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ACOUSTIC MODELLING OF MACHINES

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The vibro-acoustics modelling of machines and equipment sound fields occurring in industry has been presented in the paper. Several methods allowing the sound fiels modelling on the bases of a sound pressure and sound intensity measurements have been proposed. These methods allow to model the sound fields of actual sound sources located in a free field as well as a partially diffuse field. They are also useful in modelling of large industrial installations with linear sound sources. Mathematical dependencies, algorithms of modelling and examples of calculation results are included in the paper.

1. Introduction

Investigations of vibro-acoustic processes occurring in an environment are connected with the acoustic modelling of machines and equipment. It allows the introduction of relevant mathematical models and computer calculations. The vibro-acoustic modelling can concern the following problems:

• modelling of sound sources in machines and equipment — it allows to substitute actual sound sources by specific sources of determined directional characteristics of radiation,

• modelling for vibro-acoustic synthesis,

• modelling for vibro-acoustic research and analyses of vibro-acoustic processes.

All machines, equipment, installations present in an environment form a physical system which, by applying proper simplifications, permits the transition to a substitute acoustic model. Methods of the sound field analysis in a limited zone used nowadays are based on far reaching simplifications. The actual sound sources are substituted by point sources of spherical (or close to them) directional characteristics. That, in many cases, limits the usefulness of the results. The experience of the authors indicate that there is a possibility of modelling the complex sound sources by a set of elementary sources which is able to approximate the emission of the sound power by actual industrial sources in a much better way. One of the methods is the simple utilisation of an elementary source system (consisting of monopole sound sources) for modelling the industrial sound sources. The results of sound pressure or sound intensity measurements around the tested machines can be used for assigning parameters for such models.

2. Substitute acoustic models of sound sources determined by the sound pressure method

2.1. Optimal parameters of the sound source models

When the observation of a sound source radiation is made from a distance sufficiently long, it means from the Fraunhofer's zone, the influence of the geometrical dimensions can be neglected and the sound source can be treated as quasi pointed.

In the case of harmonic waves the spatial distribution of the sound pressure around the actual sound source can be presented by the following expression [7]:

$$p(r,\theta,\varphi) = p_0(r,0,0)\cdot R_0(\theta,\varphi), \quad [\mathrm{Pa}], \tag{1}$$

where $p_0(r,\theta,\varphi) = (A/r)e^{-ikr}$ [Pa]; r, θ, φ – spherical coordinates of the observation point, A – amplitude of the actual sound source [Pa], R_0 – radiation directivity coefficient.

The sound field generated by a set of quasi pointed substitute sources can be presented as an elementary wave superposition emitted by the individual sources.

$$p_z(r,\theta,\varphi) = \sum A_j \frac{e^{ikr_j}}{r_j} R_j(\theta,\varphi), \quad [\mathrm{Pa}], \tag{2}$$

where $A_j(e^{-ikr_j}/r_j)$ [Pa] – complex amplitude of the sound pressure in the direction (0,0), r_j – distance of the j-source from the observation point.

The radiation directivity coefficient is a value dependent on the direction and position of the source against the origin of the coordinates.

The dependency of the radiation directivity coefficient on the source position can be presented as [3]:

$$R^*(\theta,\varphi) = R(\theta,\varphi)\exp\Big[ik\,(x_0\cos\varphi\sin\theta + y_0\sin\varphi\sin\theta + z_0\cos\theta)\Big], \tag{3}$$

where $R^*(\theta,\varphi)$ — radiation directivity coefficient displaced against the origin of coordinates by a vector (x_0, y_0, z_0); $R(\theta\varphi)$ — radiation directivity coefficient located at the origin of coordinates.

Comparing the distribution of sound fields generated by an actual source and a set of substitute sources, the parameters of the set (A_n) can be selected in such a way as to increase their similarity. For that purpose we are introducing a functional:

$$K = \frac{1}{4\pi A^2}\iint\limits_S |p - p_z|^2 dS, \tag{4}$$

where $p = p_0(r,\theta,\varphi)$ — sound pressure generated by the actual source [Pa], $p_z = p_z(r,\theta,\varphi)$ — total sound pressure of the set of sources [Pa], A — actual source amplitude [Pa], S — surface of a sphere with radius r [m^2].

By analogy, the introduced functional K can be compared to the mean square functional. The spherical surface integration can be approximated by summation. As the result, the quality functional would be proportional to the square sum of the sound pressure differences.

The mean square functional, due to its well developed mathematical tools, is often applied for the determination of similarities. The assumed critera are of global nature. This means that for the sound fields of the actual source and the set of substitute sources considered to be identical their consistency in all directions is required.

The assumed criterion has also a physical interpretation. It is a relative sound power of the sound source system consisting of the actual source and the substitute sources vibrating in the reverse phase. By analogy with the active methods, it is a relative sound power of the system with an active sound compensation.

Moreover, the assumed functional is dimensionless. In the case when there is no substitute source, the functional value equals 1, while for an ideal substitute source it equals 0.

For a better understanding, a notion of the functional level is introduced here:

$$L_K = -10\log(K). \tag{5}$$

It will vary from 0, in the case when there is no substitute source, to $+\infty$ in the case of ideal consistency of the actual and substitute sources. Applying the functional assumed in such a way, we are able to determine optimal parameters for the set of the substitute sources. Assuming the notations:

$$A_{jx} = \mathrm{Re}\,(Aj) \qquad \text{and} \qquad A_{jy} = \mathrm{Im}\,(Aj) \tag{6}$$

we can formulate the equations:

$$\frac{\partial K}{\partial A_{jx}} = 0, \qquad \frac{\partial K}{\partial A_{jy}} = 0. \tag{7}$$

Those formulae lead to a linear system of algebraic equations:

$$\begin{aligned} \sum_{i=1}^{n} A_{ix}U_{ji} - \sum_{i=1}^{n} A_{iy}V_{ji} &= U_{j0}\,, \\ \sum_{i=1}^{n} A_{ix}V_{ji} - \sum_{i=1}^{n} A_{iy}U_{ji} &= V_{j0}\,, \end{aligned} \tag{8}$$

where

$$\begin{aligned} U_{ji} &= \frac{1}{4\pi r^2}\iint\limits_S \left(R_i\overline{R}_j + \overline{R}_iR_j\right) dS = U_{ij}\,, \\ V_{ji} &= \frac{1}{4\pi r^2}\iint\limits_S \left(R_i\overline{R}_j - \overline{R}_iR_j\right) dS = -V_{ij}\,. \end{aligned} \tag{9}$$

In some cases the integrals can be solved analytically. When it is assumed that the substitute sources are omnidirectional (monopole), the integrals (9) are equal to [7]:

$$U_{ji} = 2\frac{\sin(kl_{ji})}{kl_{ji}} \qquad \text{and} \qquad V_{ji} = 0, \tag{10}$$

where l — distance between the substitute omnidirectional sources [m], k — wave number.

In this method of modelling, the knowledge of the sound pressure amplitude distribution as well as the phase shift of the sound pressure at separate measuring points is required.

2.2. Optimal distribution and parameters of substitute sources in acoustic models of industrial sound sources

Measurements of the sound pressure distribution around selected industrial sound sources were carried out in the Department of Mechanics and Vibro-acoustics of the University of Mining and Metallurgy in Krak˘w. Apart from the measurements of the sound pressure amplitude, the distribution of the phase angle shift between the sound pressure at the measuring points and a single selected point were also checked. A graphic presentation of the results is given in Fig. 1.

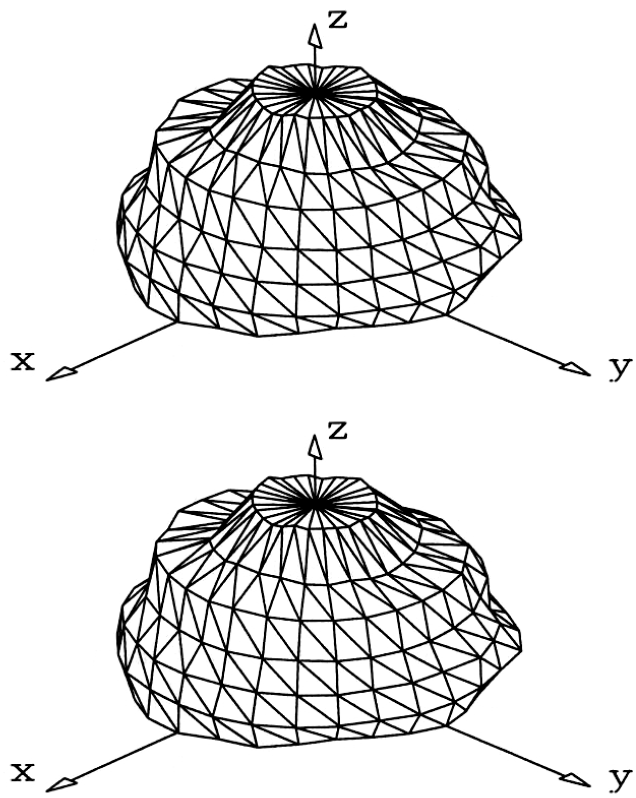

Fig. 1. Amplitude and phase characteristics at frequency = 1 kHz.

The sound field modelling of the actual sound source by an omni-directional sound field over the acoustic screen was performed on the basis of measurements of the sound pressure distribution around industrial sound sources. Formulae (8), (9) and (10) were used in the calculations.

Modelling of the actual sound sources by a greater number of substitute sources can be done in many different ways. One of the procedures and its algorithm is presented in Fig. 2.

At the beginning, the characteristic of the substitute source is assumed to be equal to that one of the actual source. Another substitute source is assumed to be located at the origin of coordinates. The criterion of the similarity gradient and the optimal parameters of the substitute source are estimated. Then the substitute source is displaced in the direction of the maximal gradient. This operation is repeated several times until the local maximum is reached. Later on, we check if the criterion is larger than required. If not, the source characteristic is changed according to the formulae in Fig. 2. A new substitute source located at the origin of coordinates is assumed and all the calculations are performed from the very beginning until the similarity criterion reaches the required value.

As the result of such a procedure, the minimal number of substitute sources in an optimal arrangement and with optimal parameters is found.

An example of the described procedure for a hand drill with the frequency of 100 Hz, is presented in Table 1.

Table 1.

Substitute source number	Location of the substitute source [m]			Sound power level of subst. source [dB]	Phase [deg]	Number of subst. sources	Similarity criterion level [dB]
	x	y	z				
1	0.02	0.11	0.00	67.50	−25.00	1	6.75
2	0.03	0.11	0.26	56.90	134.50	2	8.46
3	0.07	0.26	0.00	51.50	62.10	3	10.21
4	0.10	0.05	0.12	52.70	−134.20	4	10.73
5	−0.06	−0.09	0.27	49.10	−141.90	5	11.20
6	−0.10	0.13	0.12	50.40	−126.10	6	11.57
7	0.00	0.12	0.10	48.00	123.80	7	12.15
8	−0.10	−0.05	0.11	38.20	175.00	8	12.22
9	0.18	−0.22	0.00	38.10	−11.10	9	12.28
10	0.16	−0.03	0.00	37.10	−76.20	10	12.34

As one can see, when the hand drill is substituted by one monopoly, the functional level equals 6.75, by two → 8.46, by three → 10.21 and by ten → 12.34 [dB].

In this method it is not necessary to assume *a priori* the quantity of substitute sources. Their number depends on the distribution of the sound pressure generated by the actual source and the level set for the similarity criterion.

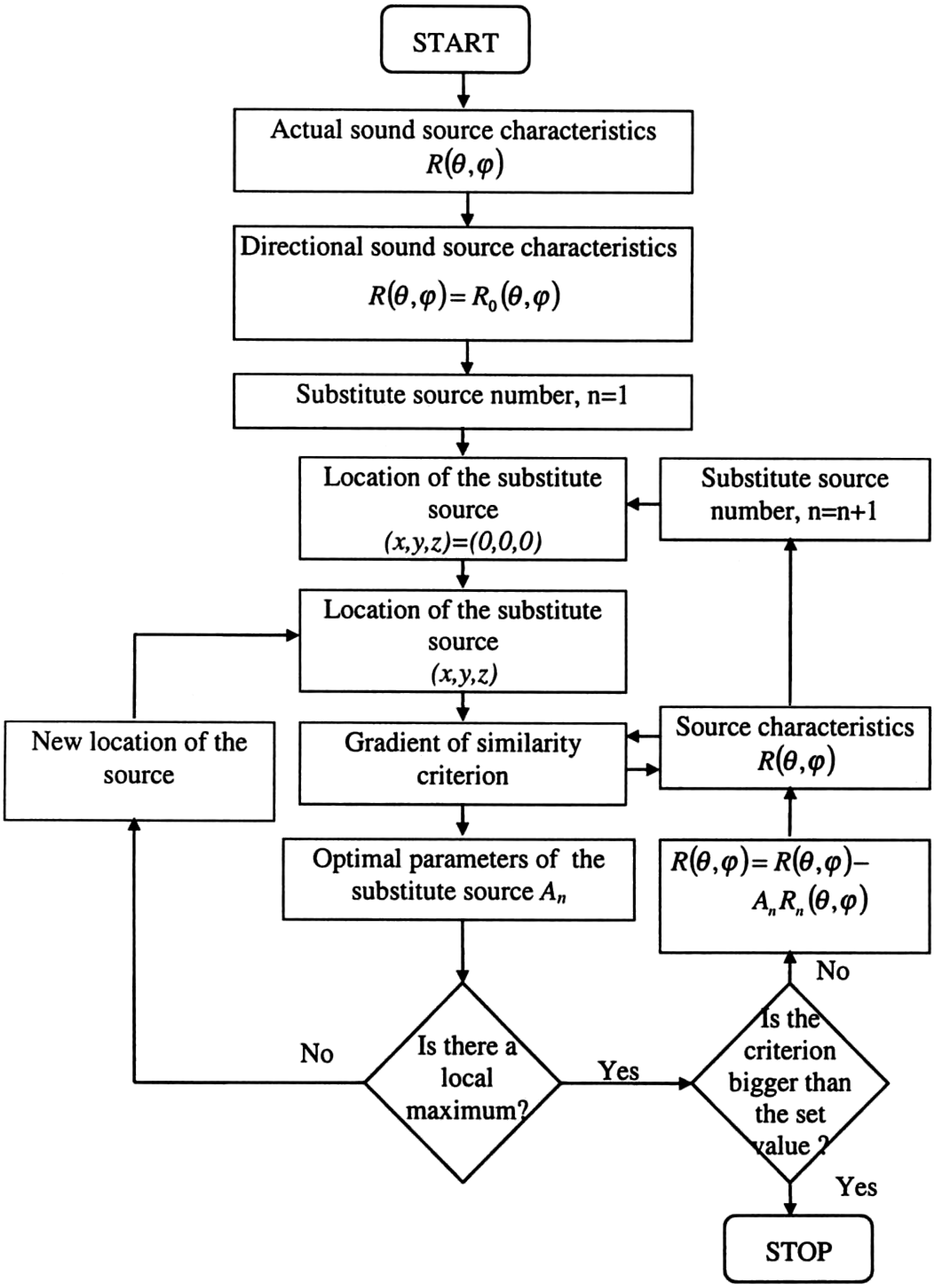

Fig. 2. The algorithm of the sound source modelling

3. Substitute models of sound sources determined by the intensity method

Measurements of the sound intensity around machines performed by a dual-microphone probe are quite often done in the industry. The results of such measurements can be utilised for the development of substitute models.

The velocity potential distribution around omnidirectional sources (monopoles) of harmonic waves can be estimated from the equation:

$$\Phi(x,y,z) = \frac{A}{r} \exp\Big[i(\omega t - kr)\Big]; \qquad [\mathrm{m}^2/\mathrm{s}], \tag{11}$$

where $r = \sqrt{x^2+y^2+z^2}$ — distance from the observation point [m], x, y, z — coordinates of the observation point [m], k — wave number [1/m], ω — angular frequency [1/s], A — source moment [m^3/s].

The sound pressure can be determined from:

$$p = \rho * \frac{\partial \Phi(x,y,z)}{\partial t} \quad [\mathrm{Pa}], \tag{12}$$

where ρ — medium density [kg/m^3].

The sound velocity in the direction of the x-axis equals:

$$v_x = -\frac{\partial \Phi(x,y,z)}{\partial x} \quad [\mathrm{m/s}] \tag{13}$$

while the sound intensity in the direction of the x-axis can be written as:

$$J_x = \frac{1}{2} p \overline{v_x} \quad [\mathrm{W/m}^2]. \tag{14}$$

After simple rearrangements, we are getting the equation for the sound intensity in the x-axis direction as:

$$J_x = \frac{A^2(i+kr)\rho\omega x}{2r^4} \quad [\mathrm{W/m}^2]. \tag{15}$$

Similarly, the equations for the sound intensity in the y-axis and z-axis directions equal:

$$J_y = \frac{A^2(i+kr)\rho\omega y}{2r^4} \qquad \text{and} \qquad J_z = \frac{A^2(i+kr)\rho\omega z}{2r^4}\,, \tag{16}$$

respectively.

The criterion of similarity (quality functional) should be introduced to enable the comparison of the sound intensity of the actual and substitute sources. A mean square criterion is very convenient due to its universal and global nature. In this case, the distribution of the sound density in every direction should be as consistent as possible. The consistence in one direction only is not sufficient. The versatility of the criterion comes from the simplicity of its mathematical description and from the linearity of the equations optimising the parameters of the substitute sources.

When restricted to the real part of the sound intensity only, the criterion of similarity has the form:

$$K = \oint |J_R - J_{zas}| dS \quad [\mathrm{W}], \tag{17}$$

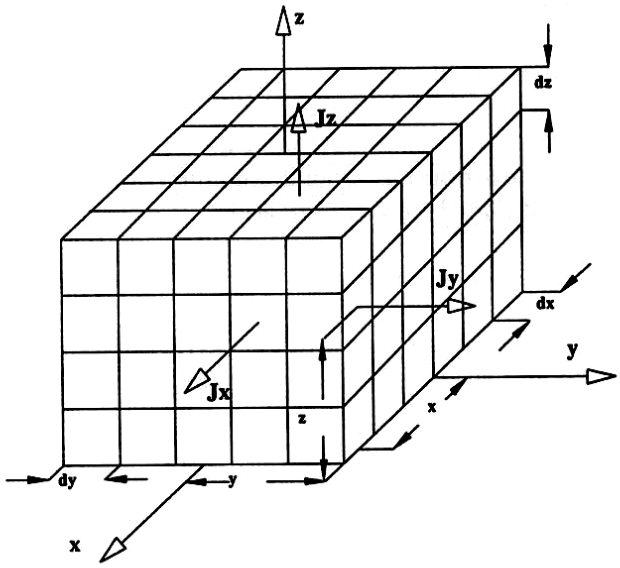

Fig. 3. The measuring grid utilised during the sound intensity measurements around machines.

where J_R — actual sound source intensity [W/m^2], J_{zas} — set of substitute sound sources intensity [W/m^2], S — elementary surface vector [m^2].

In case of discrete measurements result, e.g. those gathered at the grid points on a rectangular prism surface (Fig. 3), the criterion is as follows:

$$K = \frac{1}{N_R} \sum_{j=1}^{m} |J_R - J_{zas}| \Delta S, \tag{18}$$

where N_R — actual source sound power [W] (normative factor), m — mesh grid number, ΔS — mesh grid surface [m^2], J_{R_j}, J_{zas_j} — sound power at the j-point in the direction perpendicular to the grid surface [W/m^2] for the actual and substitute source, respectively.

A following notation of the functional quality level can be introduced:

$$L_K = -10 \log(K). \tag{19}$$

The optimal parameters A_i can be determined on the basis of the system of equations:

$$\frac{\partial K}{\partial A_i} = 0, \tag{20}$$

where A_i — complex moment of the i-source taking into account the phase shift in the sound power generated by different sources.

Applying the above method and utilising the algorithm of the optimal parameter determination, one can determine parameters of the optimal model of the actual sound source radiation with an arbitrary (chosen *a priori*) accuracy.

Table 2 illustrates results of the determination of the compressor substitute source parameters. The intensity method was applied in this example. The distribution of the substitute sources is shown in Fig. 4.

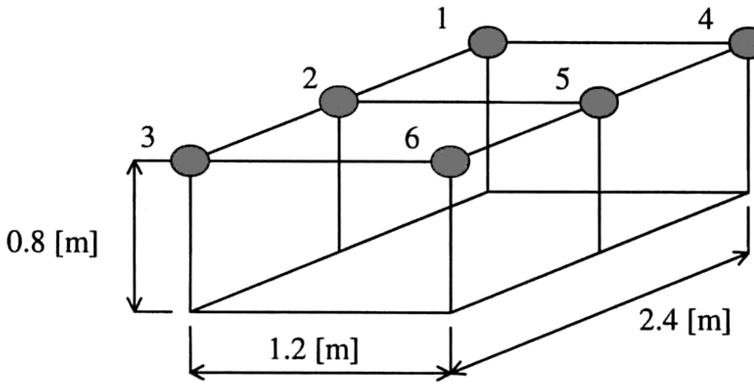

Fig. 4. Example of the substitute sources distribution scheme.

Table 2. Results of modelling of the compressor WSBW-8/220 by 6 substitute sources performed by the intensity method.

Frequency [Hz]	**103**	**130**	**163**	**205**	**259**	**326**	**410**	**516**	**649**	**818**	**1029**	**1296**
Actual source	**84.6**	**80.4**	**92.0**	**90.8**	**94.3**	**80.8**	**79.6**	**75.7**	**72.3**	**71.0**	**67.7**	**71.2**
Source No 1	47.3	48.7	50.5	50.9	52.6	48.3	43.9	43.1	40.1	39.6	38.2	37.5
Source No 2	50.0	51.5	53.1	52.9	54.5	48.8	44.4	43.3	39.6	39.5	37.8	36.7
Source No 3	51.3	54.0	56.0	59.2	61.6	67.0	54.7	67.2	60.6	58.1	55.8	59.2
Source No 4	73.3	71.1	76.5	79.2	82.8	73.0	69.6	60.4	61.3	60.0	56.9	60.6
Source No 5	78.1	77.1	87.7	88.5	89.2	74.9	74.5	70.4	67.0	67.0	62.7	68.9
Source No 6	48.6	50.6	52.7	54.8	57.0	57.3	66.4	65.5	60.3	57.8	54.0	56.2
Total of subst. sources	79.4	78.1	88.0	89.0	90.1	77.5	76.2	73.2	69.4	68.6	64.8	70.0
Similarity criterion, [dB]	**4.00**	**6.41**	**3.45**	**3.95**	**3.22**	**3.95**	**4.57**	**4.74**	**5.22**	**6.90**	**5.70**	**6.07**

4. Substitute models of pipelines

The method of the substitute sources construction presented in the previous section can be useful also in modelling of pipelines, especially in the gas reduction sequences. The reduction sequences (pipeline segments, valves, knees, reducers) were treated as a system of cylindrical sources. For the purpose of the substitute source formation, pipeline segments and linear elements of complicated shapes were substituted by a single cylindrical source of length equal that one of the actual pipe line. Knees and other small fittings (with none distinguished dimensions) were modelled by point sources.

Directional characteristics of a linear source (a system of point sources vibrating in phase and located in infinitesimal distances on the segment of length l) is given by the relationship, [8]:

$$R(\Theta) = \frac{\sin\left(\frac{kl}{2}\sin(\Theta)\right)}{\frac{kl}{2}\sin(\Theta)}, \tag{21}$$

where l — segment length [m], $k = 2\pi/\lambda$ — wave number [1/m], λ — wave length [m].

Then, the sound pressure of an arbitrary point (provided that it is located far from the radiating part, i.e. in the Fraunhofer's zone) can be calculated from:

$$p(x,y,z) = A\frac{R(x,y,z)}{r}\exp(-ikr) \quad [Pa], \tag{22}$$

where r — distance from the observation point to the end of the segment, [m].

If the location of the segment can be described by the coordinates of its beginning and end, (x_1, y_1, z_1) and (x_2, y_2, z_2), the distance from the observation point to the end of the segment will be equal to

$$r = \sqrt{(x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2} \quad [\mathrm{m}]$$

where x_s, y_s, z_s — coordinates of the middle of the segment:

$$x_s = \frac{x_1+x_2}{2}, \qquad y_s = \frac{y_1+y_2}{2}, \qquad z_s = \frac{z_1+z_2}{2}.$$

Then, sine of Θ will then be equal:

$$\sin(\Theta) = \frac{cx\cdot(x-x_s) + cy\cdot(y-y_s) + cz\cdot(z-z_s)}{R}, \tag{23}$$

where cx, cy, cz — direction cosines of the straight line on which the segment is located, given by the equations:

$$cx = \frac{x_2-x_1}{l}, \qquad cy = \frac{y_2-y_1}{l}, \qquad cz = \frac{z_2-z_1}{l}.$$

Therefore, the radiation direction characteristics will be equal to

$$R(x,y,z) = \frac{2R\cdot\sin\left(kl\frac{cx\cdot(x-x_s)+cy\cdot(y-y_s)+cz\cdot(z-z_s)}{2R}\right)}{kl(cx\cdot(x-x_s)+cy\cdot(y-y_s)+cz\cdot(z-z_s))}. \tag{24}$$

The sound pressure generated at an arbitrary point by all pipeline segments and other elements of fittings can be calculated from the equation:

$$p_z(x,y,z) = \sum_{i=1}^{n} A_i\frac{R_i(x,y,z)}{r_i}\exp(-ikr_i). \tag{25}$$

The similarity criterion for the sound fields can be given as:

$$K = \sum_{j=1}^{m}(p-p_z)^2 \quad [\mathrm{Pa}^2]. \tag{26}$$

Minimising the criterion gives the solution by the method of least squares. Therefore, the optimal parameters of sound source A_i can be found from the formula:

$$A = (S^T S)^{-1} S^T P, \tag{27}$$

where $P^T = (p_1, p_2, p_3, ...)$ — complex amplitudes (taking into account a phase shift) of the sound pressure at observation points,

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & \dots \\ s_{21} & s_{22} & s_{23} & \dots \\ s_{31} & s_{32} & s_{33} & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix},$$

where $s_{ij} = \dfrac{R_{ij}(x,y,z)}{r_{ij}} \exp(-ikr_{ij})$, i — substitute source number, j — observation point number. If we don't know the phase shifts of the sound pressure at separate points, we can assume the criterion to be:

$$K = \sum_{j=1}^{m} (p^2 - p_z^2)^2. \tag{28}$$

Using this criterion one does not get optimal parameters in a such easy way. In this case the problem requires the solution of a system of nonlinear equations.

5. Conclusions

The determination of acoustic models of machines by the pressure method requires the determination of the spatial distribution of the sound pressure. The machine under test should be placed in a dead-room and the measurements made in the Fraunhofer's zone (it means sufficiently far from the machine). This limits the applicability of the method to small machines and equipment. On the other hand, the results show smaller deviations and the procedure itself is much easier.

Large industrial installations (e.g. pipelines) are located normally in an open area. In this case it is possible to determine the sound pressure distribution around the installation in the Fraunhofer's zone on the spot. The modelling method connected strictly with the location and sizes of the actual elements seems to be optimal for such cases.

Measurements of the sound intensity around machines performed by the dual-microphone probe are very common in industry. Those results can be utilised in the construction of substitute sound models based on the intensity method. There is no need to displace the machine into a dead-room and the measurements are done close to the source. However, one has to deal with all the deviations occurring when the measurements are carried out by a dual-microphone probe in a partially diffuse field (such as normally in the industry).

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