

NONLINEARITY PARAMETER B/A OF THE LOW-SALINITY SEAWATER

E. KOZACZKA AND G. GRELOWSKA

Naval Academy
(81-919 Gdynia)

The theoretically and experimentally determined values of the nonlinearity parameter B/A , reported in the literature, were obtained predominantly for high-salinity seawater. This work contains the experimental and theoretical investigation results of nonlinearity for low-salinity seawater, especially for the Baltic Sea. The theoretical method represents a thermodynamic approach. It is based on the variation of the sound velocity with changes in pressure, temperature and salinity. Annual changes of the nonlinearity parameter B/A in the South Baltic are determined by means of this method.

The experimental method was based on the distortion of the finite amplitude sine wave emitted by a piston acoustic source. The growth of the second harmonic component is measured using a circular receiver which is coaxial with the source. In order to determine the B/A parameter, the experimental measurements were compared to the theoretical results which incorporated the nonlinear parameter. Measurements were carried out in several points of the South Baltic. The investigation results agree well in most cases. To achieve satisfactory accuracy, it is necessary to take into account the effects of both the diffraction and attenuation on the second harmonic amplitude by interpreting the measurement results.

1. Introduction

The parameter B/A determines the nonlinearity in the pressure-density relation for a liquid. The nonlinearity of the pressure-density relation and of the equation of motion causes the distortion of a finite amplitude wave as it propagates through the liquid. For the initial sine wave this distortion implicates the generation of harmonic waves, and nonlinearity induces reduction of the fundamental component. The value of the parameter B/A is of interest because it quantifies the distortion process.

The effect of distortion of the wave shape can be observed even by applying the most simplified equation for the finite amplitude wave propagation, including parameter B/A , given in the form [11]

$$\Delta p - \frac{1}{c_0^2} \frac{\delta^2 p}{\delta t^2} + \frac{\left(\frac{B}{A} + 2\right)}{2\rho_0 c_0^4} \frac{\delta^2 p^2}{\delta t^2} = 0, \quad (1)$$

where ρ_0 — equilibrium density of the medium, c_0 — speed of sound for small amplitudes, p — excess pressure, t — time.

The values of the parameter B/A for seawater, reported in the literature, were obtained predominantly for oceanic high-salinity water. This paper contains the investigation results for nonlinearity for low-salinity seawater, especially for the Baltic Sea. The nonlinearity parameter B/A has been determined using both the thermodynamic method and the acoustical one. Annual changes of the nonlinearity parameters B/A in the South Baltic have been determined by means of the thermodynamic method. To verify the data obtained in this way, a series of experimental investigations by means of the acoustical method was carried out in several points.

The nonlinearity parameter B/A for fluids is obtained from the isentropic equation of state ($S_0 = \text{const}$) in which the pressure p is expanded into a Taylor series around its equilibrium value in terms of the density (ρ):

$$p = p_0 + A \left(\frac{\rho - \rho_0}{\rho_0} \right) + \frac{B}{2} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 + \dots \quad (2)$$

Subscript "0" denotes the equilibrium value. The coefficients A and B are defined by [3]

$$A = \rho_0 \left[\left(\frac{\delta p}{\delta \rho} \right)_{S_0} \right]_{\rho = \rho_0}, \quad (3)$$

$$B = \rho_0^2 \left[\left(\frac{\delta^2 p}{\delta \rho^2} \right)_{S_0} \right]_{\rho = \rho_0}. \quad (4)$$

The ratio B/A is known as the nonlinearity parameter of medium.

2. Thermodynamic method

This method is based on the thermodynamic equations of the medium and the data obtained by measuring the changes of temperature and salinity as the functions of depth (the static pressure).

The formula for B/A was introduced by R.T. BEYER [3] and is given as follows:

$$\frac{B}{A} = 2\rho_0 c_0 \left(\frac{\delta c}{\delta p} \right)_{T, \rho = \rho_0} + \frac{2c_0 \alpha T}{C_p} \left(\frac{\delta c}{\delta T} \right)_{p, \rho = \rho_0} = \left(\frac{B}{A} \right)' + \left(\frac{B}{A} \right)'' \quad (5)$$

where T — absolute temperature, C_p — specific heat at constant pressure, $\alpha = (1/V) (\delta V / \delta T)_p$ — thermal expansion coefficient.

The first term in Eq. (5) should be determined at constant temperature, the second one — at constant pressure.

The changes in the speed of sound and thermodynamic parameters with the changes of pressure, temperature and salinity were determined by applying the relations recommended by UNESCO for computation of fundamental properties of seawater [2]. The following relations were used in the computations:

1. Equation of state for seawater $\rho(S, T, p)$ [9];
2. Speed of sound in seawater $c(S, T, p)$ [5];
3. Specific heat of seawater $C_p(S, T, p)$ [10];
4. Thermal expansion coefficient $\alpha(S, T, p)$ [6].

The range of validity of these relations is as follows:

S : from 0 to 40 PSU (Practical Salinity Unit);

T : from 0 to 40°C;

p : from 0 to 100 MPa.

Changes in the nonlinearity parameter B/A of oceanic water as a function of temperature, salinity and pressure determined by H. ENDO [8] are shown in Figs. 1–3. The thermodynamical method presented by H. ENDO was used to obtain changes in the parameter B/A for the Baltic Sea by means of Equation (5). Calculations were made basing on salinity and temperature values measured as functions of depth. Results are shown in Figs. 4–6. The average vertical distribution of the nonlinearity parameter B/A at selected stations in the South Baltic Sea region is demonstrated in the Figs. 7, 8.

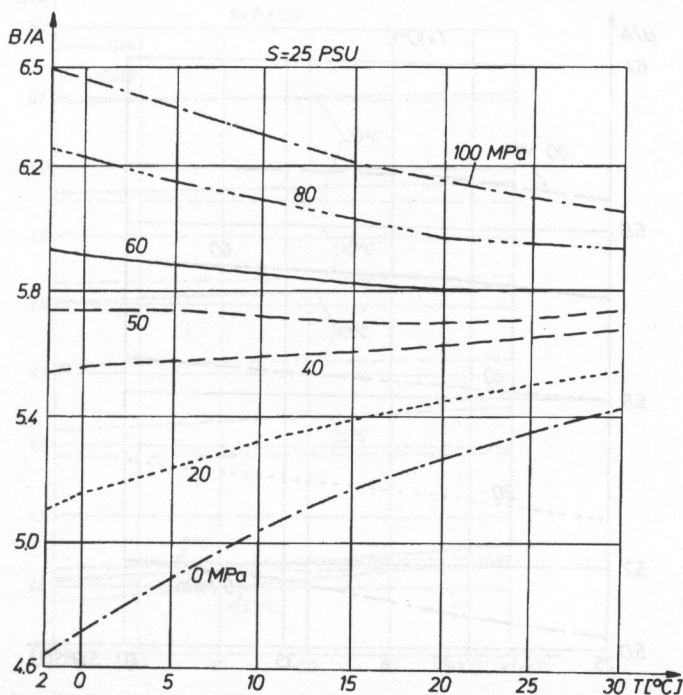


Fig. 1. Changes in the nonlinearity parameter B/A of the oceanic water as the function of temperature, for different values of pressure, given by H. ENDO [8].

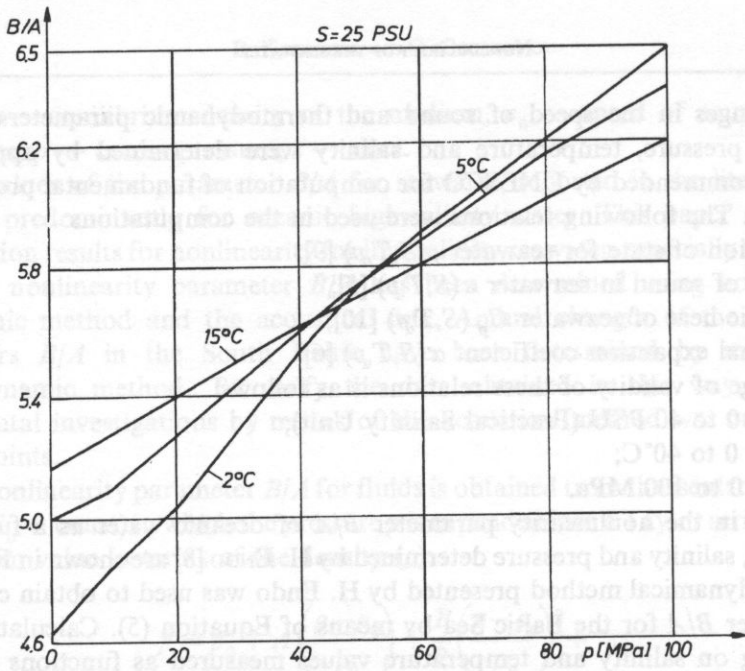


Fig. 2. Changes in the nonlinearity parameter B/A of the oceanic water as the function of pressure, for different values of temperature, given by H. ENDO [8].

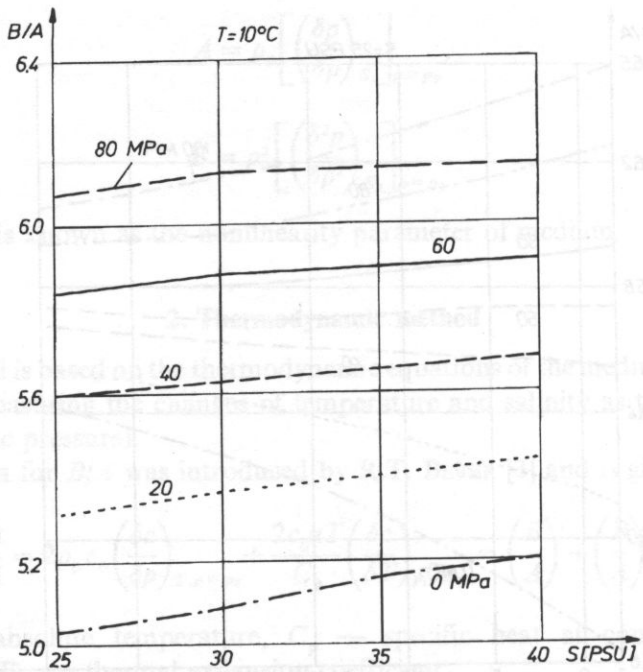


Fig. 3. Changes in the nonlinearity parameter B/A of the oceanic water as the function of salinity, for different values of pressure, given by H. ENDO [8]

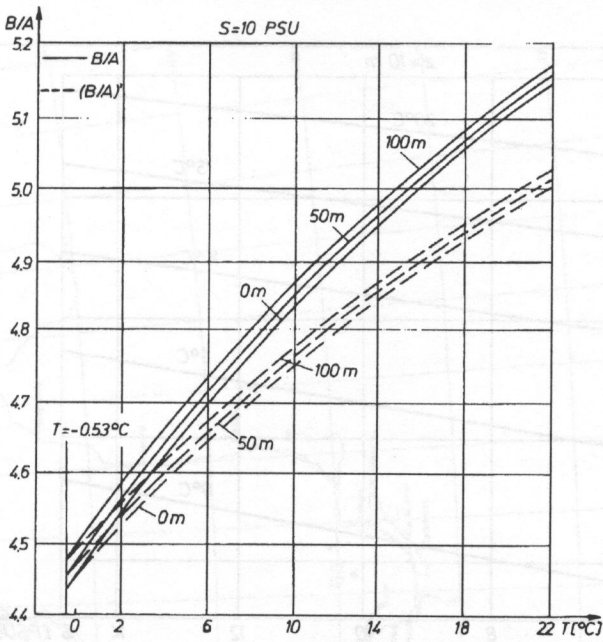


Fig. 4. Changes in the nonlinearity parameter B/A of the Baltic Sea water as the function of temperature for different values of depth.

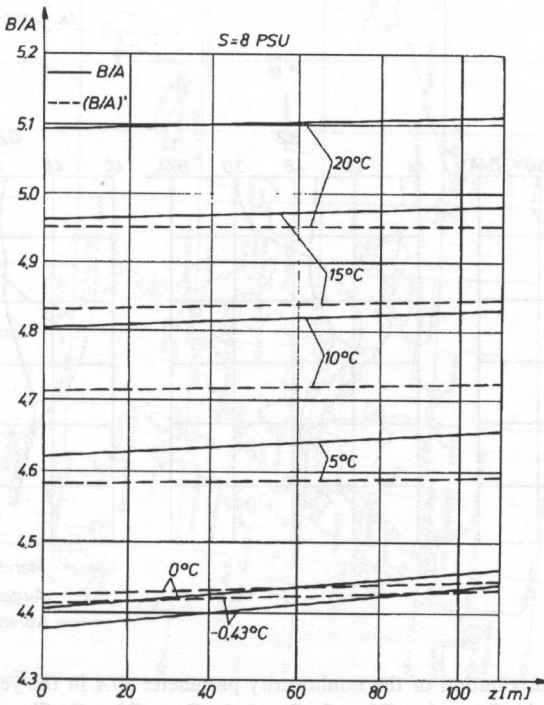


Fig. 5. Changes in the nonlinearity parameter B/A of the Baltic Sea water as the function of depth for different values of temperature.

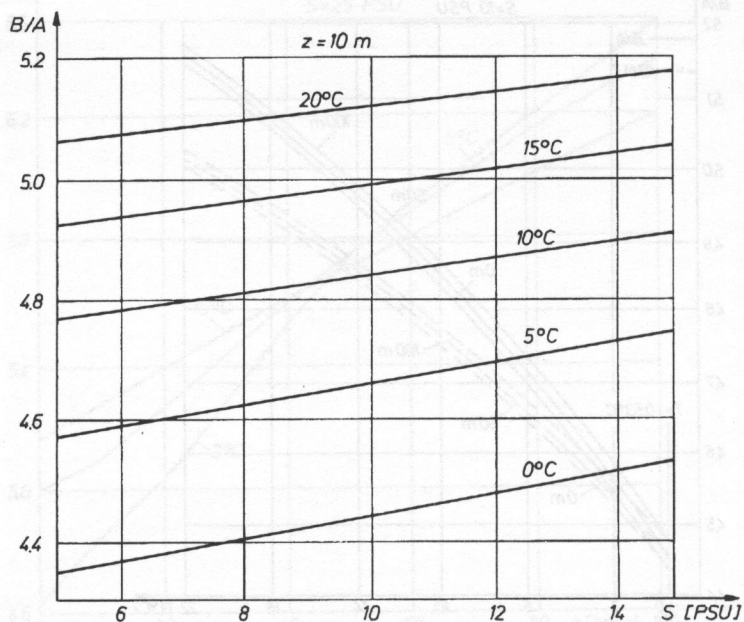


Fig. 6. Changes in the nonlinearity parameter B/A of the Baltic Sea water as the function of salinity for different values of temperature.

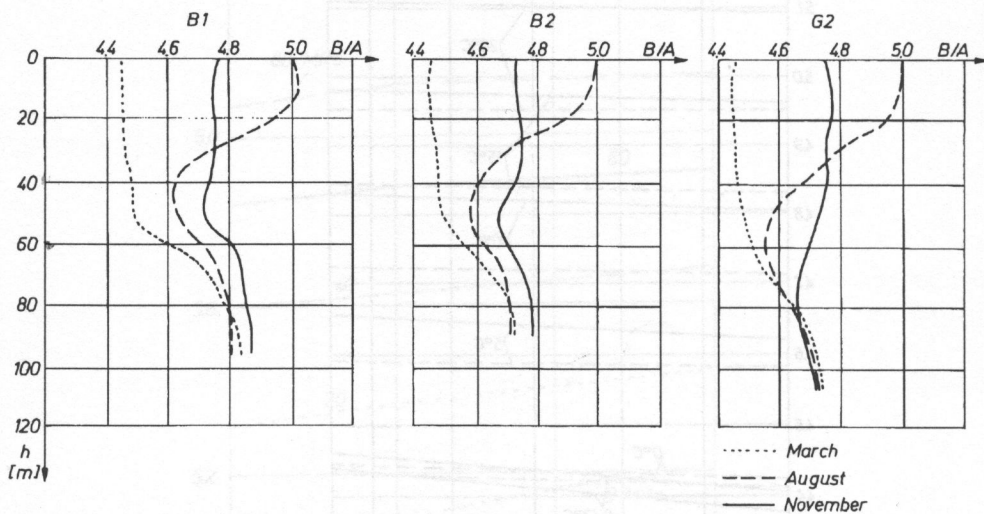


Fig. 7. The average vertical distribution of the nonlinearity parameter B/A in the years 1958–1988 at selected stations in the South Baltic Sea region; B1 — the Bornholm Deep; B2 — the Slups Furrow; G2 — the Gdańsk Deep.

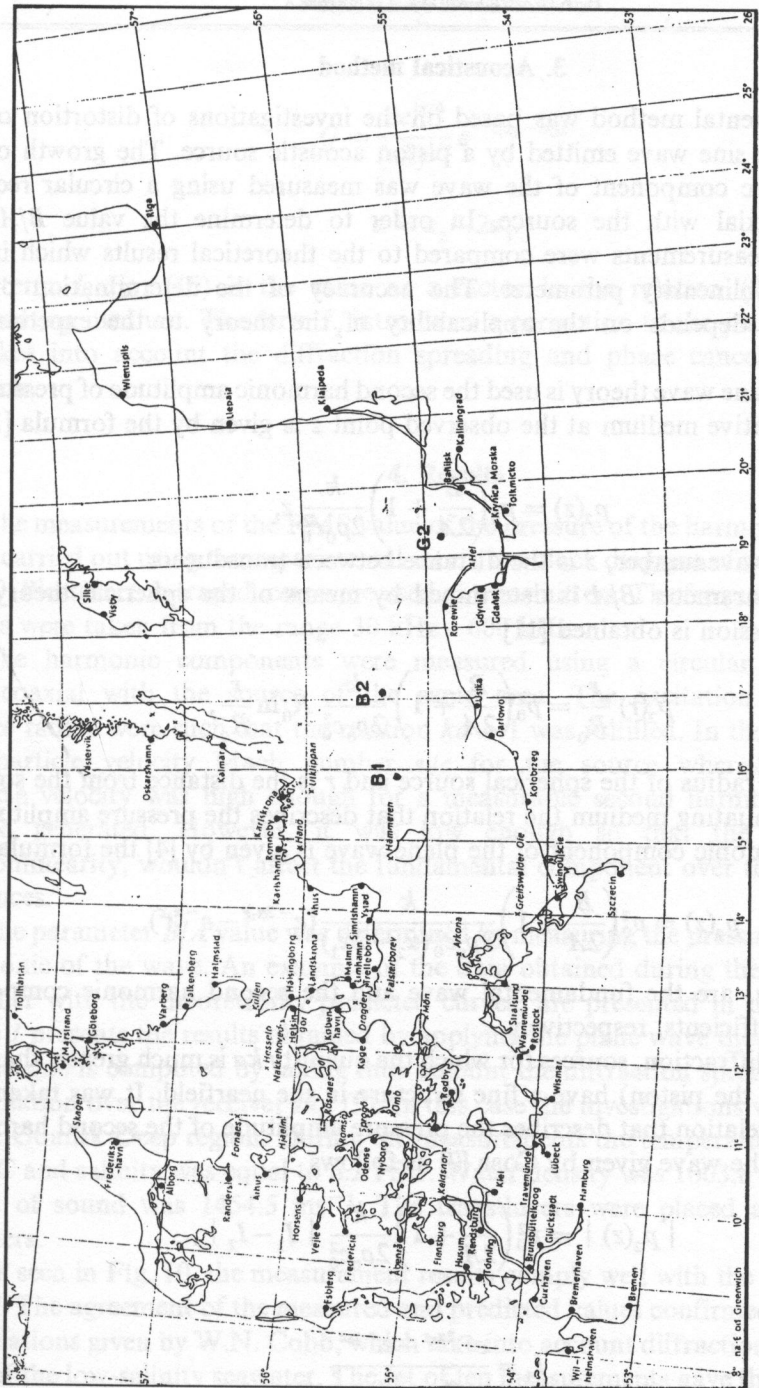


Fig. 8. Localization of the stations for which the computations of annual parameter $B/4$ changes were made by means of the thermodynamics meth

3. Acoustical method

The experimental method was based on the investigations of distortion of the finite amplitude sine wave emitted by a piston acoustic source. The growth of the second harmonic component of the wave was measured using a circular receiver which was coaxial with the source. In order to determine the value B/A , the experimental measurements were compared to the theoretical results which incorporated the nonlinearity parameter. The accuracy of the determination of the parameter B/A depends on the applicability of the theory to the experimental conditions.

When the plane wave theory is used the second harmonic amplitude of pressure p_2 , for a nondissipative medium at the observed point z is given by the formula [1]

$$p_2(z) = p_0^2 \left(\frac{B}{2A} + 1 \right) \frac{k}{2\rho_0 c_0^2} z, \quad (6)$$

where k is the wavenumber, z is the distance between transducers.

When the parameter B/A is determined by means of the spherical theory, the following expression is obtained [11]

$$p_2(r) \frac{r}{R_0} = p_0^2 \left(\frac{B}{2A} + 1 \right) \frac{k}{2\rho_0 c_0^2} R_0 \ln \frac{r}{R_0}, \quad (7)$$

where R_0 is the radius of the spherical source and r is the distance from the source.

For an attenuating medium the relation that describes the pressure amplitude of the second harmonic component of the plane wave is given by [4] the formula

$$p_2(z) = p_0^2 \left(\frac{B}{2A} + 1 \right) \frac{k}{2\rho_0 c_0^2 (\alpha_2 - 2\alpha_1)} (e^{-2\alpha_1 z} - e^{-\alpha_2 z}), \quad (8)$$

where α_1 and α_2 are the fundamental wave and the second harmonic component attenuation coefficients, respectively.

Because of diffraction, sources for which the quantity ka is much greater than 1 (a is the radius of the piston) have a fine structure in the nearfield. It was taken into account in the relation that describes the pressure amplitude of the second harmonic component of the wave given by COBB [7] as follows

$$|p_2(z)| = p_0^2 \left(\frac{B}{2A} + 1 \right) \frac{k}{2\rho_0 c_0^2} |I_1 - I_2|, \quad (9)$$

where

$$I_1 = \frac{e^{-2\alpha_1 z} - e^{-\alpha_2 z}}{\alpha_2 - 2\alpha_1}, \quad (10)$$

$$I_2 = Q \left(\int_{z/2}^z e^{2\alpha\theta} [(\Theta^2 + 4a^2)^{1/2} - \Theta]^{-1/2} d\Theta - \frac{1}{8a^2} \int_{z/2}^z e^{2\alpha\theta} [(\Theta^2 + 4a^2)^{1/2} - \Theta]^{3/2} d\Theta \right), \quad (11)$$

and

$$Q = \frac{8e^{i\pi/4}}{(\pi k)^{1/2}} e^{-2(\alpha_2 - \alpha_1)z}, \quad (12)$$

$$\alpha = \alpha_2 - 2\alpha_1. \quad (13)$$

The term I_1 Eq. (10) is the same as a factor in the relation (8) given for the attenuating medium. The term I_2 introduces a correction to the plane wave theory. It takes into account the diffraction spreading and phase cancellation over the receiver.

4. Results

The measurements of the RMS value of the pressure of the harmonic components were carried out using the experimental setup, the block diagram of which is shown in Fig. 9. Piezoelectric transducers were used as transmitters. The frequencies of primary waves were taken from the range 30 kHz–600 kHz.

The harmonic components were measured using a circular receiver which was coaxial with the source of the same area. The excitation frequency and source radius were such that the relation $ka \gg 1$ was fulfilled. In these experiments, the particle velocity Mach number v/c for the source where v denotes the particle velocity was high enough for a measurable second harmonic component to be generated. However, it was low enough so that the losses, caused by nonlinearity, wouldn't affect the fundamental component over the measurement distances.

The parameter B/A value was determined by measuring the pressure of the second harmonic of the wave. An example of the data obtained during the measurements, together with the theoretically predicted curves, are presented in the Fig. 10. The curve 1 presents the results obtained by applying the plane wave theory Eq. (6). The second one is computed by taking into account the diffraction spreading and phase cancellation over the receiver Eq. (9). In this case the investigations were carried out in the Gdańsk Deep region. During the measurements the temperature of water was 11.4°C and salinity was equal to 6.2 PSU. Water density was 1003.8 kg m⁻³ and the speed of sound was 1454.5 ms⁻¹. The transducers were placed at the depth of 2 meters.

As seen in Fig. 10, the measurement results comply well with the predicted curve No. 2. The agreement of the measured and predicted values confirmed the validity of the relations given by W.N. Cobb, which take into account diffraction effects, for the case of the low-salinity seawater. The set of ten measurements gave the average value of B/A equal to 4.72.

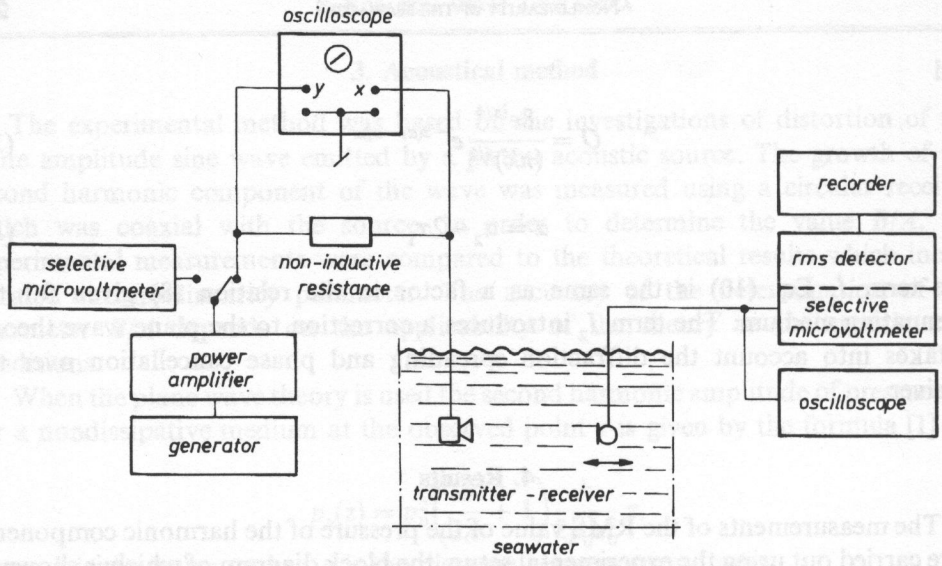


Fig. 9. The block diagram of the measurement setup for the investigation of the nonlinearity parameter B/A by means of the acoustical method.

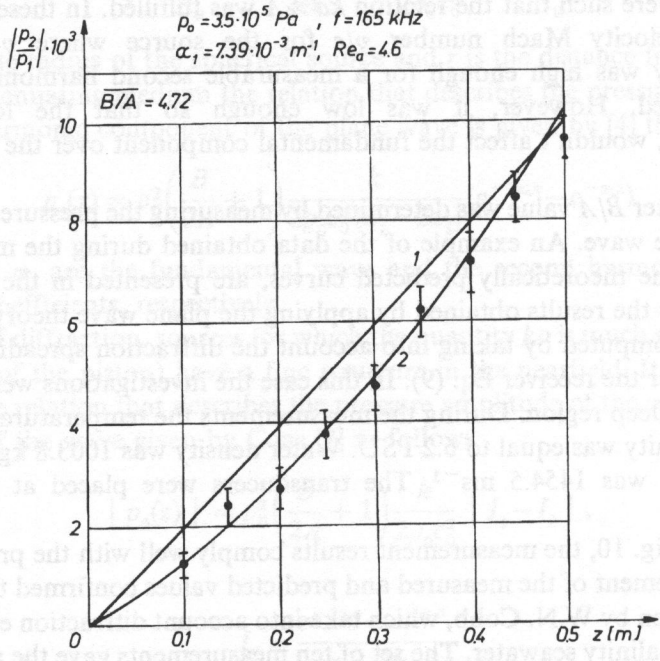


Fig. 10. The data obtained by measuring the pressure of the second harmonic of the wave and the curves by means of 1) plane wave theory, 2) diffraction theory.

5. Conclusions

1. In the Baltic Sea temperature is the main factor which can be observed to influence the value of the parameter B/A . Annual changes of the nonlinearity parameter B/A depend primarily on annual changes of the temperature.

2. In the South Baltic Sea the value of the nonlinearity parameter B/A changes mainly in its upper layer, and it varies from 4.45 to 5.03. The changes of the parameter B/A in the deep layer are considerably smaller. The value of B/A near the bottom is as follows:

- in the Bornholm Deep: from 4.80 to 4.87,
- in the Słupsk Furrow: from 4.70 to 4.78,
- in the Gdańsk Deep: from 4.69 to 4.72.

3. The investigation results obtained by means of both the methods agree well in most cases. To achieve a satisfactory accuracy, it is necessary to take into account the effects of both the diffraction and attenuation on the amplitude of the second harmonic wave in interpreting of the measurement results.

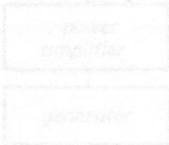
4. The experimental setup used in measurements was of a laboratory type. Actually, the preliminary work is being conducted on construction of the device for measuring the value of the nonlinearity parameter B/A in situ by means of the acoustic method.

References

- [1] L. ADLER, E.A. HIEDEMANN, *Determination of the nonlinearity parameter B/A for water and m -xylene*, J. Acoust. Soc. Am., **34**, 4, 410–412 (1962).
- [2] *Algorithms for computation of fundamental properties of seawater*, UNESCO Technical Papers in Marine Science, **44** (1983).
- [3] R.T. BEYER, *Parameter of nonlinearity in fluids*, J. Acoust. Soc. Am., **32**, 2, 719–721 (1960).
- [4] D.T. BLACKSTOCK, *Generalized Burgers equation for plane waves*, J. Acoust. Soc. Am., **77**, 6, 2050–2053 (1985).
- [5] C.T. CHEN, F.J. MILLERO, *Speed of sound in seawater at high pressures*, J. Acoust. Soc. Am., **62** (5), 1129–1135, (1977).
- [6] C.T. CHEN, F.J. MILLERO, *The specific volume of seawater at high pressures*, Deep Sea Res., **23**, 7, 595–612.
- [7] W.N. COBB, *Finite amplitude method for the determination of the acoustic nonlinearity parameter B/A* , J. Acoust. Soc. Am., **73**, 5, 1525–1531 (1983).
- [8] H. ENDO, *Calculation of nonlinearity parameter for seawater*, J. Acoust. Soc. Am., **76**, 1, July (1984).
- [9] F.J. MILLERO, C.T. CHEN, A. BRADSHAW, K. SCHLEICHER, *Summary of data treatment for the international high pressure equations of state for seawater*, UNESCO Tech. Pap. in Mar. No. 44 (1983).

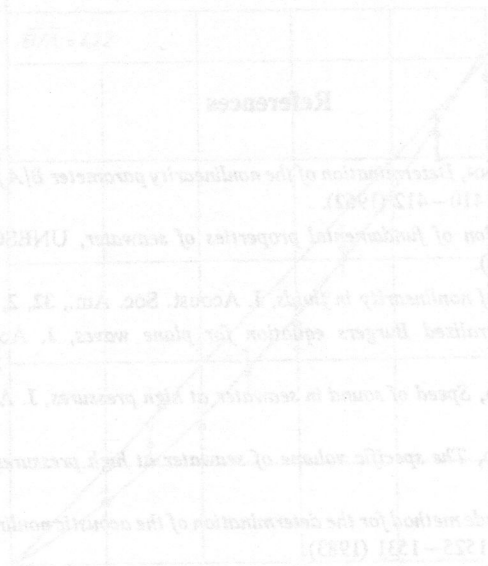
- [10] F.J. MILLERO, G. PERRON, J.F. DESNOYERS, *Heat capacity of seawater solutions from 5° to 35° C and 0.05 to 22‰ chlorinity*. *J. Geophys. Res.*, **78** (21), 4499–4506, (1973).
- [11] D.V. RUDENKO, S.I. SOLUAN, *A theoretical introduction to nonlinearity acoustics*, Nauka, Moscow 1975.

Received February 8, 1993



The investigation results obtained by means of both the methods agree well in most cases. To achieve a satisfactory accuracy, it is necessary to take into account the effects of both the diffraction and attenuation on the amplitude of the second harmonic wave in determining of the measurement results.

4. The experimental setup used in measurements was of a laboratory type. Actually, the preliminary work is being conducted on construction of the device for measuring the value of the nonlinearity parameter B/A in situ by means of the acoustic method.



- [1] L. ADLER, E.A. HERRMANN, Determination of the nonlinearity parameter B/A in water and in air. *J. Acoust. Soc. Am.*, **54**, 410–412 (1973).
- [2] Algorithm for computation of fundamental properties of seawater, UNESCO Technical Papers in Marine Science, 44 (1982).
- [3] R.T. BURGER, Frequency of nonlinearity in fluids. *J. Acoust. Soc. Am.*, **31**, 2119–2121 (1960).
- [4] D.T. BLACKSTOCK, Generalized Burgers equation for plane waves. *J. Acoust. Soc. Am.*, **77**, 620–625 (1985).
- [5] C.T. CHEN, F.J. MILLERO, Speed of sound in seawater at high pressure. *J. Acoust. Soc. Am.*, **62** (2), 1129–1132 (1977).
- [6] C.T. CHEN, F.J. MILLERO, The specific volume of seawater at high pressure. *Deep Sea Res.*, **23**, 739–742 (1976).
- [7] W.N. COLE, Finite amplitude method for the determination of the acoustic nonlinearity parameter B/A . *J. Acoust. Soc. Am.*, **73**, 1223–1231 (1983).
- [8] H. ENDO, Calculation of nonlinearity parameter for seawater. *J. Acoust. Soc. Am.*, **76**, 1, July (1984).
- [9] F.J. MILLERO, C.T. CHEN, A. BRADY, W. J. SULLIVAN, Summary of data treatment for the international high pressure standard of state for seawater, UNESCO Tech. Rep. in Mar. No. 44 (1982).