{(3)}(-ih, i\xi)}{\frac{\partial R_{0l}^{(3)}(-ih, i0)}{\partial \xi} N_{0l}(-ih)}. \tag{3.6}$$

#### 4. Relation between the sound pressure and the plate vibration

Let us assume that the plate displacement can be expressed in the form of series

$$w(r, t) = \sum_m^\infty s_m(t) w_m(r), \tag{4.1}$$

where  $w_m(r)$  are eigenfunctions described by (2.4), and  $s(t)$  is the modal amplitude in time  $t$ . Making a double differentiation with respect to the variable  $t$ , we get:

$$\ddot{w}(r, t) = \sum_m^\infty \ddot{s}_m(t) w_m(r). \tag{4.2}$$

The series (4.2) is now expressed in the oblate spheroidal coordinate system (OSCS) using the following transformation [3]

$$r = b [(1 - \eta^2)(\xi_0^2 + 1)]^{1/2}. \tag{4.3}$$

Using the properties of the OS CS and assuming  $\xi_0 = 0$ ,  $r = b(1 - \eta^2)$ , the expressions obtained become appropriate for the plate in the finite baffle:

$$\ddot{w}(\eta, t) = \sum_m^\infty \ddot{s}_m(t)w_m(\eta). \tag{4.4}$$

The eigenfunctions take the form [8]:

$$w_n(\eta) = u_{0n} \left[ J_0 \left( \gamma_n \frac{b}{a} \sqrt{1 - \eta^2} \right) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0 \left( \gamma_n \frac{b}{a} \sqrt{1 - \eta^2} \right) \right] \tag{4.5}$$

and they remain orthonormal if

$$u_{0n} = b/(aJ_0(k_n a)). \tag{4.6}$$

Considering relation (4.4), the characteristic function (3.5) can be formulated as follows:

$$W_i(-ih, t) = \sum_m \ddot{s}_m(t) \int_{-1}^1 w_m(\eta)S_{0l}(-ih, \eta)\eta d\eta. \tag{4.7}$$

In this way, the acoustic pressure acting on the surface of the considered plate can be described in terms of a spatial and time-dependent part as follows:

$$p(\eta, \xi = 0, t) = -i \frac{\rho_0}{k_0} \sum_m^\infty \ddot{s}_m(t) \sum_{l=0}^\infty W_{ml}S_{0l}(-ih, \eta)\chi_{0l}(-ih, \xi = 0), \tag{4.8}$$

where the transfer impedance  $\chi_{0l}$  was introduced

$$\chi_{0l}(-ih, \xi) = (-ih) \frac{R_{0l}^{(3)}(-ih, i\xi)}{N_{0l}(-ih) \frac{\partial}{\partial \xi} R_{0l}^{(3)}(-ih, i0)} \tag{4.9}$$

and

$$W_{ml} = \int_{-1}^1 w_m(\eta)S_{0l}(-ih, \eta)\eta d\eta. \tag{4.10}$$

Using asymptotic properties of the spheroidal functions [3]:

$$R_{0l}^{(3)}(-ih, i\xi) \xrightarrow{\xi \rightarrow \infty} (-i)^{l+1} \frac{e^{ih\xi_\infty}}{h\xi_\infty}, \tag{4.11}$$

the far-field acoustic pressure can be calculated from the following formula:

$$p(\eta, \xi_\infty, t) = -\frac{b\rho_0 e^{ih\xi_\infty}}{h\xi_\infty} \sum_m \ddot{s}_m(t) \sum_{l=0}^\infty (-i)^{l+1} \frac{W_{ml}S_{0l}(-ih, \eta)}{R_{0l}^{(3)'}(ih, 0)N_{0l}(-ih)}. \tag{4.12}$$

### 5. System discretization

Having defined several properties of the system of interest to us, it is straightforward to re-express the equation (2.1) as a set of modal equations. As mentioned earlier, the typical approximation of such a partial differential equation can be obtained from the relationship

$$w(r, t) = \sum_m^N s_m(t)w_m(r), \quad (5.1)$$

where  $w_m(r)$  represent the known eigenfunctions described by (2.4). In theory,  $N = \infty$ . However, in practice  $N$  is considered to be a finite number suitably large for the accurately modelling of the system dynamics. In a similar way let us expand the right hand side of the plate equation of motion (2.1) into series:

$$f_w(r, t) = \sum_m^N r_m(t)w_m(r), \quad (5.2)$$

$$f_s(r, t) = \sum_m^N u_m(t)w_m(r), \quad (5.3)$$

$$f_p(r, t) = \sum_m^N z_m(t)w_m(r). \quad (5.4)$$

Inserting the above expansions into equation (2.1), multiplying both sides by the orthonormal eigenfunction  $w_n(r)$ , and integrating over the surface of the structure  $S$ , the governing equation of motion can be re-expressed as:

$$\sum_{m=1}^N [\ddot{s}_m(t) + 2\mu\omega_m^2\dot{s}_m(t) + \omega_m^2s_m(t) = r_m(t) + u_m(t) + z_m(t)], \quad (5.5)$$

where

$$\left. \begin{array}{l} r_m(t) \\ u_m(t) \\ z_m(t) \end{array} \right\} = \int_S f_j(r, t)w_m(r) ds, \quad j = w, s, p, \quad m = 1, 2, \dots, N \quad (5.6)$$

mean the modal generalised forces.

### 6. Derivation of modal generalised forces

To derive the modal generalised forces  $r_m(t)$ ,  $u_m(t)$ ,  $z_m(t)$ , it is necessary to integrate the analytical expressions according to the formula (5.6). In the case of a driving force (2.10), from the integration results:

$$r_m(t) = k_{wm}r(t), \quad (6.1)$$

where

$$k_{wm} = \frac{aF_0J_1(\gamma_m)}{\gamma_mJ_0(\gamma_m)\rho H}. \quad (6.2)$$



For the point control force  $f_s(r, t)$  described by (2.11), the modal generalised force is simply equal to the value of the eigenfunction at the control force application point:

$$u_m(t) = k_{sm}u(t), \tag{6.3}$$

$$k_{sm}(t) = \frac{1}{aJ_0(\gamma_m)\rho H} \left( 1 - \frac{J_0(\gamma_m)}{I_0(\gamma_m)} \right). \tag{6.4}$$

The third component of the right hand side of Eq. (2.1) can be calculated according to (2.12) and (4.8). As the result we obtain

$$z_m(t) = \frac{b}{a} \varepsilon_1 \frac{1}{J_0(\gamma_m)} \sum_{n=1}^N \ddot{s}_n(t) c_{mn}, \tag{6.5}$$

where  $\varepsilon_1 = \rho_0/\rho H k_0$  represents the fluid-loading parameter [1, 7] and

$$c_{mn} = \sum_{l=0}^{\infty} W_{ml} \chi_{0l}(-ih) W_{nl}^T. \tag{6.6}$$

### 7. Transformation into the state-space form

The state space modelling is based on the fact that a continuous, linear system can be characterised by a set of first order differential equations. State variables are those, which comprise the smallest set of variables, which are needed to describe completely the behaviour of the dynamics of the system of interest. Grouped together, the state variables form a state vector.

As the result of the previous calculations, the following equation describing the behaviour of the  $m$ -th mode of the considered system is obtained:

$$\ddot{s}_m(t) + 2\mu\omega_m^2\dot{s}_m(t) + \omega_m^2s_m(t) = k_{wm}r(t) + k_{sm}u(t) + \sum_n^N \frac{d^2}{dt^2} s_n(t) c_{mn}. \tag{7.1}$$

The matrix notation simplifies significantly the mathematical representation of the system, and provides a form of the problem expression, which is readily amenable to a computer solution. So, we write Eq. (7.1) in the matrix form

$$(\mathbf{I} + \mathbf{C})\ddot{\mathbf{s}}(t) + 2\mu\mathbf{\Omega}^2\dot{\mathbf{s}}(t) + \mathbf{\Omega}^2\mathbf{s}(t) = \mathbf{K}_u\mathbf{u}(t) + \mathbf{K}_w\mathbf{r}(t). \tag{7.2}$$

In the above expression  $\mathbf{I}$  denotes the identity matrix,  $\mathbf{K}_s$  and  $\mathbf{K}_w$  are coefficient vectors calculated for each mode with the expression (6.2) and (6.4), respectively,  $\mathbf{C}$  represents the fluid-plate interaction matrix obtained with (6.6),  $\mathbf{\Omega} = \text{diag}[\omega_1, \omega_2, \dots, \omega_N]$ .

The modal model presented above can be expressed now in the state space format. To do so, let us define the state vector

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{s}(t) \\ \dot{\mathbf{s}}(t) \end{bmatrix}. \tag{7.3}$$

Equation (7.2) can be expressed as:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{V} \mathbf{r}(t), \tag{7.4}$$

where the dot denotes differentiation with respect to time,  $\mathbf{x}$  is the  $(n \times 1)$  state vector,  $\mathbf{u}$  is the  $(m \times 1)$  control vector, and  $\mathbf{A}$  is the  $(n \times n)$  state matrix,  $\mathbf{B}$  is the  $(n \times m)$  control input matrix and  $\mathbf{V}$  is the  $(n \times 1)$  disturbance matrix.

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -(\mathbf{I} + \mathbf{C})^{-1}\mathbf{\Omega}^2 & -2\mu(\mathbf{I} + \mathbf{C})^{-1}\mathbf{\Omega}^2 \end{bmatrix}, \quad (7.5)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} + \mathbf{C})^{-1}\mathbf{K}_s \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} + \mathbf{C})^{-1}\mathbf{K}_w \end{bmatrix},$$

The above state-space model of the considered system will be used in the process of designing the optimal feedback control so as to suppress the plate vibrations.

### 8. Computer simulation of the feedback control

The optimal linear system theory can be used now to derive the response of the considered system including the feedback control. We want to modify the dynamic response of the system by introducing a control input  $\mathbf{u}(t)$  derived from the state feedback as follows [18]:

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t), \quad (8.1)$$

where the problem is to determination of the gain matrix  $\mathbf{K}$  which facilitates our requirements. The performance index has been chosen as:

$$J = \int_0^{\infty} \left( w^2 + \alpha \dot{w}^2 + \beta \frac{u^2}{u_{\max}^2} \right) dt, \quad (8.2)$$

where  $\alpha$  and  $\beta$  are the weight coefficients.

There are different methods, which can be used to obtain a control gain matrix minimising the optimal control performance index. In this paper, the optimal LQR controller is determined by solving the problem of poles placement at desired locations given by the Ackermann's formula [18]. This algorithm is effective for a single input system when the rank of state matrix  $\mathbf{A}$  is less or equals 10. The result of the Ackermann's formula is the automatic calculation of the matrix  $\mathbf{K}$  required to place the closed-loop poles  $\Lambda_C$  in the desired locations:

$$\Lambda_C(s) = |s\mathbf{I} - \mathbf{A}_C| = |s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{K})|, \quad (8.3)$$

where  $s = i\omega$ ,  $\mathbf{I}$  is the identity matrix,  $\mathbf{A}_C$  is the close-loop system matrix.

Table 1 contains the pole values for the open and closed-loop systems calculated with the following values of the physical parameters of the plate:

$$\rho = 2700 \text{ kg/m}^3, \quad \nu = 0.33, \quad E = 7.1 \cdot 10^{10} \text{ N/m}^2, \quad \mu = 0.00011.$$

In the simulations, the model including the four first modes of the aluminium plate of a radius 0.2 m and a thickness of 1 mm was applied. In order to determine the dynamics of

Table 1.

Open-loop poles of the system [rd/sec]	Desired poles of the closed-loop system [rd/sec]
$1.0e + 003*$	
$-4.2778 + 4.4528i$	-300
$-4.2778 - 4.4528i$	-300
$-1.3516 + 3.1968i$	-45
$-1.3516 - 3.1968i$	-45
$-0.2722 + 1.5337i$	-5
$-0.2722 - 1.5337i$	-5
$-0.0180 + 0.4000i$	-15
$-0.0180 - 0.4000i$	-15

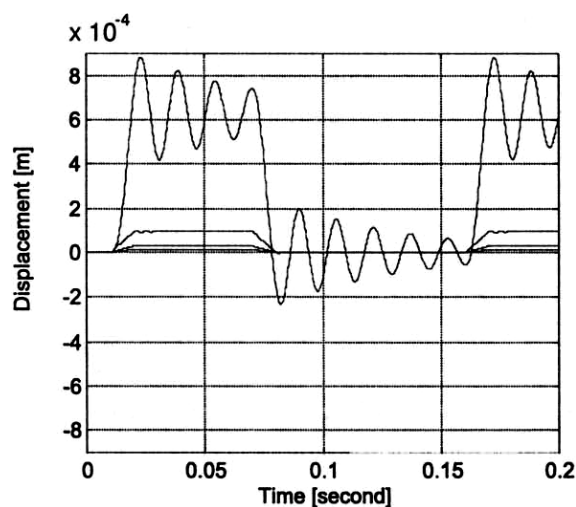


Fig. 3. The open-loop system response (four first modes) to a rectangular periodic signal with constant amplitude.

the fluid-plate system, the obtained model was first subjected to a rectangular periodic signal with constant amplitude and frequency. Figure 3 presents the behaviour of the system without control feedback.

Figure 5 presents the plate response for the gain matrix  $\mathbf{K}$  obtained. It can be seen that the control input (Fig. 4) raises during 0.01 sec until the value of 0.35, which equals 35% of maximum control signal. In this time the plate vibrations are damped sufficiently — the first mode amplitude does not exceed  $1.5 \cdot 10^{-4}$  m. The higher modes are damped almost completely. Because of the very low control signal within the time interval of 0.08 – 0.18 second, very small plate oscillations remain until next cycle. Comparing the plate vibration amplitude for the open-loop system, one can see that for the LQR controller the output signal was damped more than seven times.

The system response to a uniform periodic excitation over the plate surface, which will be realised in practice on the constructed experimental plant, has been examined as the

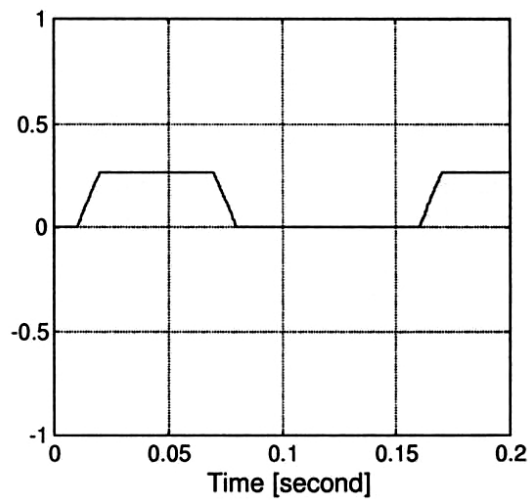


Fig. 4. Control input.

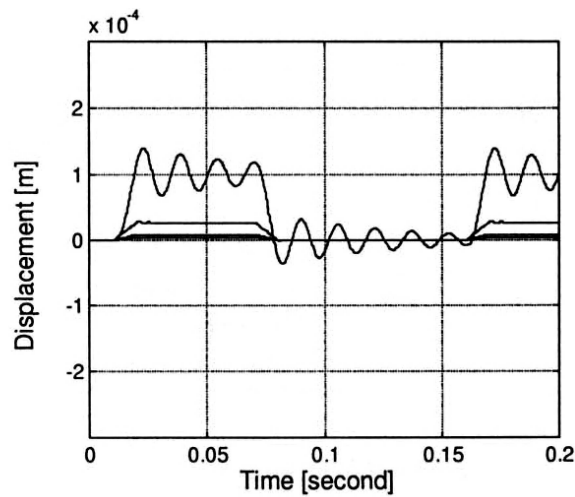


Fig. 5. Plate response.

second example. The chosen excitation of 80 Hz is close to the natural plate frequencies of the first plate mode of 63 Hz.

It can be seen in Fig. 6 that the system response is transient, however, it becomes sinusoidal when the system is allowed to run further out in time. This remaining response should be cancelled; the LQR controller designed could do it effectively as observed in Fig. 7.

The plate displacement response in Fig. 7 illustrates that for the sum of the four observed plate modes, the transient responses decay over time but due to the coupling mechanism, this modes persist as sinusoidal with a dominant frequency. The apparent

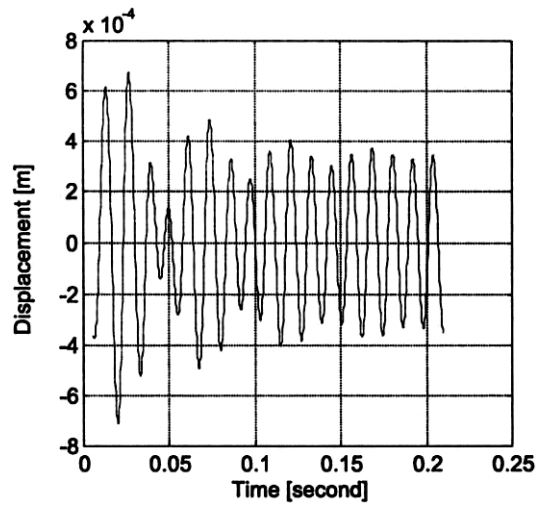


Fig. 6. The open-loop system response (the sum of four modes) to a uniform periodic excitation.

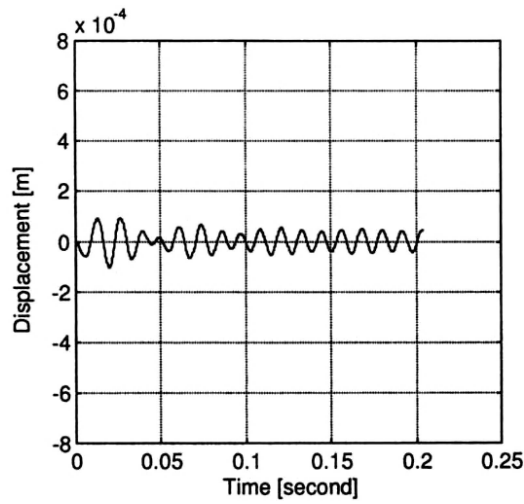


Fig. 7. The close-loop system response to a uniform periodic excitation.

beating effect is related to the difference in frequencies between the system resonant response and the forced response. The output signal is damped approximately seven times.

## 9. Conclusions

In this work the analysis of the fluid-plate system dynamics excited harmonically at low frequencies has been presented. The mathematical model of the considered system include two major difficulties: the plate vibrates in a finite baffle and the acoustic wave

radiated by the plate interacts on its surface due to the coupling mechanism. In addition the Kelvin-Voigt damping in the plate material has been taken into account.

For the system under consideration, the state-space model has been constructed. This model was used during numerical simulations of the active attenuation of plate vibrations realised in Simulink/Matlab. The optimal control theory was applied to the state equation and optimal reductions of plate vibrations were obtained for point control force located centrally using the LQR controller.

Two examples demonstrating the response of the system to two different external forces were presented. In the first case the LQR controller performance was verified by testing the behaviour of a plate driven by the rectangular periodic signal with constant amplitude and frequency. Comparing the plate vibration amplitude for the open and close-loop coupled systems, it was found that the output signal was damped more than seven times.

In a second example, a periodic forcing function with a frequency close to the plate resonance was chosen. The plate displacement response illustrates that for the sum of four observed plate modes the transient responses decay over time, but due to the coupling mechanism these modes persist as sinusoidal with a dominant frequency. These remaining mode responses should be cancelled; the designed LQR controller can do it effectively as shown in Fig. 7.

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